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# SPRING 1991 EA-1A EXAM SOLUTIONS

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$$D_x = v^x l_x$$

$$\frac{D_{x+1}}{D_x} = \frac{v^{x+1} l_{x+1}}{v^x l_x} = v f_x = v(1-g_x) = \frac{154.2870}{165.6518}$$

$$M_x - M_{x+1} = C_x \quad \frac{C_x}{D_x} = \frac{v^{x+1} d_x}{v^x l_x} = v g_x = \frac{27.0649 - 26.5371}{165.6518}$$

$$\frac{1-g_x}{g_x} = \frac{154.2870}{.5278} \Rightarrow .5278 - .5278 f_x = 154.2878 \times \\ f_x = \frac{.5278}{.5278 + 154.287} = .00341$$

(B)

2. This problem is based on actuarial equivalence of monthly life annuities and "joint and survivor" annuities. The key is that you can solve for the value of  $\frac{\ddot{a}_{xy}^{(12)}}{\ddot{a}_x^{(12)}}$  which allows you to calculate the benefit for any continuation fraction

$$300 \ddot{a}_x^{(12)} = 160 \left( \ddot{a}_x^{(12)} + 1.0 \left[ \ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)} \right] \right) \\ 300 = 160 \left( 1 + \left( \frac{\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}}{\ddot{a}_x^{(12)}} \right) \right)$$

$$\frac{300}{160} = 1 + \frac{\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}}{\ddot{a}_x^{(12)}}$$

$$\frac{140}{160} = \frac{\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}}{\ddot{a}_x^{(12)}}$$

$$300 \ddot{a}_x^{(12)} = K \left( \ddot{a}_x^{(12)} + \frac{1}{2} \left( \ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)} \right) \right)$$

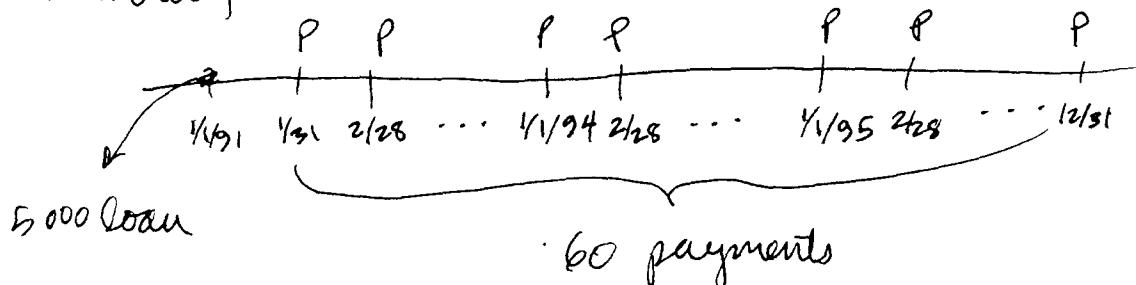
$$K = 300 / \left[ 1 + .5 \left( \frac{\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}}{\ddot{a}_x^{(12)}} \right) \right]$$

$$= 300 / (1 + 70/160) \\ = 208.70$$

(D)

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3. The loan has sixty monthly payments. The equivalent monthly interest rate is  $(1.2)^{1/12} - 1 = .949\%$  per month.



The easy way to get the total interest in the fourth year is that the interest equals total payments in that year less the principal paid. The principal paid in the fourth year equals the O/S balance at the start of the fourth year less the O/S balance at the start of the fifth year.

$$\text{O/S balance at start of fifth year } (1/1/95) = P \alpha_{12}^{24.949\%}$$

$$\text{.. " " " fourth } (1/1/94) = P \alpha_{24}^{21.949\%}$$

$$\text{Principal paid in fourth year} = P (\alpha_{24}^{21.949\%} - \alpha_{12}^{24.949\%})$$

$$5000 = P \alpha_{60}^{21.949\%} \therefore P = 109.68$$

$$\therefore \text{principal paid in fourth year} = 109.68 (\alpha_{24}^{21.949\%} - \alpha_{12}^{24.949\%})$$

$$= 2344.18 - 1238.44 = 1105.75$$

$$\text{Total payments in fourth year} = 12 (109.68) = 1316.14$$

$$\therefore \text{Total interest in fourth year} = 1316.14 - 1105.75 = 210.40$$

(B)

Alternative approach that can be used as a check.  
Interest in first payment of fourth year =  $i$  (O/S balance at start of 4th year) =  $i (P \alpha_{24}^{21.949\%}) = P (1 - v^{24})$ .

Series for interest in all 12 payments of fourth year:

$$P (1 - v^{24} + 1 - v^{23} + \dots + 1 - v^3)$$

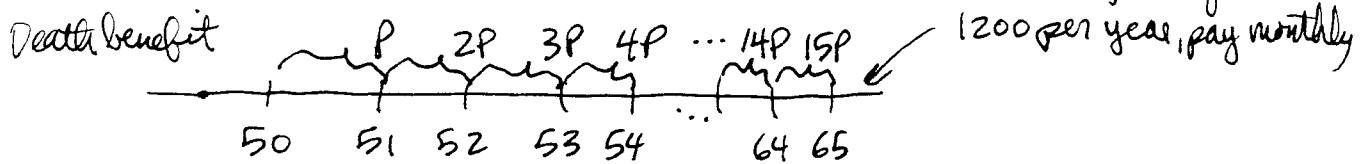
$$= P (12 - (v^{24} + v^{23} + \dots + v^2 + v) + (v^{12} + v^{11} + \dots + v^2 + v))$$

$$= P (12 - \alpha_{24}^{21.949\%} + \alpha_{12}^{21.949\%}) = 109.68 (12 - 21.37 + 11.29) = 210.40$$

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4

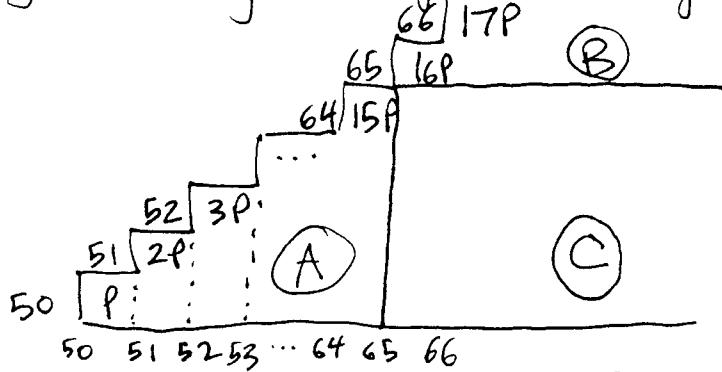
One way to miss this problem is to read it incorrectly as providing a death benefit of the sum of contributions plus interest. This annuity provides a death benefit of total premiums without interest, plus deferred monthly benefits of 100:



In basic commutation functions, the present value of annuity payments must equal the benefits shown above.

$$P \ddot{a}_{50:\overline{15}} = P \left( \frac{C_{50}}{D_{50}} \right) + 2P \left( \frac{C_{51}}{D_{50}} \right) + \dots + 15P \left( \frac{C_{64}}{D_{50}} \right) + 1200N_{65}^{(c)} \frac{D_{65}}{D_{50}}$$

The hard part of this problem is expressing this increasing death benefit in terms of  $M_x$  and  $R_x$ . Try drawing a stepwise diagram of the benefit



The area  $A+B+C$  is equal to  $P(R_{50}/D_{50})$ , which is an increasing insurance for life. You want to provide a death benefit of the A area, which equals

$$\frac{P}{D_{50}} (R_{50} - R_{65} - 15M_{65})$$

(next page)

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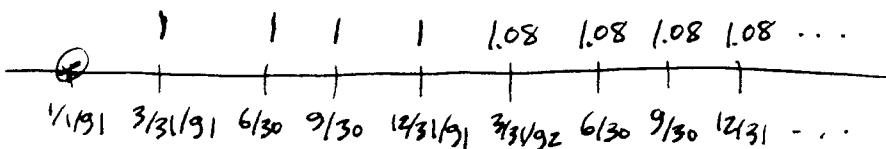
(4) continued

$$P \left( \frac{N_{50} - N_{65}}{D_{50}} \right) = \frac{P}{D_{50}} (R_{50} - R_{65} - 15M_{65}) + 1200 \frac{N_{65}^{(12)}}{D_{50}}$$

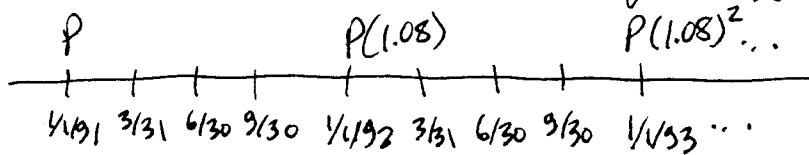
$$\begin{aligned} P &= \frac{1200 N_{65}^{(12)}}{N_{50} - N_{65} - (R_{50} - R_{65} - 15M_{65})} \\ &= \frac{1200 (10151)}{33295 - 10607 - (21392 - 8076 - 15[687])} \\ &= 619.06 \end{aligned}$$

(A) This answer is within the implied range of 550 to 650

5. The first thing you should do is to draw a time diagram that shows the stream of payments:



You can either convert the four quarterly payments to one payment per year, or consider the value of four identical perpetuities that are paid once each quarter. They are really identical ideas, so this solution will use the first approach:



$$P = 4 a_{1/1}^{(4)} 10\% = 4 \frac{1-v^4}{i^{(4)}} \quad i^{(4)} = 4 \sqrt[4]{(1.10)^{\frac{1}{4}} - 1} = 9.65\% \\ P = \frac{4(1-(1.1)^{-4})}{.0965} = 3.77$$

$$PV \text{ of perpetuity} = 3.77 \left( 1 + \frac{1.08}{1.10} + \frac{(1.08)^2}{1.10} + \dots \right)$$

$$PV = 3.77 \left( 1 + \frac{(1.0185)^{-1}}{1.0185} + \frac{(1.0185)^{-2}}{1.0185} + \dots \right) \text{ convert to single rate} \\ PV / 1.0185 = 3.77 \left( \frac{(1.0185)^1}{1.0185} + \frac{(1.0185)^2}{1.0185} + \dots \right) \text{ multiply by } (1.0185)^{-1} \\ PV (1 - (1.0185)^{-1}) = 3.77 \quad \therefore PV = 3.77 / .01818 = 207.35$$

(A)

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6. Conceptually, if termination makes no difference, then you should get the correct answer if you ignore the withdrawal decrements. To do this, you must calculate the rate of mortality, using the standard approximations:

$$\begin{aligned} q_{35}^{(d)} &= \frac{q_{35}^{(d)}}{1 - \frac{1}{2} q_{35}^{(w)}} \\ &= \frac{150/10,000}{1 - \frac{1}{2}(900/10,000)} \\ &= \frac{150}{10,000 - \frac{1}{2}(900)} = .01572 \end{aligned}$$

$$\begin{aligned} q_{36}^{(d)} &= \frac{q_{36}^{(d)}}{1 - \frac{1}{2} q_{36}^{(w)}} \\ &= \frac{152/8950}{1 - \frac{1}{2}(625/8950)} \\ &= \frac{152}{8950 - \frac{1}{2}(625)} = .01760 \end{aligned}$$

$$\begin{aligned} {}^2P_{35}^{(T)} &= (1 - q_{35}^{(d)}) (1 - q_{36}^{(d)}) \\ &= (.98428)(.98240) \\ &= .96697 \end{aligned}$$

if we really can ignore terminations, since we're only interested in surviving the mortality decrement

(B)

The more difficult alternative is to write a lengthy expression for  ${}^2P_{35}^{(T)}$ . The tricky part of this is that, since decrements occur in effect at mid-year, a participant is exposed to mortality for  $\frac{1}{2}$  year in the year they terminate from the plan.

$$\begin{aligned} {}^2P_{35}^{(T)} &= q_{35}^{(d)} \left( 1 - \frac{1}{2} q_{35}^{(d)} \right) (1 - q_{36}^{(d)}) + P_{35}^{(T)} q_{36}^{(w)} \left( 1 - \frac{1}{2} q_{36}^{(d)} \right) \\ &\quad + P_{35}^{(T)} (P_{36}^{(T)}) \end{aligned}$$

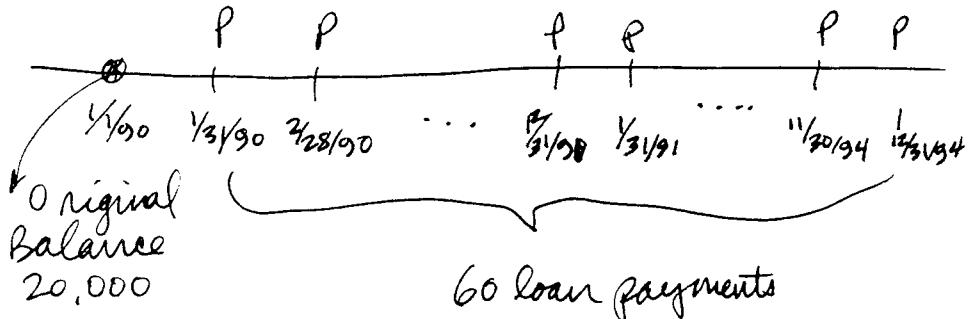
This time, we can derive  $q'x^{(d)}$  based on Bowers' approach:

$$\begin{aligned} P_x^{(1)} &= P_x^{(T)} \frac{q_x^{(d)} / q_x^{(T)}}{q_x^{(d)} / q_x^{(T)}} \\ P_{35}^{(d)} &= P_{35}^{(T)} \frac{q_{35}^{(d)} / q_{35}^{(T)}}{q_{35}^{(d)} / q_{35}^{(T)}} = .98428 \\ P_{36}^{(d)} &\therefore q_{35}^{(d)} = .01572 \\ &= \frac{(8173)}{8950} \frac{152 / (152 + 625)}{152 / (152 + 625)} = .98239 \\ &\therefore q_{36}^{(d)} = .01761 \end{aligned}$$

$$\begin{aligned} {}^2P_{35}^{(T)} &= \frac{900}{10,000} \left( 1 - \frac{.01572}{2} \right) \left( 1 - \frac{.01761}{2} \right) + \frac{8950}{10,000} \left( \frac{625}{8950} \right) \left( 1 - \frac{.01761}{2} \right) + \frac{8173}{10,000} \\ &= (477.20 + 619.50 + 8173) / 10,000 = .96697 \end{aligned}$$

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7. At 1/1/90, the series of payments looks like this:



The interest rate of 9% per annum is compounded monthly at  $.09/12$  or  $.75\%$  per month. You can equate the loan balance and the series of payments  $P$ :

$$20,000 = \text{orig loan balance} = P a_{60.75\%}$$

At 1/1/91, the remaining loan balance equals the present value of 48 payments (12 were made during 1990):

$$1/1/91 \text{ OSB} = P a_{48.75\%} = 20,000 \left( \frac{a_{48.75\%}}{a_{60.75\%}} \right)$$

This amount will be paid over the remaining 48 months, but at 12% per annum, which is compounded monthly at 1% per month. The new loan payment can be calculated as

$$1/1/91 \text{ OSB} = Q a_{48|1\%}$$

The outstanding balance of this loan at 4/1/91 reflects the fact that three payments were made at 1/31, 2/28, and 3/31:

$$\begin{aligned} 4/1/91 \text{ OSB} &= Q a_{45|1\%} = (1/1/91 \text{ OSB}) \frac{a_{45|1\%}}{a_{48|1\%}} \\ &= 20,000 \left( \frac{a_{48.75\%}}{a_{60.75\%}} \right) \left( \frac{a_{45|1\%}}{a_{48|1\%}} \right) \\ (D) &= 15,858 = 20,000 (40.18/48.17) (36.09/37.97) \end{aligned}$$

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8. Annuity A  $NSP = 11,000 \ddot{a}_{5|}$

$NSP$  is shorthand  
for net single premium

Annuity B  $NSP = 6,000 \ddot{a}_{60:5|}$

Annuity C  $NSP = 14,000 s\ddot{a}_{60|}$

The expense loadings imply that the gross single premium must provide the annuity benefit plus a percentage of the gross single premium (GSP):

Annuity B  $GSP = 6,000 \ddot{a}_{60:5|} + x(GSP)$

$$GSP = 6,000 \ddot{a}_{60:5|} / (1-x)$$

Annuity C  $GSP = 14,000 s\ddot{a}_{60|} / (1-x)$

Note that annuity B's premium can be expressed as a combination of  $\ddot{a}_{5|}$  and  $s\ddot{a}_{60|}$ :

Annuity B  $GSP = 6000 (\ddot{a}_{5|} + s\ddot{a}_{60|}) / (1-x)$

Now you can set the value of annuity A's NSP and annuity C's GSP equal to solve for  $s\ddot{a}_{60|}$ :

$$11,000 \ddot{a}_{5|} = 14,000 s\ddot{a}_{60|} / (1-x)$$

$$14 s\ddot{a}_{60|} = (1-x) 11 \ddot{a}_{5|}$$

$$s\ddot{a}_{60|} = (1-x)(11/14) \ddot{a}_{5|}$$

Now plug this into the equality between A and B:

$$11,000 \ddot{a}_{5|} = 6,000 (\ddot{a}_{5|} + s\ddot{a}_{60|}) / (1-x)$$

$$(1-x) 11,000 \ddot{a}_{5|} = 6,000 (\ddot{a}_{5|}) + 6,000 (1-x) (11/14) \ddot{a}_{5|}$$

$$11(1-x) = 6 + 6(1-x)(11/14)$$

$$11 - 11x = 6 + 4.714 - 4.714x$$

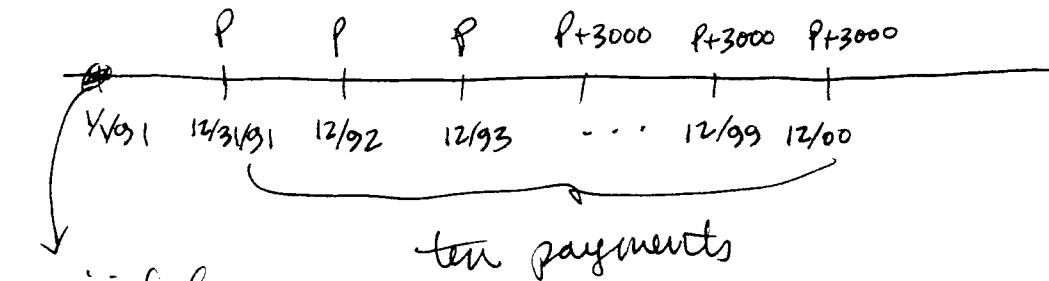
$$.286 = 6.286x$$

$$x = 4.55\%$$

(B)

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9. First draw a time diagram for the series of payments.



original loan  
balance =  $X = P \alpha_{\bar{10}i} + 3000(\alpha_{\bar{10}i} - \alpha_{\bar{3}i})$

$i$  is the annual effective interest rate  $= (1.04)^2 - 1 = 8.16\%$  per annum

The outstanding loan balance at 1-1993 equals the accumulated value of the original loan, less the payments for 1991 and 1992:

$$1-1993 \text{ OSB} = X(1.0816)^2 - P s_{\bar{2}i}$$

$$7-1993 \text{ OSB} = 1.04(X(1.0816)^2 - P s_{\bar{2}i}) = 45,000$$

Now you have two equations in the two unknowns  $X$  and  $P$ . In this problem, it will be simpler to solve for  $P$  and then substitute to solve for  $X$ .

$$X(1.0816)^2 - P s_{\bar{2}i} = 45,000 / 1.04$$

$$X - P \alpha_{\bar{2}i} = 45,000 / [(1.04)(1.0816)^2]$$

$$P \alpha_{\bar{10}} + 3000(\alpha_{\bar{10}} - \alpha_{\bar{3}}) - P \alpha_{\bar{2}} = 45,000 / (1.04)^{-5}$$

$$P (\alpha_{\bar{10}} - \alpha_{\bar{2}}) = 45,000 (1.04)^{-5} - 3000 (\alpha_{\bar{10}} - \alpha_{\bar{3}})$$

$$P = \frac{45,000 (1.04)^{-5} - 3000 (\alpha_{\bar{10}} - \alpha_{\bar{3}})}{\alpha_{\bar{10}} - \alpha_{\bar{2}}} = 5060.86$$

$$X = P \alpha_{\bar{2}} + 45,000 (1.04)^{-5}$$

$$= 5060.86 (1.779) + 45,000 (.82193)$$

$$= 45,992$$

(C)

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10. The hard part of this problem is writing out the cases for the various annuities. To avoid getting confused, you could assume you have three people at ages  $x$ ,  $y$  and  $z$ .

One approach would use reversionary annuities to directly write down the various cases:

$$\begin{aligned} & (12)(1000)\ddot{a}_{xyz} + 12(750)(\ddot{a}_{xy}-\ddot{a}_{xyz}) + 12(400)(\ddot{a}_x - \ddot{a}_{x:yz}) \\ & + 12(750)(\ddot{a}_{xz}-\ddot{a}_{xyz}) + 12(400)(\ddot{a}_y - \ddot{a}_{y:zx}) \\ & + 12(750)(\ddot{a}_{yz}-\ddot{a}_{xyz}) + 12(400)(\ddot{a}_z - \ddot{a}_{z:xy}) \end{aligned}$$

The first column pays 1000 per month if all three annuitants are alive. The second column pays 750 per month to any two annuitants after the death of the third. The third column is a reversionary annuity paid to a single participant after the death of both of the other participants.

$$\begin{aligned} & 12000 \ddot{a}_{xxx} + 3(9000)(\ddot{a}_{xx}-\ddot{a}_{xxx}) + 3(4800)(\ddot{a}_x - [\ddot{a}_{xx} + \ddot{a}_{xx} - \ddot{a}_{xxx}]) \\ & = 12000 \ddot{a}_{xxx} + 27000 \ddot{a}_{xx} + 14400 \ddot{a}_x \\ & - 27000 \ddot{a}_{xxx} - 28800 \ddot{a}_{xx} \\ & + 14400 \ddot{a}_{xxx} \\ & = -600(6.258) - 1800(7.354) + 14400(9.194) \\ & = 115,402 \end{aligned}$$

(C)

A different way to work the problem is to write down the annuity of 400 per month per participant, and figure out what needs to be subtracted. This would pay 800 if any two participants were alive, so you should subtract 50 per month for each of three annuities which represent the three cases of exactly two participants alive. The sum of these six annuities would pay 1200-150 if all three participants are alive, so there needs to be another (next page)

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(10) (Continued)

term of -50 per month when all three participants are alive:

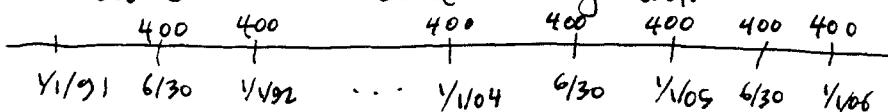
$$\begin{array}{l} 400 \ddot{a}_x \\ + 400 \ddot{a}_x \\ + 400 \ddot{a}_x \end{array} \left\{ \begin{array}{l} \text{pays } 1200 \text{ if all three alive} \\ \quad \quad \quad 800 \text{ if exactly two alive} \\ \quad \quad \quad 400 \text{ if exactly one alive} \end{array} \right. \text{(correct)}$$

$$\begin{array}{l} 400 \ddot{a}_x - 50 \ddot{a}_{xx} \\ + 400 \ddot{a}_x - 50 \ddot{a}_{xx} \\ + 400 \ddot{a}_x - 50 \ddot{a}_{xx} \end{array} \left\{ \begin{array}{l} \text{pays } 1050 \text{ if all three alive} \\ \quad \quad \quad 750 \text{ if exactly two alive} \text{ (correct)} \\ \quad \quad \quad 400 \text{ if exactly one alive} \text{ (correct)} \end{array} \right.$$

$$\begin{array}{l} 400 \ddot{a}_x - 50 \ddot{a}_{xx} \\ + 400 \ddot{a}_x - 50 \ddot{a}_{xx} - 50 \ddot{a}_{xxx} \\ + 400 \ddot{a}_x - 50 \ddot{a}_{xx} \end{array} \text{pays correct amount in all cases (monthly)}$$

$$\begin{aligned} PVB &= 12 \left[ 1200(9.154) - 150(7.354) - 50(6.258) \right] \\ &= 115,402 \end{aligned}$$

11. First you should write down what the coupon payments look like on a time diagram:



The bond would normally be redeemed for 10,000 at 12/31/05, after the last coupon payment. This would be the expression for the price based on all 30 coupon payments:

$$\begin{aligned} 12/31/06 \text{ redemption price} &= 10,000 + (.04(10,000) - 10,000(.03)) \ddot{a}_{30|03} \\ &= 10,000 + (400 - 300) \ddot{a}_{30|03} = 11960 \end{aligned}$$

Other redemption dates of 12/31/00 and 06/30/05 have a slightly different redemption value

$$12/31/00 \text{ redemption price} = 10,150 + [.04(10,000) - .03(10,150)] \ddot{a}_{20|03} = 11,571$$

$$06/30/05 \text{ redemption price} = 10,150 + [.04(10,000) - .03(10,150)] \ddot{a}_{25|03} = 11,832$$

B) 11,571 is the highest amount you can pay and be sure of earning 3%.

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12. The problem states that the annual fund balance is zero, which means the total contributions equal the total benefits. Expressed in terms of  $T_x$ , the total number of lives age  $x$  or higher.

$$1000(T_{25} - T_{65}) = K(T_0 - T_{25}) + 2K(T_{65})$$

contributions                      benefits paid  
into fund                          from fund

You are given  $T_0 = 500,000$ , and  $T_{25} = l_{25} \bar{e}_{25}$   
 $= 420,000$

In addition, you know that the number of people over 65 is twice the population below age 25:

$$\begin{aligned}T_{65} &= 2(T_0 - T_{25}) \\&= 2(500,000 - 420,000) \\&= 160,000\end{aligned}$$

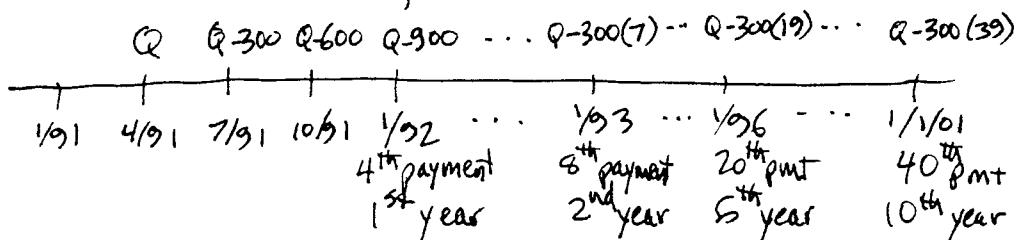
Now you can plug this information into the original equation and solve for  $K$ :

$$\begin{aligned}1000(420,000 - 160,000) &= K(500,000 - 420,000) + 2K(160,000) \\1000(260,000) &= K(80,000) + K(320,000) \\260,000,000 / 400,000 &= K \quad \therefore K = 650 \\2K &= 1300\end{aligned}$$

(D)

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13. First you should draw a time diagram to show what the annuity payments look like:



The interest rate is 2% per period, or  $\frac{8\%}{4}$ .

The present value of the payments after the 1/1/96 payment is \$50,000:

$$50,000 = v^1(Q-20(300)) + v^2(Q-21(300)) + \dots + v^{20}(Q-39(300))$$

Note the difference between the exponent and the multiplier of 300 is always 19:  $20-1=21-2=39-20$

$$\begin{aligned} v^1 50,000 &= v^2(Q-20(300)) + \dots + v^{20}(Q-38(300)) + v^{21}(Q-39(300)) \\ (1-v)(50,000) &= v(Q-20(300)) - 300v^2 - \dots - 300v^{20} - v^{21}(Q-39(300)) \\ (1-v)(50,000) + 300(v^2 + \dots + v^{20}) &= vQ - 6,000v - v^{21}Q + 11,700v^{21} \\ \underline{(1-v)(50,000) + 300(a_{20:21})} + \underline{6,000v - 11,700v^{21}} &= Q = \frac{980 + 4611 + 5882 - 7719}{.32062} \\ &= 11,711 \end{aligned}$$

(C)

14. You must remember that  $a_x = \frac{(n-\bar{a}_{n-x})}{ni}$ , where  $n = w-x$ , under DeMoivre's law. In this problem, you have to write out the few terms for  $a_{70:97}$ , since the PV of Smith's annuity is  $a_{70} - a_{70:97}$ :

$$\begin{aligned} a_{70} - a_{70:97} &= \frac{30 - \bar{a}_{30:70}}{30(7\%) - } - \left[ v \left( \frac{l_{71}}{l_{70}} \right) \left( \frac{l_{98}}{l_{97}} \right) + v^2 \left( \frac{l_{72}}{l_{70}} \right) \left( \frac{l_{99}}{l_{97}} \right) + v^3 \left( \frac{l_{73}}{l_{70}} \right) \left( \frac{l_{100}}{l_{97}} \right) \right] \\ &= \frac{30 - 13.2777}{2.10} - \left[ \frac{29}{30} \left( \frac{2}{3} \right) \frac{1}{1.07} + \frac{28}{30} \left( \frac{1}{3} \right) \frac{1}{(1.07)^2} + 0 \right] \\ &= 7.056301 - .6023 - .2717 \\ &= 7.0890 \end{aligned}$$

$\therefore$  PV of Smith's annuity =  $10,000(7.0890) = 70,890$

(B)

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- (S.) The first thing you should do is to write down what the fund value is at the end of 1991:

$$CD \text{ accumulates at } 2\% \text{ per quarter} = 100,000 (1.02)^4 = 108,243$$

Orig savings account balance

$$\text{accumulates at } \frac{1}{2}\% \text{ per month} = 100,000 (1.005)^{12} = 106,168$$

Also have monthly payments

$$\text{from mortgage, accum at } \frac{1}{2}\% \text{ per mo} = M (\$127.005)$$

$$\text{Mortgage value after 12 pmts} = 100,000 \left( \frac{1.3481.005}{1.3601.005} \right) = 99,637$$

$$\text{Less payments from savings} = -50,000 (1.005)^6 = -51,519$$

$$-55,000 (1.005)^3 = -55,829$$

$$\text{The amount of the mortgage payment } M = \frac{100,000}{\$127.005} = 1028.61$$

so the accumulated value of the twelve payments made to the savings account is  $1028.61 (\$127.005) = 12688$ .  
The total assets of the fund at 1/1/92 equal the

$$\text{Total} : 108,243 + 106,168 + 12,688 + 99,637 - 51,519 - 55,829 = 219,388$$

(C)

16. The participant is age 62 at 1-1-91, and will not retire for three years. The expression for the present value of retirement benefits and pre-retirement death benefits is

$$PVB = 12(833.33) \frac{N_{65}^{(i,2)}}{D_{62}} + 10,000 \left( \frac{M_{62} - M_{65}}{D_{62}} \right)$$

$$N_{65}^{(i,2)} = N_{65} - \frac{11}{24} D_{65} = 8872 - 4583 (965) = 8430$$

$$M_x = D_x - d N_x = D_x - (i/1+i) N_x$$

$$M_{62} = D_{62} - (0.07/1.07) N_{62} = 1251 - (0.07/1.07) 12326 = 444.63$$

$$M_{65} = D_{65} - (0.07/1.07) N_{65} = 965 - (0.07/1.07) 8872 = 384.59$$

$$PVB = [12(833.33) 8430 + 10,000 (444.63 - 384.59)] / 1251$$

$$= 67,866$$

(B)

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17. The net level reserve on a prospective basis equals the PV of future benefits less the PV of future premiums. With a zero interest rate, you can calculate the premium:

$$P_x = \frac{A_x}{\ddot{a}_x}$$

$$A_x = 1 - d \ddot{a}_x$$

$$= 1 - \left(\frac{i}{i_f}\right) \ddot{a}_x = 1 \text{ with zero interest}$$

$$\ddot{a}_x = e_x + 1$$

with zero interest

$$P_x = 1/4$$

$${}_2V_x = A_{x+2} - P_x \ddot{a}_{x+2} = 100000 \left[ 1 - \frac{1 + e_{x+2}}{41} \right]$$

$$e_x = P_x + P_x (P_{x+1}) + f_x (P_{x+1})(P_{x+2}) + \dots$$

$$e_{x+2} = P_{x+2} + P_{x+2} (P_{x+3}) + \dots$$

$$e_x = P_x + P_x (P_{x+1}) [1 + e_{x+2}]$$

$$\frac{e_x - P_x}{P_x (P_{x+1})} = 1 + e_{x+2} = \frac{40 - .98}{.98 (.98)} = 40.63$$

$$\therefore {}_2V_x = 100,000 \left[ 1 - 40.63/41 \right] = 905$$

(C)

$${}_2f_{65} = 1 - {}_2P_{65} = 1 - P_{65}(P_{66})$$

$$\sqrt{f_x \ddot{a}_{x+1}} = \ddot{a}_x \Rightarrow P_x = \frac{(\ddot{a}_x - 1)(1+i)}{\ddot{a}_{x+1}}$$

The trick to this question is that you are given the values of  $\ddot{a}_x^{(12)}$ , and you must approximate the value of  $\ddot{a}_x$

$$\ddot{a}_x^{(12)} = \ddot{a}_x - \frac{11}{24} \Rightarrow \ddot{a}_x = \ddot{a}_x^{(12)} + \frac{11}{24}$$

X	$\ddot{a}_x^{(12)}$	$\ddot{a}_x$
65	8.51	8.97
66	8.29	8.75
67	8.06	8.52

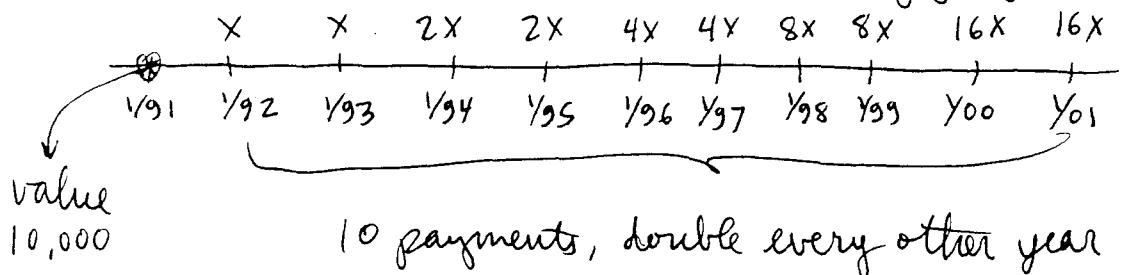
$$\begin{aligned} P_{65} &= (\ddot{a}_{65}^{(12)} - 1)(1.07) / \ddot{a}_{66} \\ &= 7.97 (1.07) / 8.75 \\ &= .9746 \end{aligned}$$

$$\begin{aligned} P_{66} &= (\ddot{a}_{66}^{(12)} - 1)(1.07) / \ddot{a}_{67} \\ &= 7.75 (1.07) / 8.52 \\ &= .9733 \end{aligned}$$

$$\textcircled{B} \quad \therefore 1 - {}_2P_{65} = 1 - .9746(.9733) = 5.1\%$$

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19. The first thing you should do is to write down a time diagram for the series of payments:



The easy way to solve for  $X$  is to consider this as an annuity of 10 payments equal to  $16X$ , less a series of annuities of shorter duration:

$$10,000 = 16X a_{10\%} - 8X a_{8\%} - 4X a_{6\%} - 2X a_{4\%} + a_{2\%}$$

$$\therefore X = \frac{10,000}{[16a_{10\%} - 8a_{8\%} - 4a_{6\%} - 2a_{4\%} + a_{2\%}]} \\ = 271$$

(B)

You must be careful to check the series of payments resulting from the differences in the annuities to be sure the correct annuity payment results each year. For example, in the first year, the net payment is  $16-8-4-2-1=1$ .

Another way to work the problem is to consider the sum as a direct calculation of annuities, all for a two year period:

$$10,000 = X a_{2\%} + v^2 (2X) a_{2\%} + v^4 (4X) a_{2\%} + v^6 (8X) a_{2\%} + v^8 (16X) a_{2\%}$$

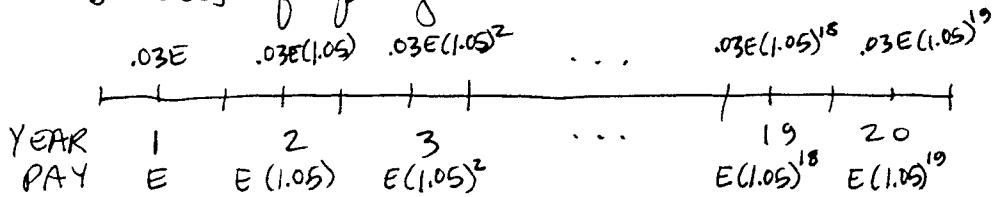
$$2v^2(10,000) = X [2v^2(a_{2\%}) + v^4(4a_{2\%}) + v^6(8a_{2\%}) + v^8((6a_{2\%}) + v^{10}(32a_{2\%}))]$$

$$10,000 (2v^2-1) = X [32v^{10}a_{2\%} - a_{2\%}]$$

$$X = \frac{10,000 (2v^2-1)}{a_{2\%} (32v^{10}-1)} = \frac{7468.77}{27.60} = 271$$

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20. You should write down a time diagram for this series of payments:



The accumulated value at the end of 20 years is

$$(1.07)^5 (.03E)(1.05)^{19} + (1.07)^{15} (.03E)(1.05)^{18} + \dots + (1.07)^{18.5} (.03E)(1.05)^1 + (1.07)^{19.5} (.03E)$$

Note that the sum of the exponents is equal to 19.5

Now this accumulated value must be expressed as a percentage of the earnings in the final year:

$$\begin{aligned} q_0 &= \frac{.03E [(1.07)^5 (1.05)^{19} + \dots + (1.07)^{18.5} (1.05) + (1.07)^{19.5}]}{E (1.05)^{19}} \\ &= .03 \left[ (1.07)^5 + \frac{(1.07)^{15}}{1.05} + \dots + \frac{(1.07)^{18.5}}{(1.05)^{18}} + \frac{(1.07)^{19.5}}{(1.05)^{19}} \right] \\ &= (1.07)^5 (.03) \left[ 1 + \left(\frac{1.07}{1.05}\right)^1 + \dots + \left(\frac{1.07}{1.05}\right)^{18} + \left(\frac{1.07}{1.05}\right)^{19} \right] \\ &= (1.07)^5 (.03) \left[ 1 + 1.01905 + \dots + (1.01905)^{18} + (1.01905)^{19} \right] \\ &= (1.07)^5 (.03) \$2071.905 q_0 = 74.79. \end{aligned}$$

(D)

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21. The equation of value for the PV of premiums and the PV of benefits is

$$P \ddot{a}_{35} + .05 P (Ia_{36}) = 95,000 A_{35} + 5,000 IA_{35}$$

$$P \left( \frac{N_{35}}{D_{35}} \right) + .05 P \left( \frac{S_{36}}{D_{35}} \right) = 100,000 \left( \frac{M_{35}}{D_{35}} \right) + 5000 \left( \frac{R_{36}}{D_{35}} \right)$$

$$\begin{aligned} P &= \frac{100,000 M_{35} + 5000 R_{36}}{N_{35} + .05 S_{36}} \\ &= \frac{100,000 (2916.60)}{314,517.9} + \frac{5000 (92,461.08)}{314,517.9} \\ &= 1425.56 \end{aligned}$$

(E) The second year's premium is  $1.05 (1425.56) = 1496.84$

The PV of death benefits can be expressed in commutation functions equally well as  $95000 \left( \frac{M_{35}}{D_{35}} \right) + 5000 \left( \frac{R_{35}}{D_{35}} \right)$ .

22. The probability that at least one person dies in 1994 is one minus the probability that neither dies in 1994. An alternative approach is to consider all possible cases where one or both die during 1994. Both of these approaches must give the same numerical answer.

Try to determine 1 minus the probability neither dies in 1994:

(a) both may die before 1994

(b) both may die after 1994

(c) one may die before 1994 and the other may die after 1994

$$\begin{aligned} \text{For case (a), we have } 3f_{60}(3f_{65}) &= (1-3P_{60})(1-3P_{65}) \\ &= (1-943/1000)(1-828/900) = .00456 \\ \text{(b), we have } 4f_{60}(4f_{65}) &= (922/1000)(802/900) = .8216 \\ \text{(c) we have } 3f_{65}(4P_{60}) &= (1-828/900)(922/1000) = .0738 \\ &+ 3f_{60}(4P_{65}) = (1-943/1000)(802/900) = \frac{.0508}{.9507} \end{aligned}$$

The final probability we want is  $1 - .9504 = .0493$

(C)

The alternate solution is to specify cases where one dies in 1994:

(a) one may die during 1994, and the other does not

(b) both may die during 1994

$$\begin{aligned} \text{For case (a), we have } (3f_{65}-4P_{65})[1-(3f_{60}-4P_{60})] &= \frac{(828-802)}{900} \left(1 - \frac{943-922}{1000}\right) \\ &+ (3f_{60}-4P_{60})[1-(3f_{65}-4P_{65})] = \left[1 - \frac{828-802}{900}\right] \left(\frac{943-922}{1000}\right) \\ &= .0283 + .0204 \end{aligned}$$

$$\begin{aligned} \text{(b), we have } (3f_{60}-4P_{60})(3f_{65}-4P_{65}) &= \left(\frac{943-922}{1000}\right) \left(\frac{828-802}{900}\right) \\ &= .0006 \end{aligned}$$

The total probability is  $.0283 + .0204 + .0006 = .0493$

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$$\begin{aligned}
 23. \quad e_x &= p_x + 2p_x + 3p_x + \dots \\
 &= (1-q_x) + (1-q_x)(1-q_{x+1}) + (1-q_x)(1-q_{x+1})(1-q_{x+2}) + \dots \\
 1.25 &= .8 + .8(1-K) + .8(1-K)(.2) + .8(1-K)(.2)(\emptyset) \\
 &= .8 + .8 - .8K + .16 - .16K \\
 .96K &= 1.76 - 1.25 \\
 K &= \frac{.51}{.96} = .531
 \end{aligned}$$

(C)

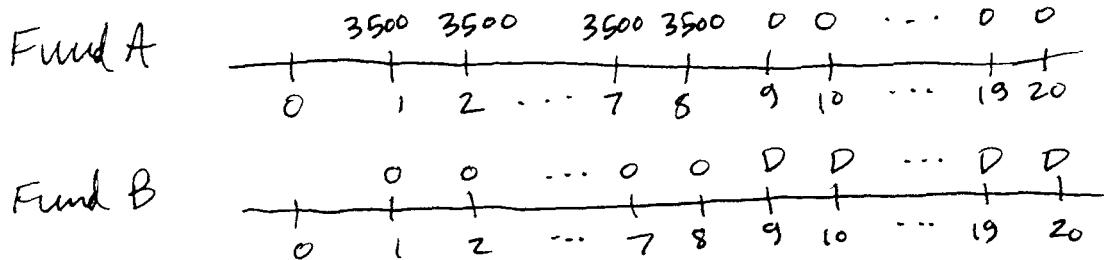
24. This problem requires that you know  $v p_x \ddot{a}_{x+1} = a_x$

$$\begin{aligned}
 vp_{51} \ddot{a}_{52} &= a_{51} & \ddot{a}_{52} &= (1+i)(\ddot{a}_{51} - 1) / p_{51} \\
 vp_{50} \ddot{a}_{51} &= a_{50} & \ddot{a}_{51} &= (1+i)(\ddot{a}_{50} - 1) / p_{50} \\
 && &= (1.07)(11.07873) / (1-.005616) \\
 && &= 11.9212 \\
 \ddot{a}_{52} &= (1.07)(10.9212) / (1-.006196) \\
 && &= 11.7585
 \end{aligned}$$

(C)

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25. You should write down a time diagram that shows the deposits for funds A and B:



At the end of 20 years, the original loan would accumulate to  $(1.10)^{20} (25,000) = 168,187$ .

The accumulated values for each fund should be determined separately, since each one earns a different rate of interest:

$$\text{Fund A : } 3500 \left( S_{20|7\%} - S_{12|7\%} \right) = 80875$$

$$\text{Fund B : } D S_{12|5\%} = 15.9171 D$$

In order for the payments to funds A and B to pay off the loan at the end of 20 years, the accumulated values of the two funds must add up to the accumulated value of the loan:

$$80875 + 15.9171 D = 168,187$$

$$D = \frac{87,312}{15.9171} = 5485$$

D