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# SPRING 1993 EA-1A EXAM SOLUTIONS

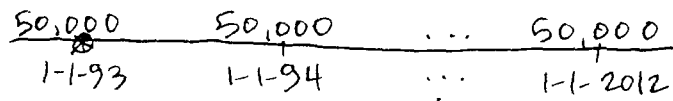
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Revision History:

02/23/99	Enhanced problem 9	added faster method of solution
	Enhanced problem 11	added faster method of solution

# Spring 1993 EA-1A Solutions

1 Here are the payments for the 20 year annuity

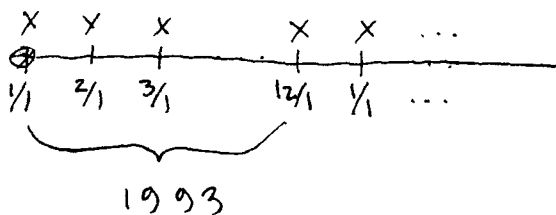


$$PV = 50,000 \ddot{a}_{20|j}$$

$$= 524,703$$

$$1+j = \left(1 + \frac{.08}{2}\right)^2 = 1.0816$$

Here is the payment stream for the perpetuity:



$$PV = X + X(1+k)^{-1} + X(1+k)^{-2} + \dots$$

$$(1+k)^{-1} PV = X(1+k)^{-1} + X(1+k)^{-2} + \dots$$

$$PV = \frac{X}{1-(1+k)^{-1}} \quad \text{(result after subtraction)}$$

$$(1+k)^{12} = 1.0816 \quad 1+k = 1.0066$$

$$\frac{X}{1-(1.0066)^{-1}} = PV = 524,703$$

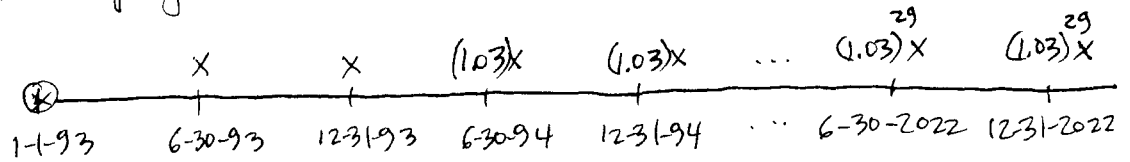
$$X = .0066(524,703)$$

$$= 3,419$$

(B)

# Spring 1993 EA-1A Solutions

- 2 write out the diagram which shows all the loan payments:



The present value of the payment stream is 100,000, calculated at 1-1-93:

$$100,000 = \frac{X}{(1+j)^1} + \frac{X}{(1+j)^2} + \frac{(1.03)X}{(1+j)^3} + \frac{(1.03)X}{(1+j)^4} + \dots + \frac{(1.03)^{29}X}{(1+j)^{59}} + \frac{(1.03)^{29}X}{(1+j)^{60}}$$

$$100,000 = [(1+j) + 1] \left[ \frac{X}{(1+j)^2} + \frac{(1.03)X}{(1+j)^4} + \dots + \frac{(1.03)^{29}X}{(1+j)^{60}} \right]$$

The above expression assumes that  $j$  is the interest rate for a six month period:

$$(1+j)^2 = \left(1 + \frac{.08}{4}\right)^4 = 1.0824 \quad \therefore (1+j) = 1.0404$$

Now you can replace  $(1+j)^2$  in the denominators with 1.0824:

$$\begin{aligned} 100,000 &= 2.0404 \left[ \frac{X}{1.0824} + \frac{1.03X}{(1.0824)^2} + \dots + \frac{(1.03)^{29}X}{(1.0824)^{30}} \right] \\ &= \frac{2.0404 X}{1.0824} \left( 1 + \frac{1.03}{1.0824} + \dots + \left(\frac{1.03}{1.0824}\right)^{29} \right) \\ &= 1.8850 X \ddot{a}_{\overline{30}|k} \quad \text{where } 1+k = 1.0824/1.03 \\ \therefore X &= 100,000 / [1.8850 \ddot{a}_{\overline{30}|5.09\%}] \\ &= 3,318 \end{aligned}$$

$$\text{Last payment} = 3,318 (1.03)^{29} = 7,819$$

(C)

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- 3 In general, for a life subject to multiple decrements to survive a year, they must survive all the decrements. If you know the absolute rates of decrement (as in this problem) you can say that

$${}_n p_x = [{}_n p_x^{(1)}] [{}_n p_x^{(2)}] [{}_n p_x^{(3)}] \dots$$

This is equivalent to expressing the result as the opposite of the sum of the probabilities

$${}_n p_x = [1 - {}_n q_x^{(1)} - {}_n q_x^{(2)} - {}_n q_x^{(3)} - \dots]$$

The relationships between rates and probabilities do not have to be known in order to work this problem; the answer can be calculated very quickly using the first formula shown above:

$$\begin{aligned} {}_7 q_x^{(T)} &= 1 - {}_7 p_x^{(T)} = 1 - (.8)(.8)(.6) \\ &= 1 - .3840 \\ &= .6160 \end{aligned}$$

(A)

# Spring 1993 EA-1A Solutions

- 4 The annuity  $a_{65:65:\overline{1}|}$  pays \$1 at the end of the year if both lives currently age 65 are alive at that time:

$$a_{65:65:\overline{1}|} = v \cdot p_{65} \cdot p_{65}$$

Since  $D_x = v^x l_x$ ,  $\frac{D_{x+1}}{D_x} = v \cdot p_x$

$$v \cdot p_{65} = \frac{D_{66}}{D_{65}} = \frac{880}{950} = .9263$$

$$\therefore p_{65} = .9263(1.07) = .9912$$

$$\therefore a_{65:65:\overline{1}|} = .9263(.9912) = .9181$$

(B)

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- 5 The bond has coupon payments for the years 1987 through 2004 inclusive, which is 18 years. You can use the alternate price formula, which directly gives you the amortized value of the bond as well

$$\begin{aligned}\text{Standard price formula: } P &= Fr a_{\overline{n}|i} + K \\ &= Fr a_{\overline{n}|i} + C v^n \\ &= Fr a_{\overline{n}|i} + C (1 - i a_{\overline{n}|i}) \\ &= C + (Fr - Ci) a_{\overline{n}|i}\end{aligned}$$

The original purchase price of the bond could be calculated:  
 1-1-87 price =  $100,000 + (100,000(.01) - 100,000(.0075)) a_{\overline{18}|.75\%}$

The figures in parentheses are the monthly coupons of 1,000 less the yield rate times the maturity value. The annuity at the .75% per month yield rate plus the redemption value gives the bond price.

At 1-1-93, the amortized value can be calculated using the same formula, but the annuity only includes the 12 remaining years of monthly coupons:  
 1-1-93 amort value =  $100,000 + (1000 - 750) a_{\overline{144}|.75\%}$   
 $= 121,968$

The new buyer wants to earn 15% per annum, compounded monthly

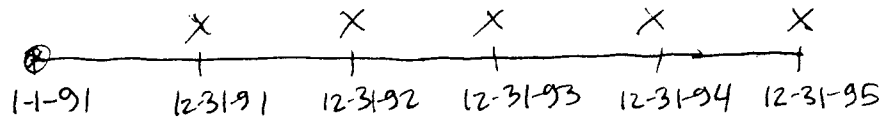
$$\begin{aligned}1-1-93 \text{ purchase price} &= 100,000 + (1000 - 100,000(.0125)) a_{\overline{144}|1.25\%} \\ &= 100,000 - 250 a_{\overline{144}|1.25\%} \\ &= 83,343\end{aligned}$$

$$\Delta = \text{loss} = 121,968 - 83,343 = 38,625$$

(C)

# Spring 1993 EA-1A Solutions

- 6 Excluding the pre-payment penalty, this is the time diagram for the loan:



$$5,000 = X a_{\overline{5}|10\%} \text{ at } 1-1-91$$

$$X = \frac{5000}{a_{\overline{5}|10\%}} = 1,319$$

At 12-31-92, the borrower paid the  $95\%$  balance, which is  $X \ddot{a}_{\overline{4}|10\%} = 4,599$ . Now you must solve for the interest rate that equates the original loan amount with the actual loan payments and prepayment penalty:

$$\begin{array}{ccccc} 5,000 & = & 1,319 v & + & (4,599 + 225) v^2 \\ 1-1-91 & & 12-31-91 & & 12-31-92 \end{array}$$

Since you are given the interest rate ranges in the answer, you can use trial and error

$$\begin{array}{ll} \text{At } 11.5\%, & PV = 1319 / (1.115) + 4824 / (1.115)^2 = 5063 \\ 11.75\%, & PV = 5043 \\ 12.00\%, & PV = 5023 \\ 12.25\%, & PV = 5003 \end{array}$$

The yield must be slightly greater than 12.25%

(E)

# Spring 1993 EA-1A Solutions

7 You should be careful about the relationships between the various commutation functions:

$$N_x = \sum D_x$$

$$S_x = \sum N_x$$

$$M_x = \sum C_x$$

$$R_x = \sum M_x$$

$$A_x = \frac{M_x}{D_x}$$

$$M_x = D_x - d N_x$$

$$A_{28} = \frac{M_{28}}{D_{28}}$$

$$= \frac{D_{28} - d(N_{28})}{D_{28}}$$

Since you are given the  $S_x$  values for ages 27 and higher, you can calculate values for  $N_{28}$  and  $D_{28}$ :

Age $x$	$S_x$	$N_x$	$D_x$
27	4,582,339	318,293	19,786
28	4,264,046	298,507	18,640
29	3,965,539	279,867	
30	3,685,672		

$$A_{28} = \frac{D_{28} - i v (N_{28})}{D_{28}}$$

$$= \frac{18,640 - \frac{.06}{1.06}(298,507)}{18,640}$$

$$= .0935$$

One trick is the error of calculating values at age 27 instead of age 28! (C)



# Spring 1993 EA-IA Solutions

- 8 The first step is calculation of the annual payment amount. Then you can determine how many years Smith must survive to receive \$5,500, and also the probability.

1-1-93 Age 60

$$10,000 = X a_{60}$$

$$= X \frac{N_{61}}{D_{60}}$$

$$\therefore X = 10,000 \frac{D_{60}}{N_{61}} = 10,000 \left( \frac{N_{60} - N_{61}}{N_{61}} \right)$$

$$= 10,000 \left( \frac{27,665 - 25,182}{25,182} \right) = 986$$

For Smith to receive \$5,500 in annuity payments, he would have to survive six years or more, and receive payments at age 66 ( ${}_6p_{60}$ ).

$$D_{66} = N_{66} - N_{67} = 15,184 - 13,608 = 1,576$$

$$D_{60} = N_{60} - N_{61} = 27,665 - 25,182 = 2,483$$

$$\frac{D_{60}}{D_{66}} = \frac{v^{60} l_{60}}{v^{66} l_{66}} = \frac{(1+i)^6}{{}_6p_{60}}$$

$${}_6p_{60} = (1.06)^6 D_{66} / D_{60}$$

$$= 1.4185 (1,576 / 2,483)$$

$$= .9004$$

(D)

Spring 1993 EA-1A

Solution added 2/23/99

- 9) There are two approaches to working this problem. The shortest technique is to realize that the mortality table corresponds to De Moivre's law after age 66. This allows you to quickly calculate the value of  $a_{66}$ , and then calculate  $\ddot{a}_{65}$ .

The potential pitfall to this approach would be to apply De Moivre's law starting at age 65. This gives an answer that luckily, is in the same answer range. On another problem, you might not get lucky!

To see that De Moivre's law applies, calculate  $l_{65}, l_{66}$ , etc

Assume $l_{65} = 100$		✓ De Moivre's law
$1p_{65} = .90 - .02(0) = .90$	$l_{66} = 90$	$d_{66} = 2$
$2p_{65} = .90 - .02(1) = .88$	$l_{67} = 88$	$d_{67} = 2$
$3p_{65} = .90 - .02(2) = .86$	$l_{68} = 86$	$d_{68} = 2$
$\vdots$	$\vdots$	$\vdots$
$45p_{65} = .90 - .02(44) = .02$	$l_{110} = 2$	$d_{110} = 2$
$46p_{65} = .90 - .02(45) = 0$	$l_{111} = 0$	$\Rightarrow w = 111$

Under De Moivre's law,  $a_x = \frac{n - \ddot{a}_{n|i}}{n(i)}$  where  $n = w - x$

$$a_{66} = \frac{45 - \ddot{a}_{45|.07}}{45(.07)} \quad \text{since } 111 - 66 = 45$$

$$a_{66} = 9.66 = \frac{45 - 14.5579}{45(.07)}$$

$$\begin{aligned} \ddot{a}_{66} &= 1 + v p_{66} \ddot{a}_{66} \\ &= 1 + (1.07)^{-1} (.90) (1 + 10.66) \\ &= 9.9699 \end{aligned}$$

The more direct solution on the next page requires more work (E)

# Spring 1993 EA-1A Solutions

- (9) The first step is to write the formula for  $\ddot{a}_{65}$  based on "first principles".

$$\ddot{a}_{65} = 1 + v p_{65} + v^2 {}_2p_{65} + \dots$$

You are given the general formula for  ${}_np_{65}$ :

$${}_1p_{65} = .90$$

$${}_2p_{65} = .90 - .02 = .88$$

$${}_3p_{65} = .90 - .04 = .86$$

$\vdots$

$${}_{44}p_{65} = .90 - .86 = .04$$

$${}_{45}p_{65} = .90 - .88 = .02$$

$${}_{46}p_{65} = .90 - .90 = -0-$$

$$\ddot{a}_{65} = 1 + \frac{.90}{(1.07)^1} + \frac{.88}{(1.07)^2} + \dots + \frac{.04}{(1.07)^{44}} + \frac{.02}{(1.07)^{45}}$$

$$\frac{\ddot{a}_{65}}{1.07} = \frac{1}{(1.07)^1} + \frac{.90}{(1.07)^2} + \dots + \frac{.04}{(1.07)^{44}} + \frac{.02}{(1.07)^{45}} + \frac{.02}{(1.07)^{46}}$$

$$\begin{aligned} \ddot{a}_{65} \left( \frac{.07}{1.07} \right) &= 1 - \frac{.10}{1.07} - \frac{.02}{(1.07)^2} - \dots - \frac{.02}{(1.07)^{44}} - \frac{.02}{(1.07)^{45}} - \frac{.02}{(1.07)^{46}} \\ &= 1 - \frac{.08}{1.07} - .02 \left[ \frac{1}{1.07} + \frac{1}{(1.07)^2} + \dots + \frac{1}{(1.07)^{46}} \right] \\ &= .9252 - .02 a_{\overline{46}|.07} \end{aligned}$$

$$\begin{aligned} \ddot{a}_{65} &= \frac{1.07}{.07} \left( .9252 - .02 (13.6500) \right) \\ &= 9.9699 \end{aligned}$$

(E)

# Spring 1993 EA-1A Solutions

- 10 First, you should write down expressions for the annuities for Smith and Brown:

$$\text{Smith - age 62 - PV} = 12K a_{62}^{(12)}$$

$$\text{Brown - age 45 - PV} = 12(800) a_{45:\overline{5}|}^{(12)}$$

Since Brown's present value equals 75% of Smith's, you have

$$12(800) a_{45:\overline{5}|}^{(12)} = .75(12)K a_{62}^{(12)}$$

Since  $\ddot{a}_x^{(12)} = \ddot{a}_x - \frac{11}{24}$ , you should use  $a_x^{(12)} = \ddot{a}_x - \frac{13}{24}$   
and  $\ddot{a}_x^{(12)} - \frac{1}{12} = a_x^{(12)}$

$$800 \left( \frac{N_{45} - \frac{13}{24} D_{45} - N_{50} + \frac{13}{24} D_{50}}{D_{45}} \right) = .75 \left( \frac{N_{62} - \frac{13}{24} D_{62}}{D_{62}} \right) K$$

$$K = \frac{800}{.75} \left( \frac{D_{62}}{D_{45}} \right) \left( \frac{N_{45} - \frac{13}{24} D_{45} - N_{50} + \frac{13}{24} D_{50}}{N_{62} - \frac{13}{24} D_{62}} \right)$$

$$\begin{array}{lll} D_{45} = N_{45} - N_{46} & D_{50} = N_{50} - N_{51} & D_{62} = N_{62} - N_{63} \\ = 5,691 - 5,245 & = 3,752 - 3,442 & = 1,206 - 1,084 \\ = 446 & = 310 & = 122 \end{array}$$

$$\begin{aligned} K &= 1066.67 \left( \frac{122}{446} \right) \left( \frac{5691 - \frac{13}{24}(446) - 3752 + \frac{13}{24}(310)}{1,206 - \frac{13}{24}(122)} \right) \\ &= 477.46 \end{aligned}$$

(B)

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Solution added 2/23/99

- 11 In this problem, the key to solution is knowing an expression for the reserve. This page will use the prospective reserve definition to get the answer:

$${}_tV_x = PV(\text{Future benefits}) - PV(\text{Future premiums})$$

$${}_2V_{40} = 1000 A_{42} - 1000 P_{40} \ddot{a}_{42}$$

$$= 1000 (A_{42} - P_{40} \ddot{a}_{42})$$

Now use some insurance identities

$$A_x = 1 - d \ddot{a}_x \quad P_x = (A_x / \ddot{a}_x) = (1 / \ddot{a}_x) - d$$

$${}_2V_{40} = 1000 (1 - d \ddot{a}_{42} - \ddot{a}_{42} (1 / \ddot{a}_{40} - d))$$

$$= 1000 (1 - d \ddot{a}_{42} - \ddot{a}_{42} / \ddot{a}_{40} + d \ddot{a}_{42})$$

$$= 1000 (1 - \ddot{a}_{42} / \ddot{a}_{40})$$

You are given a value for  $\ddot{a}_{40}$ , and you can use another identity to calculate the value of  $\ddot{a}_{42}$ :

$$a_x = v P_x \ddot{a}_{x+1} \quad \ddot{a}_x = 1 + v P_x \ddot{a}_{x+1}$$

$$\ddot{a}_{40} = 1 + v P_{40} \ddot{a}_{41} \quad \ddot{a}_{41} = 1 + v P_{41} \ddot{a}_{42}$$

First you must use  $\ddot{a}_{40}$  and  $q_{40}$  to derive  $\ddot{a}_{41}$ , then use  $\ddot{a}_{41}$  and  $q_{41}$  to derive  $\ddot{a}_{42}$ .

$$\ddot{a}_{41} = (\ddot{a}_{40} - 1.0)(1+i) / p_{40}$$

$$= (13.37 - 1.0)(1.07) / (1 - .0021)$$

$$= (12.37)(1.07) / .9979$$

$$= 13.2638$$

$$\ddot{a}_{42} = (\ddot{a}_{41} - 1.0)(1+i) / p_{41}$$

$$= (13.2638 - 1.0)(1.07) / (1 - .0023)$$

$$= 12.2638(1.07) / .9977$$

$$= 13.1525$$

Now, substitute this value in the original expression

$${}_2V_{40} = 1000 (1 - \ddot{a}_{42} / \ddot{a}_{40})$$

$$= 1000 (1 - 13.1525 / 13.37)$$

$$= 16.27$$

(C)

# Spring 1993 EA-1A Solutions

- (11) The definition of the net level premium reserve is that it equals the PV of future death benefits less the PV of future premiums.

At issue age 40,  $PVFB - PVFP = \text{zero}$ .  
At age 42, we have

$$PVFB = (A_{42}) 1000$$

$$PVFP = \left( 1000 \frac{A_{40}}{\ddot{a}_{40}} \right) \ddot{a}_{42}$$

An easier method of solution would use the retrospective definition, which defines the reserve as the accumulated value of past premiums less the accumulated value of past death benefits:

$${}_2V_{40} = \left( \frac{(1+i)^2}{{}_2p_{40}} + \frac{1+i}{p_{41}} \right) \text{Premium} - 1000 \left( (1+i) {}_6q_{40} + {}_6p_{40} q_{41} \right)$$

$$\begin{aligned} \text{Premium} &= 1000 \frac{A_{40}}{\ddot{a}_{40}} = 1000 \frac{(1-d) \ddot{a}_{40}}{\ddot{a}_{40}} \\ &= 1000 \frac{(1-(.07/1.07) 13.37)}{13.37} \\ &= 9.37 \end{aligned}$$

$$\begin{aligned} {}_2V_{40} &= \left( \frac{(1.07)^2}{(.9979)(.9977)} + \frac{1.07}{.9977} \right) 9.37 - 1000 \left( (1.07)(.0021) + .9979(.0023) \right) \\ &= 20.83 - 4.54 \\ &= 16.29 \end{aligned}$$

(C)

# Spring 1993 EA-1A Solutions

- 12 The first step is to write down an expression for the present value of each of the annuities:

$$A: 12(1000) \ddot{a}_X^{(12)}$$

$$B: 12(750) \ddot{a}_X^{(12)} + 12(500) (\ddot{a}_Y^{(12)} - \ddot{a}_{XY}^{(12)})$$

$$C: 12(K) \ddot{a}_X^{(12)} + 12(.75K) (\ddot{a}_Y^{(12)} - \ddot{a}_{XY}^{(12)})$$

To solve these equations more easily, divide all three of them by 12, and replace the annuity  $\ddot{a}_X^{(12)}$  by  $X$ , and replace the annuity  $\ddot{a}_Y^{(12)} - \ddot{a}_{XY}^{(12)}$  by  $Y$ :

$$1000 X = 750 X + 500 Y$$

$$1000 X = K(X) + .75K(Y)$$

Use substitution on the first equation to express  $Y$  in terms of  $X$ :

$$250 X = 500 Y \quad \therefore Y = .5X$$

$$1000 X = K(X) + .75K(.5X)$$

$$1000 = K + .375K \quad (\text{divide through by } X) \\ = 1.375K$$

$$\therefore K = 727.27$$

©

# Spring 1993 EA-1A Solutions

- 13 At age 55, the present value of future benefits equals the present value of future premiums. The annuitant will pay premiums for 10 years, then start receiving benefits at age 65 for 10 years certain, and life thereafter

$$P \cdot \ddot{a}_{55:\overline{10}|} = 12(300)_{10} | \ddot{a}_{55:\overline{10}|}^{(12)}$$

$$P \left( \frac{N_{55} - N_{65}}{D_{55}} \right) = 3600 \left[ \left( \frac{N_{75}^{(12)}}{D_{55}} \right) + \ddot{a}_{\overline{10}|}^{(12)} \left( \frac{D_{65}}{D_{55}} \right) \right]$$

Since  $N_{75}^{(12)} = N_{75} - \frac{11}{24}D_{75}$ , you can plug in all the commutation functions to calculate P:

$$P = \frac{3600 (N_{75}^{(12)} + D_{65} \ddot{a}_{\overline{10}|}^{(12)})}{N_{55} - N_{65}}$$

$$N_{75}^{(12)} = 247 - \frac{11}{24}(36) = 230.50$$

$$\begin{aligned} \ddot{a}_{\overline{10}|}^{(12)} &= (1+i)^{\frac{1}{12}} \frac{(1-v^{10})}{i^{(12)}} \\ &= 1.0057 \left( \frac{1 - .5083}{.0678} \right) \\ &= 7.2871 \end{aligned}$$

$$\begin{aligned} \left[ 1 + \frac{i^{(12)}}{12} \right]^{12} &= 1+i \\ i^{(12)} &= \left[ (1+i)^{\frac{1}{12}} - 1 \right] 12 \\ &= 6.78\% \end{aligned}$$

$$\begin{aligned} P &= \frac{3600 (230.50 + 100(7.2871))}{2547 - 919} \\ &= 2121 \end{aligned}$$

(D)



# Spring 1993 EA-1A Solutions

- 14 The symbol  $A'_{30:\overline{30}|}$  is a term insurance for a life age 30, for 30 years. You must derive a value for this based on the insurance functions you are given:

$$A'_{30:\overline{30}|} = \frac{M_{30} - M_{60}}{D_{30}}$$

Given  $A_{30:\overline{30}|} = \frac{M_{30} - M_{60} + D_{60}}{D_{30}} = .2$  (endowment insurance)

Given  $A_{60} = \frac{M_{60}}{D_{60}} = .4$

Given  $A_{30} = \frac{M_{30}}{D_{30}} = .1$

$M_{60} = .4 D_{60}$  based on  $A_{60}$  definition

$$\begin{aligned} A_{30:\overline{30}|} &= .2 = \frac{M_{30}}{D_{30}} - \frac{M_{60}}{D_{30}} + \frac{D_{60}}{D_{30}} \\ &= .2 = .1 - \frac{.4 D_{60}}{D_{30}} + \frac{D_{60}}{D_{30}} \\ \therefore .1 &= .6 \frac{D_{60}}{D_{30}} \quad \frac{D_{60}}{D_{30}} = \frac{1}{6} = .1667 \end{aligned}$$

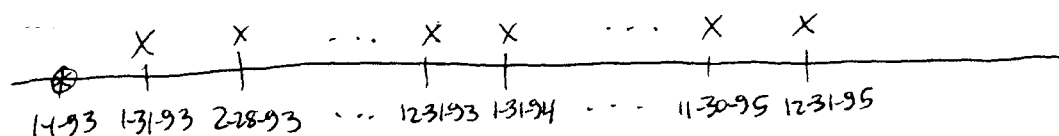
$$\begin{aligned} A_{30:\overline{30}|} &= .2 = \frac{M_{30}}{D_{30}} - \frac{M_{60}}{D_{30}} + \frac{D_{60}}{D_{30}} \\ &= .2 = .1 - \frac{M_{60}}{D_{30}} + .1667 \end{aligned}$$

$$\therefore \frac{M_{60}}{D_{30}} = .0667$$

$$\therefore A'_{30:\overline{30}|} = \frac{M_{30}}{D_{30}} - \frac{M_{60}}{D_{30}} = .1 - .0667 = .0333 \quad \textcircled{B}$$

# Spring 1993 EA-1A Solutions

- 15 To solve this problem, you must set up the amortization schedule for the loan. Here is the time diagram for the 36 payments:



The Present value of payments at 1-1-93 is the original loan amount,  $= X(a_{\overline{36}|.01})$ .

The amortization schedule allows you to identify the portion of each payment attributable to interest and principal:

<u>Time</u>	<u>Amount of Payment</u>	<u>Interest Repaid</u>	<u>Principal Repaid</u>	<u>Outstanding Principal</u>
0				$X(a_{\overline{36} .01})$
1	X	$X(1-v^{36})$	$X \cdot v^{36}$	$X(a_{\overline{35} .01})$
2	X	$X(1-v^{35})$	$X \cdot v^{35}$	$X(a_{\overline{34} .01})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
12	X	$X(1-v^{25})$	$X \cdot v^{25}$	$X(a_{\overline{24} .01})$

$$\begin{aligned}
 \text{Principal repaid in 1993} &= 3,000 = X(v^{36} + v^{35} + \dots + v^{25})v \\
 &= Xv^{25}(\ddot{a}_{\overline{12}|.01}) \\
 X &= \frac{3000}{.7798(11.3676)} \\
 &= 338.43
 \end{aligned}$$

$$\begin{aligned}
 \text{Original loan amount} &= X(a_{\overline{36}|.01}) \\
 &= 338.43(30.1078) \\
 &= 10,189
 \end{aligned}$$



# Spring 1993 EA-IA Solutions

- 16 You are given numerous details about the life annuity policy issued at age 50. The problem requires you to calculate the net single premium for an insurance policy that is issued at age 51 at 1-1-94.

you must use the identity that relates insurance and annuity benefits:

$$A_x = 1 - d \cdot \ddot{a}_x$$

$$134,400 = 12,000 a_{50} \quad a_{50} = 11.20$$

$$A_{51} = 1 - d(\ddot{a}_{51}) = 1 - (0.07/1.07)\ddot{a}_{51}$$

$$a_{50} = v p_{50} + v^2 {}_2p_{50} + \dots$$

$$a_{51} = v p_{51} + v^2 {}_2p_{51} + \dots$$

$${}_1p_{50} a_{51} = v {}_2p_{50} + v^2 {}_3p_{50} + \dots$$

$${}_1p_{50} \ddot{a}_{51} = {}_1p_{50} + v({}_2p_{50}) + v^2({}_3p_{50})$$

$$v {}_1p_{50} \ddot{a}_{51} = v {}_1p_{50} + v^2 {}_2p_{50} + v^3 {}_3p_{50} \\ = a_{50}$$

$$\therefore \ddot{a}_{51} = (1+i) a_{50} / {}_1p_{50} = (1+i) a_{50} / (1 - q_{50}) \\ = (1.07)(11.20) / .99 \\ = 12.1051$$

$$\therefore A_{51} = 1 - (0.07/1.07) 12.1051 \\ = .2081$$

$$100,000 A_{51} = 20,808$$

(D)

# Spring 1993 EA-1A Solutions

- 17 As a first step, you should draw a time diagram for each of the series of annuity payments:

$$A \Rightarrow \begin{array}{ccccccc} 5,000 & 5,000(1.07) & 5,000(1.07)^2 & \dots \\ \text{---} & \text{---} & \text{---} & \text{---} \\ 1/93 & 1/94 & 1/95 & \dots \\ 60 & 61 & 62 & \dots \end{array}$$

$$B \Rightarrow \begin{array}{ccccccc} & & & & K & K(1.07) & K(1.07)^2 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1/93 & 1/94 & 1/95 & \dots & 1/98 & 1/99 & 1/00 \\ 60 & & & & 65 & 66 & 67 \end{array}$$

The key to working this problem is that the interest rate is 7%, and it will cancel out the 7% increases in benefits:

$$\begin{aligned} PV(A) &= 5,000 (1 + 1.07 v p_{60} + (1.07)^2 v^2 p_{60} + \dots) \\ &= 5,000 (1 + p_{60} + {}_2p_{60} + \dots) \\ &= 5,000 (1 + e_{60}) = 5,000 (16.5) = 82,500 \end{aligned}$$

$$\begin{aligned} PV(B) &= \frac{D_{65}}{D_{60}} (K) (1 + 1.07 v p_{65} + (1.07)^2 v^2 p_{65} + \dots) \\ &= \frac{D_{65}}{D_{60}} (K) (1 + e_{65}) \\ &= \frac{868-774}{1484-1339} (K) (13.9) = 9.01K \end{aligned}$$

Since  $PV(A) = PV(B)$  you can solve for  $K$

$$\begin{aligned} 9.01K &= 82,500 \\ \therefore K &= 9,155 \end{aligned}$$

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## Spring 1993 EA-1A Solutions

18 The stationary population based on 12,500 births is  $12,500 \left( \frac{T_0}{l_0} \right) = 600,000$

$$\therefore \frac{T_0}{l_0} = \frac{600,000}{12,500} = 48 = e.$$

The proportion of the population under age  $x$  is

$\frac{T_0 - T_x}{T_0}$  by definition. You are given that

$$\frac{T_0 - T_{20}}{T_0} = .35 \quad \therefore \frac{T_{20}}{T_0} = .65 \quad \therefore \frac{T_{20}}{l_0} = .65(48) = 31.20$$

If the birth rate is increased to 13,000<sup>at 1-1-70</sup>, then the total population under age 20 at 1-1-90 is

$$\begin{aligned} 13,000 \left( \frac{T_0 - T_{20}}{l_0} \right) &= 13,000 \left( \frac{T_0}{l_0} - \frac{T_{20}}{l_0} \right) \\ &= 13,000 (48 - 31.20) \\ &= 218,400 \end{aligned}$$

The population over age 20 at 1-1-90 is based on the original 12,500 birth rate:

$$\begin{aligned} 12,500 \left( \frac{T_{20}}{l_0} \right) &= 12,500 (31.20) \\ &= 390,000 \end{aligned}$$

The total stationary population is 608,400 (C)

# Spring 1993 EA-1A Solutions

- 19) This question is similar to one that appeared on a prior exam. The key to the problem is knowing how to work with reversionary annuities.

The answer is the present value of the annuities that would be paid if  $x$  is the only survivor, plus the annuity for  $x$  as 1 of exactly two survivors, plus the annuity for  $x$  as one of exactly three survivors:

12,000  $a_x - a_{x:\overline{y}z}$  is the annuity payable to  $x$  after the death of the last of  $y$  and  $z$ .

6,000  $a_{xy} - a_{xy:z}$  plus  
6,000  $a_{xz} - a_{xz:y}$  are the annuities payable to  $x$  after the death of  $y$  or  $z$ , with exactly two survivors.

4,000  $a_{xyz}$  is the annuity payable for exactly three survivors. Now you must plug in the annuity payments and PV factors:

$$12,000(a_x - a_{x:\overline{y}z}) = [a_x - (a_{xy} + a_{xz} - a_{xyz})] 12,000 \\ = 12,000(13.18 - 12.16 + 11.78 - 11.06) = 3,600$$

$$6,000(a_{xy} + a_{xz} - 2a_{xyz}) = 6,000(12.16 + 11.78 - 2(11.06)) \\ = 10,920$$

$$4,000 a_{xyz} = 4,000(11.06) = 44,240$$

$$\text{Sum of all three} = 3,600 + 10,920 + 44,240 = 58,760 \quad \textcircled{B}$$

# Spring 1993 EA-1A Solutions

- 20 There are two ways to work this problem. The quickest way is to use the standard approximation:

$$q'_x^{(1)} \doteq \frac{q_x^{(1)}}{1 - \frac{1}{2} q_x^{(2)}}$$

By definition,  $z p_{20}'^{(d)} = \left[ p_{20}'^{(d)} \right] \left[ p_{21}'^{(d)} \right]$   
 $= \left[ 1 - q_{20}'^{(d)} \right] \left[ 1 - q_{21}'^{(d)} \right]$

$$q_{20}'^{(d)} \doteq q_{20}^{(d)} / \left[ 1 - \frac{1}{2} q_{20}^{(u)} \right] = .001311 / (1 - \frac{1}{2}(.05))$$

$$= .001345$$

$$q_{21}'^{(d)} \doteq q_{21}^{(d)} / \left[ 1 - \frac{1}{2} q_{20}^{(u)} \right] = .001267 / (1 - \frac{1}{2}(.05))$$

$$= .001299$$

$$z p_{20}'^{(d)} \doteq (1 - .001345)(1 - .001299) = .998655(.998701)$$

$$= .997358$$

(C)

An alternative calculation method can be used to check the arithmetic in this solution. Bowers formula is not an approximation:

$$p'_x^{(1)} = \left[ p_x^{(T)} \right] \frac{q_x^{(1)}}{q_x^{(T)}}$$

$$p_{20}'^{(d)} = \left[ p_{20}^{(T)} \right] \frac{q_{20}^{(d)}}{q_{20}^{(T)}} = (1 - .001311 - .05) \frac{.001311}{.051311} = .998655$$

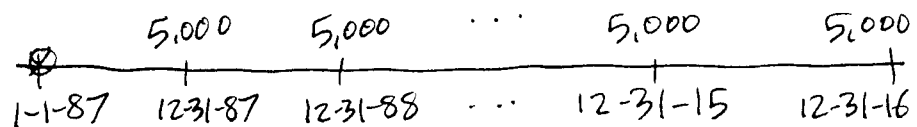
$$p_{21}'^{(d)} = \left[ p_{21}^{(T)} \right] \frac{q_{21}^{(d)}}{q_{20}^{(T)}} = (1 - .001267 - .05) \frac{.001267}{.051267} = .998700$$

$$\therefore z p_{20}'^{(d)} = (.998655)(.998700) = .997357$$

## Spring 1993 EA-1A Solutions

- 21 The key to this problem is to not do unnecessary calculations. You should keep writing down representations and formulas based on the information.

The original 30 year loan looks like this:



The original loan balance is 5,000 at 7%.  
 At 1-1-93, there have been six loan payments (12-31-87 through 12-31-92), and the outstanding loan balance is 5,000 at 7%.

With the additional 15,000 loan, the renegotiated loan balance can be calculated:

$$\text{New loan} = 15,000 + 5,000 \text{ at } 7\% = 72,347$$

$$\text{The new loan payment is } 72,347 / a_{\overline{18}|7\%} = 7,192$$

Here is a partial amortization schedule

<u>Date</u>	<u>Payment</u>	<u>Interest</u>	<u>Principal</u>	<u>O/S loan balance</u>
1-1-93				7192 ( $a_{\overline{18} 7\%}$ )
12-31-93	7192	$7192(1-v)^{18}$	$7192v^{18}$	7192 ( $a_{\overline{17} 7\%}$ )
12-31-94	7192	$7192(1-v)^{17}$		
		= 4,915		

The interest in the 12-31-94 payment is 4,915

(B)



# Spring 1993 EA-1A Solutions

22 You can write down the values for  $q_{107}$ ,  $q_{108}$  and  $q_{109}$  easily:

$x$	$l_x$	$d_x$	$q_x$
107	—	—	.30 given
108	200	100	.50
109	100	100	1.00
110	0	—	

Now you can calculate  $A_{107}$  using the expression in terms of  $q_x$ :

$$\begin{aligned}
 A_{107} &= v q_{107} + v^2 {}_1p_{107} q_{107} + v^3 {}_2p_{107} q_{107} + \dots \\
 &= \frac{.30}{1.08} + \frac{.70(.50)}{(1.08)^2} + \frac{.70(.50)1.0}{(1.08)^3} + \dots \\
 &= .27778 + .3001 + .2778 \\
 &= .8557
 \end{aligned}$$

(B)

## Spring 1993 EA-1A Solutions

- 23 This is an easy problem if you remember that the premiums are paid monthly. You must calculate the monthly life annuity at age 50 using the standard approximation:

$$(12P) \ddot{a}_{50}^{(12)} = 12000 A_{50}$$

$$\ddot{a}_{50}^{(12)} = \ddot{a}_{50} - \frac{11}{24} = \frac{3,752,218}{310,647} - \frac{11}{24} = 11.6204$$

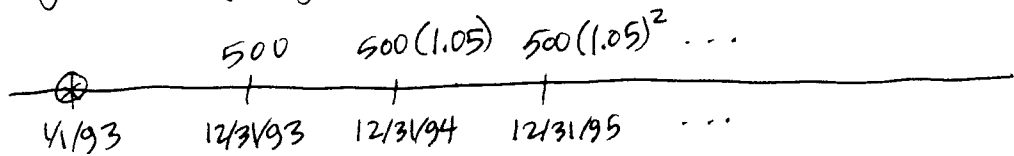
$$A_{50} = 1 - d \ddot{a}_{50} = 1 - \frac{.07}{1.07} \left( \frac{3,752,218}{310,647} \right) = .2098$$

$$P = \frac{1000 A_{50}}{\ddot{a}_{50}^{(12)}} = \frac{1000(.2098)}{11.6204} = 18.05$$

(E)

# Spring 1993 EA-1A Solutions

24 The first step is writing a diagram of the payment amounts



$$PV \text{ at } 1/1/93 = P = v \cdot 500 + v^2(500)(1.05) + v^3(500)(1.05)^2 + \dots$$

$$v(1.05)P = v^2(500)(1.05) + v^3(500)(1.05)^2 + \dots$$

$$P - v(1.05)P = v \cdot 500$$

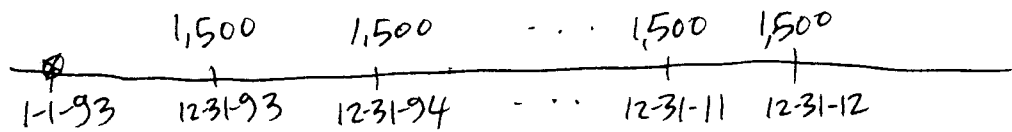
$$P = \frac{v \cdot 500}{1 - v(1.05)} = \frac{500}{1.08 - 1.05} = 16,667$$

(D)

The answer is identical to the value of a perpetuity of 500 per year at an interest rate of 3% per annum. 3% is equal to the difference between the rate of interest and the rate of increase in payments.

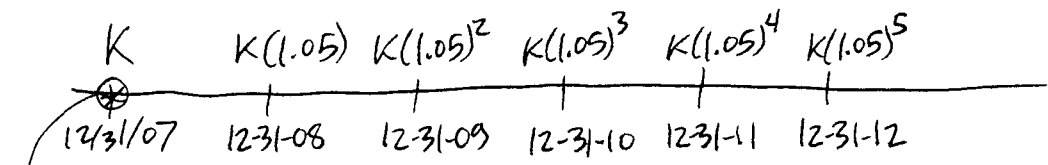
# Spring 1993 EA-1A Solutions

25 Here is a diagram of the original loan:



The original loan balance at 1-1-93 is 1,500 ( $\ddot{a}_{\overline{20}|.07}$ ). Immediately prior to the 12-31-07 loan payment, the outstanding loan balance is  $1,500 (\ddot{a}_{\overline{6}|.07}) = 7,650$ .

Here is a diagram of the renegotiated loan:



$$\begin{aligned} PV = 7,650 &= K + vK(1.05) + v^2K(1.05)^2 + \dots + v^5K(1.05)^5 \\ &= K \left[ 1 + \frac{1.05}{1.07} + \left(\frac{1.05}{1.07}\right)^2 + \dots + \left(\frac{1.05}{1.07}\right)^5 \right] \\ &= K \ddot{a}_{\overline{6}|j} \quad \text{where } j = \frac{1.07 - 1}{1.05} = 1.90\% \end{aligned}$$

$$K = \frac{7,650}{\ddot{a}_{\overline{6}|1.90\%}} = 1,335.94$$

The payment due at 12-31-12 is  $K(1.05)^5$   
 $= 1,335.94(1.05)^5$   
 $= 1,705$

(C)