SoftwarePolish

Rick Groszkiewicz
2964 Nestle Creek Drive
Marietta, GA 30062-4857
Voice/fax (770) 971-8913
email: rickg@softwarepolish.com

SPRING 1992 EA-1A EXAM SOLUTIONS

Revision History:

02/23/99	Enhanced problem 11	added faster method of solution

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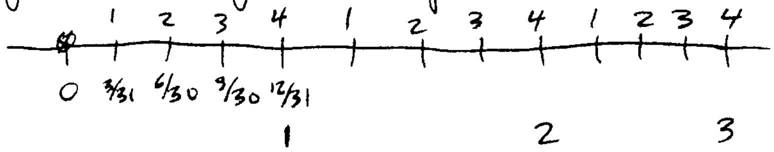
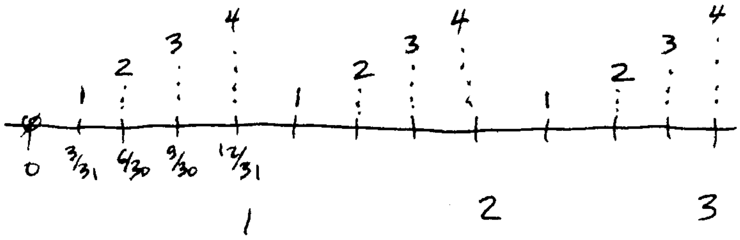
Note to EA-1A seminar attendees:

Here are the solutions to the 1992 EA-1A exam. Please review the solution to number 3 carefully. This is an unusual usage of the reversionary annuity! You should memorize the short answer, since it takes a very long time to simplify the summation!

Spring 1992 EA-1A

1 The easiest way to value the annuity is to think of it as four separate annual annuities:





There are four annuities, each of which has ten annual payments for the same amount:

$$PV = v^{\frac{1}{4}}\,\ddot{a}_{\overline{10}|.07} + v^{\frac{2}{4}}(2)\,\ddot{a}_{\overline{10}|.07} + v^{\frac{3}{4}}(3)\,\ddot{a}_{\overline{10}|.07} + v^{\frac{4}{4}}(4)\,\ddot{a}_{\overline{10}|.07}$$

$$= (.9832 + 2(.9667) + 3(.9505) + 4(.9346))\,7.5152$$

$$= 71.44$$

D

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2 This problem ~~tests~~ your knowledge of two identities. The present value of the death benefit can be expressed as

$$10{,}000\, A_{65} = 10{,}000\, M_{65}/D_{65} = 10{,}000\,(C_{65}+M_{66})/D_{65}$$

The identities you must use to calculate the value of the insurance are

$$M_x = D_x - d\cdot N_x \qquad \text{and} \qquad N_x^{(12)} = N_x - \frac{11}{24} D_x$$

$$D_x = v^x l_x = (v^{x+1} l_{x+1})(1+i)\frac{l_x}{l_{x+1}} = D_{x+1}\frac{(1+i)}{p_x}$$

$$N_x = N_{x+1} + D_x$$

You can calculate D_{65} directly, as well as N_{66}. This gives you N_{65} and M_{65}, and then you are done:

$$D_{65} = D_{66}(1.08)/p_{65} = 47{,}775.55(1.08)/(1-.022562)$$
$$= 52{,}788.61$$

$$N_{66} = N_{66}^{(12)} + \frac{11}{24} D_{66} = 382{,}153.96 + \frac{11}{24}(47{,}775.55)$$

$$= 404{,}041.92$$

$$N_{65} = N_{66} + D_{65} = 456{,}830.53$$

$$M_{65} = D_{65} - d\cdot N_{65} = 52{,}788.61 - \frac{.08}{1.08}(456{,}830.53)$$
$$= 18{,}949.31$$

$$10{,}000\frac{M_{65}}{D_{65}} = 3589.66$$

A

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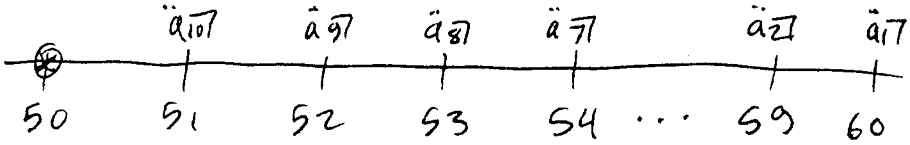
3 The present value is a series of annuities payable upon death. Each annuity is paid for one year less than the preceding one:



$$PV = \left(\sum_{t=0}^{9} v^{t+1} \, {}_tp_x \, q_{x+t} \, \ddot{a}_{\overline{10-t}|i} \right) 1000$$

Unfortunately, this expression can't be evaluated directly. With a lot of work, it can be shown that this is equal to

$$(a_{\overline{10}|i} - a_{50:\overline{10}|i})\,1000$$

$$= \left(7.0236 - \frac{N_{50}-N_{60}}{D_{50}}\right) 1000$$

$$= (7.0236 - 6.7679)\,1000 = 255.64$$

D

Logically, this is a reversionary annuity. The death benefit is equal to $\ddot{a}_{\overline{10}|}$ at the end of the year, except that no payments are made unless the participant is dead.

Another way to derive the formula shown above is to re-write the original expression:

$$\begin{aligned} PV = {} & v(1-p_{50})(1+v+v^2+\dots+v^8+v^9) \\ & + v^2 p_{50}(1-p_{51})(1+v+v^2+\dots+v^7+v^8) \\ & + v^3 p_{50}p_{51}(1-p_{52})(1+v+v^2+\dots+v^6+v^7) \\ & + \dots \\ & + v^9 p_{50}p_{51}\dots p_{57}(1-p_{58})(1+v^1) + v^{10} p_{50}p_{51}\dots p_{58}(1-p_{59})(1) \end{aligned}$$

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(3) Continued

$$
\begin{aligned}
\rho V = \ & v\,(1 + v + \cdots + v^{9}) \\
& - p_{50}\,(v + v^2 + v^3 + \cdots + v^{10}) \\
& + p_{50}\,(v^2 + v^3 + \cdots + v^{10}) \\
& - p_{50}p_{51}\,(v^2 + v^3 + \cdots + v^{10}) \\
& + p_{50}p_{51}\,(v^3 + v^4 + \cdots + v^{10}) \\
& - p_{50}p_{51}p_{52}\,(v^3 + v^4 + \cdots + v^{10}) \\
& \cdots \qquad \cdots \\
& + p_{50}p_{51}\cdots p_{57}\,(v^9 + v^{10}) \\
& - p_{50}p_{51}\cdots p_{57}p_{58}\,(v^9 + v^{10}) \\
& + p_{50}p_{51}\cdots p_{57}p_{58}\,(v^{10}) \\
& - p_{50}p_{51}\cdots p_{57}p_{58}p_{59}\,(v^{10})
\end{aligned}
$$

$$
\begin{aligned}
\rho V &= a_{\overline{10}|i} - v\,p_{50} - v^2 p_{50}p_{51} - \cdots - v^9 p_{50}p_{51}\cdots p_{58} - v^{10} p_{50}p_{51}\cdots p_{58}p_{59} \\
&= a_{\overline{10}|i} - a_{50:\overline{10}|i}
\end{aligned}
$$

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4 This problem tests your knowledge of several identities:

$$\ddot{s}_{x:\overline{15}|} = \frac{N_x - N_{x+15}}{D_{x+15}} = \frac{D_x}{D_{x+15}}\left(\ddot{a}_{x:\overline{15}|}\right)$$

$$A_{x:\overline{15}|} = 1 - d\,\ddot{a}_{x:\overline{15}|}$$

$$A^{1}_{x:\overline{15}|} = A_{x:\overline{15}|} - \frac{D_{x+15}}{D_x}$$

$$\therefore \quad .30 = 1 - d\,\ddot{a}_{x:\overline{15}|} - \frac{D_{x+15}}{D_x}$$

$$\frac{D_{x+15}}{D_x} = 1 - .05(7) - .30 = .35$$

$$\therefore \quad \ddot{s}_{x:\overline{15}|} = 7.0/.35 = 20$$

E

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5 The time weighted return is based on ratios of market values. You must determine the market value both before and after each cash flow.

Let MV1B = market value before first cash flow
MV2A = market value after second cash flow

Then $1+TWI = \left(\frac{MV1B}{MVstart}\right)\left(\frac{MV2B}{MV1A}\right)\left(\frac{MV3B}{MV2A}\right)\left(\frac{MV4B}{MV3A}\right)\left(\frac{MV\,at\,end}{MV4A}\right)$

Date	3/31	6/30	9/30	12/31
Cash flow	-10,000	+70,000	-10,000	-10,000
MV before	210,000	216,000	286,000	270,000
MV after	200,000	286,000	276,000	260,000

← All these given in the problem

$1+TWI = \frac{210{,}000}{200{,}000} \cdot \frac{216{,}000}{200{,}000} \cdot \frac{286{,}000}{286{,}000} \cdot \frac{270{,}000}{276{,}000} \cdot \frac{260{,}000}{260{,}000}$

$= (1.05)(1.08)(1.00)(.9783)(1.00)$

$= 1.1093$

I. Time weighted return is 10.93%

The dollar-weighted return is calculated by expressing the rate of interest earned on each cash flow, including the beginning value

$200{,}000(1+i) - 10{,}000\left(1+\tfrac{3}{4}i\right) + 70{,}000\left(1+\tfrac{2}{4}i\right) - 10{,}000\left(1+\tfrac{1}{4}i\right) - 10{,}000 = 260{,}000$

$i\left(200{,}000 - 10{,}000\left(\tfrac{3}{4}\right) + 70{,}000\left(\tfrac{2}{4}\right) - 10{,}000\left(\tfrac{1}{4}\right)\right) = 260{,}000 - (200{,}000 - 10{,}000 + 70{,}000 - 20{,}000)$

II $i = 20{,}000 / (200{,}000 - 7{,}500 + 35{,}000 - 2{,}500) = 8.89\%$

III Assuming uniform distribution of cash flows,

$III = \frac{2I}{A+B-I} = \frac{40{,}000}{200{,}000 + 260{,}000 - 20{,}000} = 9.09\%$

A

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6 In a stationary population with an entry age of 25, the number of deaths each year is $l_{25} - l_{65}$ (based on retirement at age 65).

The average age at death in this stationary population is given as 55, and is equal to

$$55 = 25 + \frac{T_{25} - T_{65} - 40 l_{65}}{l_{25} - l_{65}}$$

You know that $l_{25} - l_{65} = 2$, and also that $T_{25} - T_{65} = 100$

so you can solve for l_{65}, and then l_{25}:

$$55 = 25 + \frac{100 - 40 l_{65}}{2}$$

$$30 = 50 - 20 l_{65}$$

$$l_{65} = 1$$

$$l_{25} - l_{65} = 2 = l_{25} - 1 \quad \therefore l_{25} = 3$$

B

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7 You can write down a time diagram for the payment stream:

10	30	50	70	90	110 ...	110	...
0	1	2	3	4	5 ...	6	...

$$P = 10 + 30v + 50v^2 + 70v^3 + 90v^4 + 110v^5 + 110v^6 + \dots$$

To solve for the value of the perpetuity, you can multiply both sides by $1+i$ and ~~then~~ subtract

$$(1+i)P = 10(1+i) + 30 + 50v + 70v^2 + 90v^3 + 110v^4 + 110v^5 + \dots$$

$$iP = 10(i+1) + 20 + 20v + 20v^2 + 20v^3 + 20v^4$$

$$P = \frac{30.8 + 20(v + v^2 + v^3 + v^4)}{.08}$$

$$= \frac{30.8 + 20\,a_{\overline{4}|.08}}{.08} = 1213.03$$

C

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8 This problem tests your knowledge of several identities:

$$A_x = 1 - d \cdot \ddot{a}_x \qquad a_x = vp_x \, \ddot{a}_{x+1}$$

$$\ddot{a}_x = \frac{1-A_x}{d} \qquad (\ddot{a}_x - 1) = vp_x\,(a_{x+1} + 1)$$

$$= \frac{1-.3645}{.045/1.045} \qquad \frac{\ddot{a}_x - 1}{v\,p_x} = 1 + a_{x+1}$$

$$= 14.7577$$

$$a_{x+1} = \frac{(1+i)(\ddot{a}_x - 1)}{p_x} - 1$$

$$= \frac{1.045(13.7577)}{1-.007} - 1$$

$$= 13.4782$$

B

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9 The easiest way to determine the total interest is that it equals the total payments less the original loan amount. You must calculate the payment amount X. Be careful of the irregular timing of the first two payments.

The best approach may be to calculate the O/S loan amount at 1/1/93:

$$1/1/93 \text{ O/S loan} = 10{,}000\,(1.01)^{12} - 600\,(1.01)^{10} - 600\,(1.01)^{8}$$
$$= 9{,}955.76$$

$$X\,\ddot{a}_{\overline{12}|.01} = 9{,}955.76 \Rightarrow X = 875.80$$

Total loan payments $= 2(600) + 12(875.80) = 11{,}709.59$

Total loan interest $= 11{,}709.59 - 10{,}000 = 1{,}709.59$

A

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10 The net annual premium will be paid from ages 57 through 64, for a ~~total~~ of 8 payments:

$$P \cdot \ddot{a}_{57:\overline{8}|} = 12(250)\left(\ddot{a}^{(12)}_{\overline{10}|} + {}_{10|}\ddot{a}^{(12)}_{65}\right)\frac{D_{65}}{D_{57}}$$

$$P\,\frac{(N_{57}-N_{65})}{D_{57}} = 3000\left(\ddot{a}^{(12)}_{\overline{10}|} + \frac{N^{(12)}_{75}}{D_{65}}\right)\frac{D_{65}}{D_{57}}$$

$$P = 3000\,\frac{\left(D_{65}\cdot \ddot{a}^{(12)}_{\overline{10}|} + N^{(12)}_{75}\right)}{N_{57}-N_{65}} \qquad N^{(12)}_{75} = N_{75} - \frac{11}{24}D_{75}$$

$$= 3000\,\frac{\left(100(7.219) + 247 - \frac{11}{24}(36)\right)}{2111-919}$$

$$= 2397$$

C

Spring 1992 EA-1A   Added solution 2/23/99

11 There are two ways to approach this problem. This page shows how to use the bond price formula at two successive dates to calculate the answer without knowing the term of the bond. The alternate approach on the next page calculates the term of the bond first.

With an unknown term, you can still draw a time line diagram for the bond

	80	80	80	80	80	80	80	... 1000
1-92	1-93	1-94	1-95	1-96	1-97	1-98	1-99	...

You receive a coupon of $80 = .08(1000)$ each year, and after n years the bond is redeemed for 1000. You can write down formulas for the bond price at both 1-1-97 and 1-1-98:

$$\text{1-1-97 price} = Z = 80\, a_{\overline{n}|10\%} + 1000(1.10)^{-n}$$

$$\text{1-1-98 price} = Z + 10.25 = 80\, a_{\overline{n-1}|10\%} + 1000(1.10)^{-n+1}$$

$$1.10\,Z = 80\, \ddot{a}_{\overline{n}|10\%} + 1000(1.10)^{-n+1} \quad \text{multiply by 1.10}$$

$$1.10Z - 80 = 80\, a_{\overline{n-1}|10\%} + 1000(1.10)^{-n+1}$$

$$\therefore\ 1.10Z - 80 = Z + 10.25$$

$$.10Z = 90.25$$

$$Z = 902.50 = \text{1-1-97 price}$$

Now you can express the 1-1-92 price in terms of the 1-1-97 price

$$\text{1-1-92 price} = 80\, a_{\overline{n+5}|10\%} + 1000(1.10)^{-n-5}$$

$$= (\text{1-1-97 price})\,v^5 + 80\, a_{\overline{5}|10\%}$$

$$= (902.50)(1.10)^{-5} + 80(3.7908)$$

$$= 863.64$$

B

This is a relatively fancy way of getting the answer. A more straightforward approach is on the next page

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(11) In this problem, you don't know the term of the original bond. You must solve for it using the information you have been given.

The best bond formula to use to solve this problem is the alternate formula:

$$P = Fr \cdot a_{\overline{n}|i} + Cv^n \qquad a_{\overline{n}|i} = \frac{1-v^n}{i} \Rightarrow v^n = 1 - i \cdot a_{\overline{n}|i}$$

$$= Fr \cdot a_{\overline{n}|i} + C(1 - i \cdot a_{\overline{n}|i})$$

$$= C + (Fr - Ci)\,a_{\overline{n}|i} \qquad \text{at } 1\text{-}1\text{-}92$$

$$Z = C + (Fr - Ci)\,a_{\overline{n-5}|i} \qquad \text{at } 1\text{-}1\text{-}97$$

$$Z + 10.25 = C + (Fr - Ci)\,a_{\overline{n-6}|i} \qquad \text{at } 1\text{-}1\text{-}98$$

$$10.25 = (Fr - Ci)(a_{\overline{n-6}|} - a_{\overline{n-5}|})$$

$$= (80 - 100)\left(\frac{v^{n-5} - v^{n-6}}{i}\right)$$

$$= -200\,(v^{n-5})(1 - (1+i))$$

$$= +20\,v^{n-5}$$

$$v^{n-5} = .5125 = (1.1)^{5-n}$$

$$\log .5125 = (5-n)(\log 1.1)$$

$$5 - n = \frac{\log .5125}{\log 1.1} = -7.013$$

$$n = 12.013 \Rightarrow \text{use 12 years for formula!}$$

$$P = C + (Fr - Ci)\,a_{\overline{n}|i}$$

$$= 1000 + (80 - 100)\,a_{\overline{12}|10\%}$$

$$= 1000 - 20\,(7.495)/1.10$$

$$= 863.73$$

(B)

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12 This is an annuity that reduces upon the death of either the participant or the beneficiary. The best way to express the present value is to use a reversionary annuity factor:

$$PV = 12{,}000\left(\ddot{a}^{(12)}_{60:65}\right) + 9{,}000\left(\ddot{a}^{(12)}_{60F} - \ddot{a}^{(12)}_{60:65}\right) \text{ F alive, M dead}$$

both alive

$$+\ 9{,}000\left(\ddot{a}^{(12)}_{65M} - \ddot{a}^{(12)}_{60:65}\right) \text{ M alive, F dead}$$

$$= 9{,}000(11.2993 + 9.3452) - 6{,}000(8.1620)$$

$$= 136{,}828$$

C

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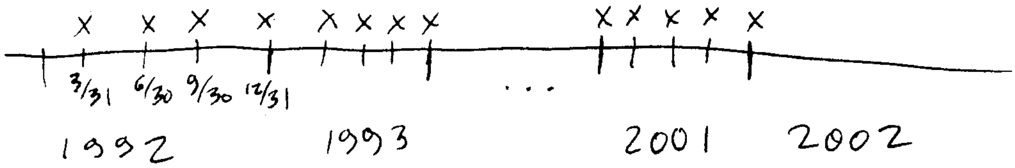
13 With a sinking fund, the annual loan interest is paid directly. There are separate payments made into a sinking fund that earns a different rate of interest. The accumulated value of the sinking fund at the end of the term of the loan equals the original amount of the loan.

The sinking fund payments are quarterly, so it will be convenient to convert 6% per annum, payable semiannually to a quarterly rate:

$$(1.03)^{\frac{1}{2}} = 1.014889 \Rightarrow 1.4889\% \text{ per quarter}$$



As of 1/1/2002, the sinking fund balance is equal to

$$X \cdot s_{\overline{40|}1.4889\%} = SFB^{1/1/02}$$

With 100 sinking fund payments in total, we have

$$X \cdot s_{\overline{100|}1.4889\%} = 10{,}000{,}000$$

$$\therefore SFB^{1/1/02} = 10{,}000{,}000\left(\frac{s_{\overline{40|}1.4889}}{s_{\overline{100|}1.4889}}\right) = 2{,}382{,}192$$

B

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14 The probability is $1 - {}_2p_x$. The key is to derive the relationship between successive values of the expectation of life:

$$e_{x+1} = p_{x+1} + {}_2p_{x+1} + \ldots$$

$$e_x = p_x + {}_2p_x + \ldots$$

$$= p_x(1 + p_{x+1} + {}_2p_{x+1} + \ldots)$$

$$= p_x(1 + e_{x+1})$$

$$p_x = \frac{e_x}{1+e_{x+1}} = \frac{20.2}{20.6} = .9806 \qquad p_{x+1} = \frac{e_{x+1}}{1+e_{x+2}} = \frac{19.6}{20.0} = .9800$$

$$\therefore {}_2p_x = p_x(p_{x+1})$$

$$= .9806(.9800)$$

$$= .96097$$

$$1 - {}_2p_x = .03903$$

D

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15 This problem is unique because the population of husbands is increasing due to marriages of bachelors. The 82,252 husbands at age 36 equals the 75,252 plus 7,546 new marriages less the 546 deaths.

To calculate the probability of death for a husband, the denominator must take into account the exposure of the 7546 former bachelors for 1/2 of a year on the average:

$$q^{hl}_{35} \doteq \frac{546}{75{,}252 + \frac{1}{2}(7{,}546)} = .0069$$

$$p^{hl}_{35} = 1 - q^{hl}_{35} = .9931$$

(B)

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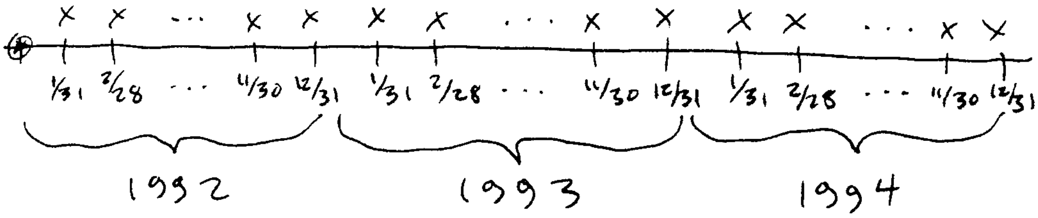
16 The first step is calculation of the original loan payment. Next, you should determine the outstanding loan balance at 1-1-93. Then you can calculate the new loan payment, and the effect of the change over the life of the loan.



$20{,}000 = x \cdot a_{\overline{36}|1.5\%}$

$x = 20{,}000 / a_{\overline{36}|1.5\%} = 723.05$

At 1-1-93, there are 24 payments left.

1-1-93 O/S loan balance $= 723.05 \, (a_{\overline{24}|1.5\%})$
$= 14{,}483$

New payment $Y = 14{,}483 / a_{\overline{24}|1.0\%}$
$= 681.76$

Effect of new payment is a reduction in the last 24 loan payments:

$24\,(681.76 - 723.05) = -990.90$

(C)

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17 The special restriction affects payments due at 12/31/92 and 12/31/93. Without the special restriction, the present value can be calculated as

$$10{,}000\, q^{M}_{65}\left(v p^{F}_{64} + v^{2}{}_{2}p^{F}_{64} + v^{3}{}_{3}p^{F}_{64} + \ldots\right) \quad \text{male dies 1992}$$
$$+10{,}000\, p^{M}_{65}\, p^{F}_{64}\, q^{M}_{66}\left(v^{2} p^{F}_{65} + v^{3}{}_{2}p^{F}_{65} + \ldots\right) \quad \text{male dies 1993}$$
$$+10{,}000\, {}_{2}p^{M}_{65}\, {}_{2}p^{F}_{64}\, q^{M}_{67}\left(v^{3} p^{F}_{66} + v^{4}{}_{2}p^{F}_{66} + \ldots\right) \quad \text{male dies 1994}$$
$$+ \ldots \quad \ldots$$

The actual payments made prior to 12/31/94 can be represented by this expression:

$$10{,}000\, q^{M}_{65}\left(\overset{12/31/92}{v p^{F}_{64}} + \overset{12/31/93}{v^{2}{}_{2}p^{F}_{64}}\right) + 10{,}000\, p^{M}_{65}\, p^{F}_{64}\, q^{M}_{66}\left(\overset{12/31/93}{v^{2} p^{F}_{65}}\right)$$

For female mortality

$l_x = 104 - x$

$p_x = \dfrac{l_{x+1}}{l_x} = \dfrac{103-x}{104-x}$

$q_x = \dfrac{1}{104-x}$

For male mortality

$l_x = 98 - x$

$p_x = \dfrac{l_{x+1}}{l_x} = \dfrac{97-x}{98-x}$

$q_x = \dfrac{1}{98-x}$

PV of pre 12/31/94 payments:

$$10{,}000\left(\frac{1}{33}\right)\left((1.07)^{-1}\frac{39}{40} + (1.07)^{-2}\frac{38}{40}\right) + 10{,}000\left(\frac{32}{33}\right)\left(\frac{39}{40}\right)\left(\frac{1}{32}\right)\left((1.07)^{-2}\frac{38}{39}\right)$$

$$= \frac{10{,}000}{33}(1.740982) + \frac{10{,}000}{33}(.829767)$$

$$= 779$$

E

Spring 1992 EA-1A

18 One way to work this problem is to rephrase it to "the probability that three or less members of the population die before age 71".

Pr (Ø members die) $= (.97)^{100} = .04755$
Pr (1 member dies) $= 100(.97)^{99}(.03) = .1471$
Pr (2 members die) $= \frac{100(99)}{2}(.97)^{98}(.03)^2 = .2252$

Pr (3 members die) $= \frac{100(99)(98)}{3 \times 2}(.97)^{97}(.03)^3 = .2275$

The sum of these probabilities is .6472.

B

If you feel uneasy about how large these values are, you can continue to calculate probabilities and see that the sum does approach 1.00 asymptotically:

			Σ all prob
Pr (4 members die)	$= \frac{100(99)(98)(97)}{4 \times 3 \times 2 \times 1}(.97)^{96}(.03)^4$	$= .17061$	.8179
Pr (5 members die)	$= .17061 \frac{(96)}{(5)} \frac{(.03)}{(.97)}$	$= .1013$	.9192
Pr (6 members die)	$= .1013 \frac{(95)}{(6)} \left(\frac{.03}{.97}\right)$	$= .0496$	.9688
Pr (7 members die)	$= .0496 \left(\frac{94}{7}\right)\left(\frac{.03}{.97}\right)$	$= .0206$	.9894

and so forth. Clearly, the probabilities are declining rapidly, at an increasing rate.

B

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19) The hardest part of this problem is writing down an expression for the death benefit.

$$P \cdot \ddot{a}_{45:\overline{20}|} = 6000\left(\ddot{a}^{(12)}_{\overline{5}|3\%} + {}_{5|}\ddot{a}^{(12)}_{65}\right)\frac{D_{65}}{D_{45}} + \text{(return of prem. death ben)}$$

	P	$2P$	$3P$	$4P$		$18P$	$19P$	$20P$
45	46	47	48	49	$\cdots$	63	64	65

If the participant dies while age 45, the first year's premium is refunded at the end of the year of death. If they die in the second year, there are two premiums to refund at the end of the year of death.

PV of death benefits = increasing, temporary insurance
$= P(R_{45} - R_{65} - 20M_{65})/D_{45}$

R_{45} provides the increasing insurance, plus payments upon death at the rate of $21P$ for age 65, $22P$ for age 66, etc. By subtracting R_{65}, there would be a flat layer of $20P$ after age 65. This is eliminated by the $20M_{65}$ term.

$$P(N_{45}-N_{65})/D_{45} = 6000\left(\ddot{a}^{(12)}_{\overline{5}|3\%}\cdot D_{65} + N^{(12)}_{70}\right)/D_{45} + P(R_{45}-R_{65}-20M_{65})/D_{45}$$

$$\therefore P = \frac{6000\left(\ddot{a}^{(12)}_{\overline{5}|3\%}\cdot D_{65} + N_{70} - \frac{11}{24}D_{70}\right)}{N_{45}-N_{65} - (R_{45}-R_{65}-20M_{65})}$$

$$= \frac{6000\left(4.6538 \overset{(996)}{+} 6217 - \frac{11}{24}(706)\right)}{44{,}455 - 10{,}607 - (26{,}751 - 8{,}076 - 20(687))}$$

$= 6000\,(10{,}528.59)/28{,}913$

$= 2185$

B

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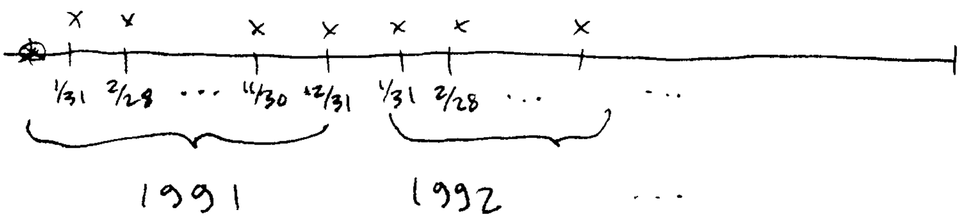
20 With a level interest rate, this would be a simple problem. With the varying rates, you have three different values for $a_{\overline{60}|i}$, and they are discounted to 1-1-91 at varying rates.



With a single rate, $100{,}000 = X\, a_{\overline{180}|j}$

$$= X\left(a_{\overline{60}|j} + v^{60} a_{\overline{60}|j} + v^{120} a_{\overline{60}|j}\right)$$

With interest rates given in the problem, you have

$$100{,}000 = X\left(a_{\overline{60}|1\%} + (1.01)^{-60} a_{\overline{60}|.75\%} + (1.01)^{-60}(1.0075)^{-60} a_{\overline{60}|.5\%}\right)$$

$$= X\left(44.9550 + .5505(48.1734) + .5505(.6387)(51.7256)\right)$$

$\therefore X = 1115.36$

D

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21 The interest rate can vary each year. At the end of each year, a loan payment is made. Then a new payment is calculated by amortizing the outstanding loan balance over the remaining life of the loan at the new interest rate.

The first payment occurs at 12/31/90. By dividing the interest paid by the outstanding balance at the beginning of the year, you can back into the interest rate for the year:

$$170{,}000 = X \cdot a_{\overline{20}|i}$$

$$170{,}000\,i = 11{,}900 \Rightarrow i = 7\%$$

$$1/1/91 \text{ Loan balance} = 170{,}000\left(\frac{a_{\overline{19}|i}}{a_{\overline{20}|i}}\right) = 165{,}853$$

$$165{,}853\,j = 13{,}268 \Rightarrow j = 8\%$$

$$165{,}853 = Y \cdot a_{\overline{19}|8\%} \Rightarrow Y = 17{,}270 \text{ for } 1991$$

Since the interest rate is not changed for 1992, the same payment will be calculated for 1992:

$$1/1/92 \text{ Loan balance} = 165{,}853\left(\frac{a_{\overline{18}|8\%}}{a_{\overline{19}|8\%}}\right)$$

$$12/31/92 \text{ Payment} = \frac{1/1/92 \text{ Loan balance}}{a_{\overline{18}|8\%}} = \frac{165{,}853}{a_{\overline{19}|8\%}}$$

$$= 17{,}270$$

D

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22 You must write down the expressions for the present value of future benefits at both 1/1/91 and 1/1/92. Then solve for the unknown items. The problem is simplified somewhat by the participant still being in the five year certain period.

$$1\text{-}1\text{-}91 \quad PV = 12{,}000\left(\ddot{a}^{(12)}_{\overline{5}|8\%} + N^{(12)}_{x+5}/D_x\right) = 120{,}000$$

$$1\text{-}1\text{-}92 \quad PV = 12{,}000\left(\ddot{a}^{(12)}_{\overline{4}|8\%} + N^{(12)}_{x+5}/D_{x+1}\right)$$

The first step is to calculate $\ddot{a}^{(12)}_{\overline{5}|8\%}$ and solve for the value of $\dfrac{N^{(12)}_{x+5}}{D_x}$:

$$\ddot{a}^{(12)}_{\overline{5}|8\%} = \frac{1}{12}\ddot{a}_{\overline{60}|.6434\%} = 4.1637$$

$$N^{(12)}_{x+5}/D_x = \frac{120{,}000}{12{,}000} - 4.1637 = 5.8363$$

Since you are given the probability of survival and the interest rate, you can calculate

$$D_x/D_{x+1} = \frac{(1+i)}{p_x} = \frac{1.08}{.995} = 1.0854$$

$$1\text{-}1\text{-}92\ PV = 12{,}000\left(\ddot{a}^{(12)}_{\overline{4}|8\%} + 5.8363(1.0854)\right)$$
$$= 117{,}466$$

D

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23 There are two ways to solve this problem. One is to know that the ratio of the principal portion of each payment will increase with the monthly interest rate:

9/30 int. portion = 94.473% principal portion = 5.527%
10/31 int. portion = 94.418% principal portion = 5.582%

$$\text{Ratio} = 1+i = 1+\frac{x}{12} = \frac{5.582}{5.527} = 1.00995 \Rightarrow x = 11.94\%$$

A

The more exacting approach is to derive the monthly loan amortization schedule:

Payment Number	Total Payment	Interest Portion	Principal Portion	O/S Balance After payment	Payment Date
1	Z	$Z(1-v^{360})$	Zv^{360}	$Z\cdot a_{\overline{359}\mid i}$	1/31/92
2	Z	$Z(1-v^{359})$	Zv^{359}	$Z\cdot a_{\overline{358}\mid i}$	2/28/92
⋮	⋮	⋮	⋮	⋮	⋮
69	Z	$Z(1-v^{292})$	Zv^{292}	$Z\cdot a_{\overline{291}\mid i}$	9/30/97
70	Z	$Z(1-v^{291})$	Zv^{291}	$Z\cdot a_{\overline{290}\mid i}$	10/31/97

This tells you that $v^{291} = .05582$

$v = .9901\%$

$1+i = 1.009966$

$x = 11.96\%$

A

The reason for the difference in the value of x is that the input values for the principal portion only contain 4 digits of precision. The value of 1+i appears to have 7 digits of precision!

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24 This problem tests your knowledge of the identity

$$A_x = 1 - d\ddot{a}_x$$

$$P_x = \frac{A_x}{\ddot{a}_x} = \frac{1}{\ddot{a}_x} - d$$

$$\frac{1}{\ddot{a}_x} - \frac{.08}{1.08} = .05$$

$$\frac{1}{a_x + 1} = .05 + .0741$$

$$a_x + 1 = 8.0597$$

$$a_x = 7.0597$$

C

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25 The question is mostly one of interpreting the various functions you are given. The quantity $A^{1}_{50:\overline{2}|}$ is simply a two year term insurance at age 50.

$$A^{1}_{50:\overline{2}|} = \frac{M_{50} - M_{52}}{D_{50}}$$

$$A_{50} = \frac{M_{50}}{D_{50}} \qquad A_{52} = \frac{M_{52}}{D_{52}}$$

$$\therefore A^{1}_{50:\overline{2}|} = A_{50} - \frac{D_{52}}{D_{50}}(A_{52})$$

$$A_{50:\overset{1}{\overline{2}|}} = \frac{D_{52}}{D_{50}}$$

$$\therefore A^{1}_{50:\overline{2}|} = .1771 - .8472(.1966)$$
$$= .0105$$

C