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# SPRING 1999 EA-1A EXAM SOLUTIONS

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Revision History:

12/09/02      Corrected problem 14      Alternate solution was incorrect

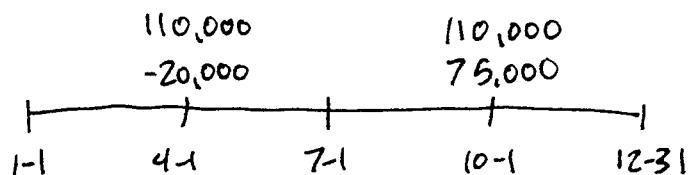
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Spring 1999 EA-1A

- | This is a typical problem involving time weighted and dollar weighted rates of return. The key to the problem is writing down the cash flows and market values on a time line. You need to develop the market values before and after the cash flows to calculate the time weighted return:

Before cashflows

Cash flows



After cash flows    100,000    90,000    95,000    185,000    180,000

$$\text{Time weighted : } 1+t = \frac{110,000}{100,000} \left( \frac{110,000}{90,000} \right) \left( \frac{180,000}{185,000} \right) = 1.308 \quad t = 30.8\%$$

3mos      6mos      3mos

For the dollar weighted return, the total interest earned during the year is the difference between beginning and ending market value and the cash flows:

$$\begin{aligned} \text{Total interest} &= 180,000 - (100,000 + 75,000 - 20,000) = 25,000 \\ &= 100,000 \left(\frac{4}{4}\right)d - 20,000 \left(\frac{3}{4}\right)d + 75,000 \left(\frac{1}{4}\right)d \end{aligned}$$

Rewrite this as an exposure formula

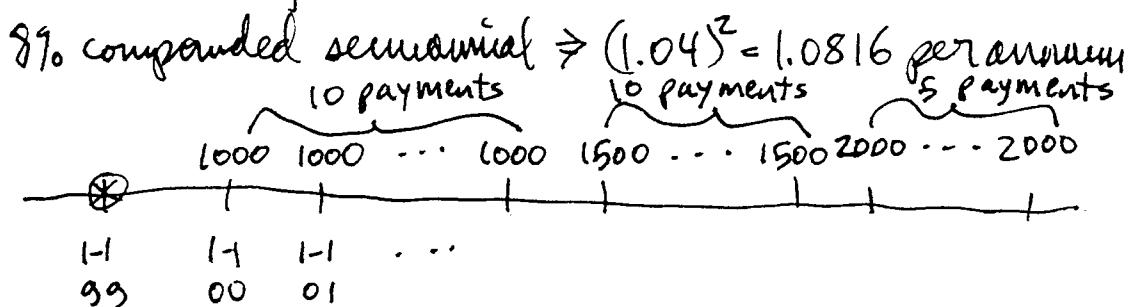
$$\begin{aligned} d &= \frac{25,000}{100,000 \left(\frac{4}{4}\right) - 20,000 \left(\frac{3}{4}\right) + 75,000 \left(\frac{1}{4}\right)} \\ &= 25,000 / (100,000 - 15,000 + 18,750) = 24.1\% \end{aligned}$$

If you assume mid-year cash flows, then the weights on both cash flows are  $\frac{1}{2}$ :

$$\begin{aligned} m &= \frac{25,000}{100,000 \left(\frac{2}{2}\right) - 20,000 \left(\frac{1}{2}\right) + 75,000 \left(\frac{1}{2}\right)} \\ &= 19.6\% \quad \therefore t > d > m \quad \text{(B)} \end{aligned}$$

Spring 1999 EA-IA

- 2 The key to working this problem, and many others, is to write down all payments on a time line. In addition, you need to convert the interest rate so it is convertible per payment period of the annuity.



There are several ways to value the annuity. The most direct approach is to value each of the three annuities, and discount back to 1-1999:

$$PV = 1000 a_{\overline{10}|8.16\%} + v^{10} (1500 a_{\overline{10}|8.16\%}) + v^{20} (2000 a_{\overline{5}|8.16\%})$$

The technique which requires the fewest calculations looks at the payments as a series of layers "carved out" of an ultimate 2000 payment:

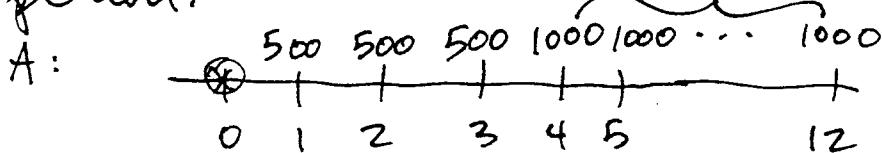
$$\begin{aligned} PV &= 2000 a_{\overline{25}|8.16\%} - 500 a_{\overline{20}|8.16\%} - 500 a_{\overline{10}|8.16\%} \\ &= 2000(10.5305) - 500(9.7023) - 500(6.6619) \\ &= 12,879 \end{aligned}$$

(B)

You will get exactly the same answer using the other expression shown earlier. One thing I prefer to use is immediate annuities for all solutions on the EA-IA exam. Since I use an HP-12C, this keeps me from changing the annuity due/annuity immediate setting, which helps prevent careless errors.

Spring 1999 EA-1A

- 3 The key to this problem is writing down the payments on a time line, and converting the interest rate to a compounding period equal to the annuity payment period.

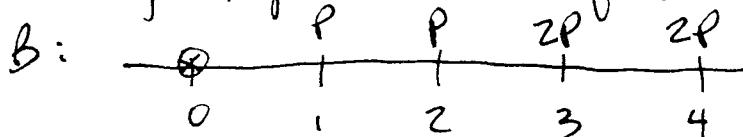


The interest rate is  $.08/12$  per month, or  $.6667\%$ /mo. You can calculate the present value as either the sum of two annuities, or the difference:

$$\begin{aligned}
 PV &= 500 a_{\overline{3}|.6667\%} + v^3 (1000) a_{\overline{9}|.6667\%} \quad \text{add layers,} \\
 &= 1000 a_{\overline{12}|.6667\%} - 500 a_{\overline{3}|.6667\%} \quad \text{or subtract them} \\
 &= 1000(11.4958) - 500(2.9604) \\
 &= 10,016
 \end{aligned}$$

Since the annuities are actuarially equivalent, the present value of annuity B is also equal to 10,016.

Once you write the payments on a time line and write the expression for the present value of annuity B, you can solve for the payment  $P$ :



On a quarterly basis, the interest rate is  $(1.00667)^3 = 1.0201$

$$\begin{aligned}
 PV &= P a_{\overline{4}|1.0201} + v^2 P a_{\overline{2}|1.0201} \quad \text{add layers,} \\
 &= 2P a_{\overline{4}|1.0201} - P a_{\overline{2}|1.0201} \quad \text{or subtract them} \\
 &= 2P(3.8065) - P(1.9412) \\
 &= 5.6718 P \\
 &= 10,016 \quad \therefore P = 10,016 / 5.6718 \\
 &\qquad\qquad\qquad = 1,766 \quad \textcircled{A}
 \end{aligned}$$

Spring 1999 EA-1A

- 4 Modified duration is defined as regular duration divided by  $1+i$ . The regular duration is the weighted average year of payment where the weight is the present value of the payment:

$$\text{modified duration} = \frac{\bar{d}}{1+i} \quad \text{where } \bar{d} = \frac{\sum_{t=1}^n t V^t R_t}{\sum_{t=1}^n V^t R_t}$$

The interest rate is convertible monthly as  $\frac{10.8\%}{12} = .90\%/\text{month}$

Apply the duration formula to the sequence of 360 loan payments. If  $R$  is the level loan payment,

$$\text{modified duration} = \frac{R(v + 2v^2 + 3v^3 + \dots + 360v^{360})}{R(1.009)(v + v^2 + \dots + v^{360})}$$

To evaluate this, you must know (or derive) the formula for an increasing annuity:

$$Ia\bar{n}i = v + 2v^2 + \dots + nv^n = \frac{\ddot{a}\bar{n}i - nv^n}{i}$$

Use this to evaluate the numerator of the

$$\begin{aligned} \text{modified duration} &= \frac{\ddot{a}\bar{360}7.9\% - 360v^{360}}{(0.009)(1.009) \ddot{a}\bar{360}7.9\%} \\ &= \frac{1.009(106.6960) - 360(.0397)}{.009(1.009)(106.6960)} \\ &= 93.3515/.9689 \\ &= 96.35 \end{aligned}$$

(C)

Spring 1999 EA-1A

- 5 The key to this problem is writing down (very carefully) the information you are given on a time line. You also need to be able to calculate the time weighted and dollar weighted return values. This is the hardest problem on time / dollar weighted return in many years!

The difficult part of the problem is the definition of the cash flows for a quarter. You need to carefully put the benefit payments on the first day of a quarter, and the contributions on the last day:

| Contributions       |           | 30,000    | 85,000    | y         | 27,617 |
|---------------------|-----------|-----------|-----------|-----------|--------|
|                     | 1-1       | 3.31      | 6-30      | 9-30      | 12-31  |
| Ending market value | 1,750,000 | 1,775,626 | 1,907,549 | 1,937,033 | z      |
| Benefit Payments    | -22,000   | -43,000   | -27,994   | -39,228   |        |

To calculate the dollar weighted return for the year, you must develop values for both y and z. To do this, you'll calculate the time weighted returns for the third and fourth quarters. First, you can get the value of j because the time weighted return for a year is the product of the four quarterly returns:

$$1.0879 = (1.02)(1.0519)(1-.0226)(1+j)$$

$$1+j = 1.0879 / [1.0102(1.0519)(.9774)] \\ = 1.0475$$

Now you need to set up the formula for the 4th quarter time weighted return. This is based on the

Spring 1999 EA-1A

(5) asset growth between the cash flows:

$$1.0475 = \frac{\text{MVA before 12-31 cash flows}}{\text{MVA after 09-30 cash flows}}$$

$$= \frac{z - 27,617}{1,937,033 - 39,228}$$

$$z - 27,617 = 1,987,863$$

$$z = 2,015,480$$

Now set up a similar expression for the third quarter time weighted return, which allows you to calculate the value of  $y$ :

$$1.0226 = \frac{\text{MVA before 09-30 cash flows}}{\text{MVA after 06-30 cash flows}}$$

$$= \frac{1,937,033 - y}{1,907,549 - 27,994}$$

$$1,937,033 - y = 1,831,077$$

$$y = 99,956$$

Finally, you can set up the formula for the dollar weighted rate of return. Simply weight each net cash flow by the period remaining to the end of the year:

$$(1+i)(1,750,000 - 22,000) + (1+.75i)(30,000 - 43,000) + (1+.5i)(85,000 - 27,994)$$

$$+ (1+.25i)(99,956 - 39,228) + 27,617 = 2,015,480$$

$$1,987,863 = (1+i)(1,728,000) + (1+.75i)(-13,000) + (1+.5i)(57,006) + (1+.25i)(60,728)$$

$$i = \frac{155,129}{\frac{4}{4}(1,728,000) + \frac{3}{4}(-13,000) + \frac{2}{4}(57,006) + \frac{1}{4}(60,728)}$$

This formula shows the "exposure" approach for the net cash flows

$$i = 155,129 / 1,761,935 = 8.80\%$$

D

Spring 1999 EA-1A

- 6 The key to working this problem is writing down all the payments on a time line. You need to be careful about the interest rates which are stated as nominal annual rates either compounded monthly or compounded annually. You also need to be careful when you count the payments that you correctly determine the balance "just prior to the 53<sup>rd</sup> scheduled loan repayment".

| Savings pmts  | P     | P         | P         | P         |                   |
|---|-------|-----------|-----------|-----------|-------------------|
| loan pmts   | Q Q   | ... Q Q Q | ... Q Q Q | ... Q Q Q |                   |
|   | + + + | + + +     | + + +     | + + +     |                   |
| 1-1 2-1 3-1 ... 1-1 2-1 3-1 ... 1-1 2-1 3-1 ... 1-1 2-1 3-1 ... 1-1 2-1 3-1 4-1 5-1 6-1 |       |           |           |           |                   |
| 1999 2000 2001 2002 2003  |       |           |           |           |                   |
| payment number  | 1 2   | 12 13     | 24 25     | 36 37     | 48 49 50 51 52 53 |

6-1-03 is the date of the 53<sup>rd</sup> loan payment. The outstanding loan balance at that date is  $Q \ddot{a}_{\overline{20-52}} = Q \ddot{a}_{68}$ .

At 1-1-99, you have  $Q \dot{a}_{\overline{20}} = 10,000$ . The loan interest rate on a monthly basis is .5654%:

$$(1+j)^{12} = 1.07 \quad 1+j = (1.07)^{1/12} \quad j = (.07)^{1/12} - 1 = .005654$$

$$6-1-03 \text{ loanbalance} = Q \ddot{a}_{68.5654\%} = 10,000 \left( \frac{\ddot{a}_{68.5654}}{\dot{a}_{\overline{20}.5654}} \right)$$

$$= 6513.97 = 10,000 (1.005654)(56.323) / 86.9540$$

The savings account rates is 9% compounded monthly, on 9.38% per annum:  $(1+.09/12)^{12} = 1.0938$ . The value of the savings account at 6-1-03 is

$$(P 5479.38\%) (1.0075)^5 = P (4.5989) (1.0381) = P (4.7739)$$

This should pay off the loan, plus the 10% prepayment penalty

$$P (4.7739) = (1.10) (6513.97) \Rightarrow P = 1500.94$$

D

Spring 1999 EA-1A

- 7 The key point of this problem is that the total interest paid equals the total payments minus the principal payments of 100,000. You must determine how many fewer payments will be made due to the additional payment of 3000. You should also calculate the amount of the final payment, which is lower.

$P \ a_{360} | 1\% = 100,000 \Rightarrow P = 100,000 / 97.2183 = 1028.61 / \text{mo}$   
After paying this amount for five years (94 - 98) there are still 300 payments left ( $360 - 5 \times 12$ ).

$$1 - 1.01^5 \text{ o/s balance} = 1028.61 \ a_{300} | 0.01 = 97,663$$

$$\text{After paying } 3000, 1 - 1.01^5 \text{ o/s balance} = 94,663$$

Now solve for the number of remaining payments of 1028.61 using your calculator. The HP-12C only returns integer values for  $n$ , which is an overstatement: 255 for  $n$ , which corresponds to  $a_{n=1} = 94,663 / 1028.61 = 92.03$ . If I plug 255 in and calculate  $a_{255} | 1\% = 92.09$ , so the last payment will be lower.

Based on the extremely wide answer ranges, you can skip calculation of the final payment. You may be off by as much as 1028. Total payments =  $(255+60)1028.61 + 3000$ , or 327,013, which produces 227,013 in interest B

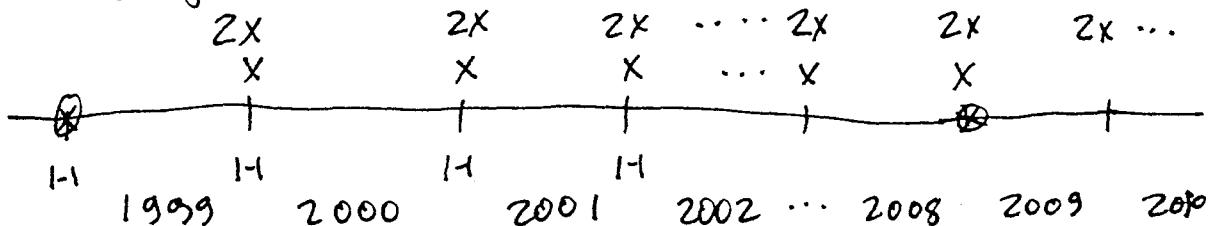
Or you can calculate the final payment

$$94,663 = 1028.61 (a_{254} | 0.01) + z (1.01)^{-255} \Rightarrow z = 215.36$$

$$\text{Total payments} = (254+60)1028.61 + 215.36 + 3000 = 326,200, \text{ of } 226,200. \text{ Interest}$$

Spring 1999 EA-1A

8 The key to working this problem is knowing the formulas for a perpetuity immediate, and for an accumulated immediate annuity. Here are the payments shown on a time line



$$1-1-99 \text{ Perpetuity: } P = 2X/i$$

$$1-1-99 \text{ Annuity: } P/2 = X s_{\bar{10}i}$$

$$2X/i = X s_{\bar{10}i} \quad \text{after substituting for } P$$

$$2/i = s_{\bar{10}i}$$

$$\frac{1}{i} = \frac{(1+i)^{10}-1}{i}$$

$$2 = (1+i)^{10}$$

$$1+i = 1.0718$$

(A)

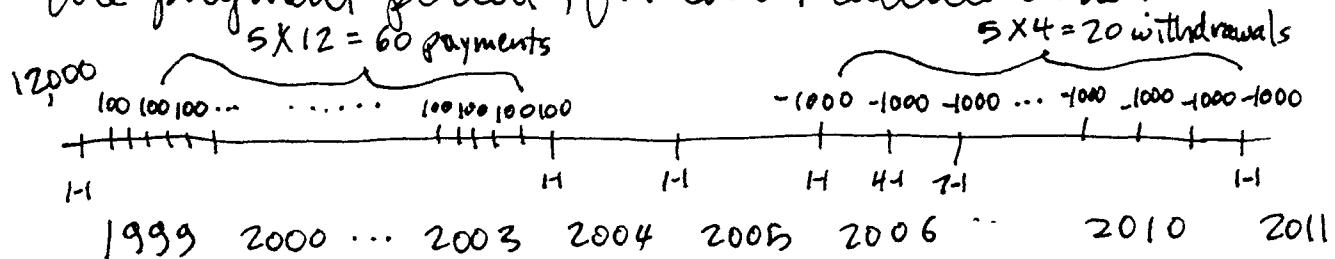
Another "method" would be to use trial and error with the answer ranges once you got down to  $\frac{1}{i} = s_{\bar{10}i}$ :  $i = \frac{1}{s_{\bar{10}i}}$

|       |               |                 |
|-------|---------------|-----------------|
| $i$   | $\frac{1}{i}$ | $s_{\bar{10}i}$ |
| 7.25% | 13.73         | 13.98           |

As  $i$  increases,  $\frac{1}{i}$  will decrease and  $s_{\bar{10}i}$  will increase, so the answer must be less than 7.25%.

Spring 1999 EA-1A

- 9 The key to working this problem is writing down all the deposits and withdrawals on a time line. You also should convert the interest rate to match the payment period, for easier calculations.



The interest rate of 8% per annum, compounded monthly equals  $.08/12 = .667\%$  per month. This will be used to accumulate the deposits:

$$\begin{aligned}
 \text{1-1-2011 accumulated value} &= 100 \sum_{n=1}^{84} 1.00667^n + 12000(1.00667)^{84} \\
 &= 7347.69(1.7474) + 12000(2.6034) \\
 &= 44,080
 \end{aligned}$$

For the withdrawals, the equivalent quarterly interest rate is  $(1.00667)^3 - 1 = 2.01\%$ . Now accumulate the withdrawals to the same date

$$\begin{aligned}
 \text{1-1-2011 accumulated value} &= 1000 \sum_{n=1}^{207} 1.0201^n \\
 &= 1000(1.0201)(24.3297) \\
 &= 24,820
 \end{aligned}$$

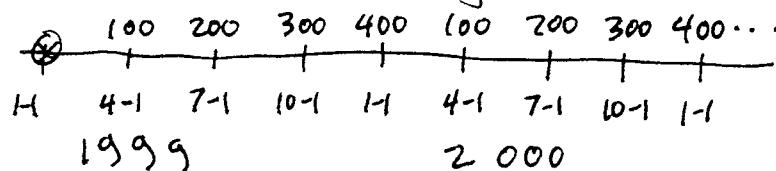
The fund balance is  $44,080 - 24,820 = 19,261$

(E)

Note that I use an HP-12C calculator, and always have it set to calculate annuity immediate values for EA-1A. That is why I multiplied by 1.0201 earlier:  $(1+i)\bar{s}_n = \bar{s}_{n+1}$

## Spring 1999 EA-1A

- 10 This is a typical perpetuity problem. The key to working it is writing down the payments on a time line, then deciding which method to use:



Method 1: The trickiest way to work this is to simply write down the formula based on first principles, and try to reduce it to simplest terms.

Method 2: Replace the four payments with a single payment on 1/1. Then calculate the PV of the annual perpetuity

Method 3: Consider the series of payments as 4 perpetuities. Calculate the PV of one of them, then all four together.

The first step is calculation of the equivalent quarterly rate of interest:  $(1+j)^4 = 1.10 \quad j = 2.411\%$

This would be used for method 1 as follows:

$$P = 100v + 200v^2 + 300v^3 + 400v^4 + 100v^5 + 200v^6 + 300v^7 + 400v^8 + \dots$$

$$v(P) = 100v^2 + 200v^3 + 300v^4 + 400v^5 + 100v^6 + 200v^7 + 300v^8 + \dots$$

$$P(1-v) = (100v + 100v^2 + 100v^3 + 100v^4 - 300v^5 + 100v^6 + 100v^7 + 100v^8 - 300v^9 + \dots)$$

$$= 100(v + v^2 + \dots) - 400(v^5 + v^9 + \dots)$$

= perpetuity immediate for 100 per quarter at 2.411%

+ perpetuity immediate for 400v per annum at 10.0%

$$P(1-0.9765) = \frac{100}{0.02411} - \frac{400(0.9765)}{0.10}$$

$$P(.0235) = 4147 - 3906$$

$$P = 10,244$$

(B)

Spring 1999 EA-1A

- (10) For method 2, replace the four payments with one:

~~$\frac{Q}{d}$~~        $\frac{Q}{d}$       ...

1-1-99      1-1-2000

$$P = \frac{Q}{d} = \frac{Q}{.10/1.10}$$

$$\begin{aligned} Q &= 100(1.10)^{\frac{1}{4}} + 200(1.10)^{\frac{2}{4}} + 300(1.10)^{\frac{-3}{4}} + 400(1.10)^{\frac{-4}{4}} \\ &= 97.65 + 190.69 + 279.30 + 363.64 \\ &= 931.28 \end{aligned}$$

$$P = \frac{931.28}{.10(.9091)} = 10,244$$

For method 3, calculate the PV of the annual payments of 100 at 4-1-99. The PV of the payments of 200 at 7-1-99 will be twice as large, and so on. Then you calculate the PV of all four perpetuities at 1-1-99:

~~$\frac{100}{d}$~~        $\frac{100}{d}$       ...

1-1      4-1      1-1      4-1

$$4-1 \quad PV = \frac{100}{d} = \frac{100}{.10/1.10} = 1100 \quad \text{for one perpetuity}$$

$$\begin{aligned} 1-1 \quad PV \text{ of all four} &= 1100(1.10)^{\frac{-1}{4}} + 2(1100)(1.10)^{\frac{-3}{4}} + 3(1100)(1.10)^{\frac{-5}{4}} + 4(1100)(1.10)^{\frac{-6}{4}} \\ &= 1074 + 2098 + 3072 + 4000 \\ &= 10,244 \end{aligned}$$

The point of this demonstration is that any method will produce the correct answer. Methods 2 and 3 may be more straightforward in application than method 1.

Spring 1999 EA-1A

- 11 The key to working this problem is handling the reinvestment and yield rates correctly. The bond pays 3% coupons every six months, which are invested to earn 2.5% every six months. You need to accumulate these values to the bond's redemption date of 1-1-2009. Then you should discount everything back to 1-1-1999 at the 2.75% semiannual yield rate to get the price of the bond.

This is a 10 year bond with 20 coupons. If the coupons were reinvested at the yield rate, you can calculate the price using the standard formula:

$$\begin{aligned} \text{1-1-99 price} &= F + \frac{C}{r} [1 - (1 + r)^{-n}] \\ &= 100,000 + \frac{3,000}{0.0275} [1 - (1 + 0.0275)^{-20}] \end{aligned}$$

Since the coupons do not earn 2.75%, in this problem you need to accumulate everything to the redemption date, then discount it back:

$$\begin{aligned} \text{1-1-2009 value} &= 100,000 + 100,000(0.03) S_{20|2.5\%} \\ &= 100,000 + 3,000(25.5447) \\ &= 176,634 \end{aligned}$$

$$\begin{aligned} \text{1-1-1999 value} &= 176,634 (1.0275)^{-20} \\ &= 102,669 \end{aligned}$$

(D)

Spring 1999 EA-1A

- 12 The key to working this problem is knowing how to use identities to solve for the value of  $y$ . You are given information on the premium for a whole life policy:

$$\text{Annual Premium} = \frac{45}{1000} = \frac{A_{30}}{\ddot{a}_{30}}$$

$$\text{Single premium} = 1000 A_{30} = Y$$

The most useful identity is  $A_x = 1 - d \ddot{a}_x$  in this problem

$$\begin{aligned} .045 &= \frac{A_{30}}{\ddot{a}_{30}} = \frac{1 - d \ddot{a}_{30}}{\ddot{a}_{30}} = \frac{1}{\ddot{a}_{30}} - d \\ \frac{1}{\ddot{a}_{30}} &= .045 + d \\ \ddot{a}_{30} &= (.045 + .06/1.06)^{-1} \\ &= 9.8422 \end{aligned}$$

Now you can substitute for the value of  $Y$

$$\begin{aligned} Y &= 1000 A_{30} \\ &= 1000 \left( \frac{45 \ddot{a}_{30}}{1000} \right) \\ &= 45 (9.8422) \\ &= 442.90 \end{aligned}$$

(B)

This is a very straightforward identity question!

Spring 1999 EA-1A

- 13 There are two techniques for solving this problem. If you feel comfortable with reversionary annuities, you can enumerate the cases and write down the present value of the inheritance directly.

An alternative is to use a method that is fairly visual and intuitive. This is called the "Method of detached coefficients." In this solution method you set up a sparse matrix which can easily be solved.

First you write down a general formula for the present value. Since we are solving for a payment value of  $X$ , I'll use  $A, B, C, D \dots$  as the total payments coefficients. I'll also define the beneficiaries as three different individuals, just to be sure all the cases are accounted for:

$$PV = (A \ddot{a}_a^{(12)} + B \ddot{a}_b^{(12)} + C \ddot{a}_c^{(12)} + D \ddot{a}_{ab}^{(12)} + E \ddot{a}_{ac}^{(12)} + F \ddot{a}_{bc}^{(12)} + G \ddot{a}_{abc}^{(12)}) / 12$$

I intentionally wrote the PV formula in a general fashion. Now we can construct the matrix of payment amounts:

|              |                             |        |            |
|--------------|-----------------------------|--------|------------|
| All 3 alive: | $A + B + C + D + E + F + G$ | = 1200 | $= 3(400)$ |
| 2 only alive | $A + B + D$                 | = 900  | $= 2(450)$ |
| " "          | $A + C + E$                 | = 900  |            |
| " "          | $B + C + F$                 | = 900  |            |

Spring 1999 EA-1A

- (13) a only alive :  $A = X$   
 b only alive :  $B = X$   
 c only alive :  $C = X$

Now you can work backwards to solve for D, E, and F, which all equal  $900 - 2X = 900 - A - B = 900 - B - C$  etc  
 Finally, you can calculate the value of G

$$\begin{aligned} G &= 1200 - (A+B+C) - (D+E+F) \\ &= 1200 - 3X - 3(900 - 2X) \\ &= 3X - 1500 \end{aligned}$$

Now you can express the present value in terms of the annuity values given, and solve for X:

$$\frac{140,000}{12} = (X + X + X) \ddot{a}_y^{(12)} + 3(900 - 2X) \ddot{a}_{yy}^{(12)} + (3X - 1500) \ddot{a}_{yyy}^{(12)}$$

one alive      two alive      three alive

$$11,666.67 = 3X(9.194) + (2700 - 6X)(7.354) + (3X - 1500)(6.258)$$

$$11,666.67 - 2700(7.354) + 1500(6.258) = 3X(9.194 - 2(7.354) + 6.258)$$

$$1197.87 = X(3)(.7440)$$

$$X = 536.68$$

(C)

The alternative technique is to use reversionary annuities to write down expressions for the present values based on exactly one, two or three beneficiaries being alive:

$$\begin{aligned} 140,000 &= 12 \left[ 3(400) \ddot{a}_{abc}^{(12)} + 2(450) \left( \ddot{a}_{ab}^{(12)} - \ddot{a}_{abc}^{(12)} \right) + X \left( \ddot{a}_a^{(12)} - \ddot{a}_{a:bc}^{(12)} \right) \right. \\ &\quad \left. 2(450) \left( \ddot{a}_{bc}^{(12)} - \ddot{a}_{abc}^{(12)} \right) + X \left( \ddot{a}_b^{(12)} - \ddot{a}_{b:ac}^{(12)} \right) \right. \\ &\quad \left. 2(450) \left( \ddot{a}_{ac}^{(12)} - \ddot{a}_{abc}^{(12)} \right) + X \left( \ddot{a}_c^{(12)} - \ddot{a}_{c:ab}^{(12)} \right) \right] \end{aligned}$$

Spring 1999 EA-1A

- (13) I intentionally used the ages a, b and c to keep all the cases separate. It can be difficult to account for each case correctly when everyone's annuity has  $\ddot{a}_y^{(12)}$  in it!

The final reversionary annuity terms must be expanded:

$$\ddot{a}_a - \ddot{a}_{a:b:c} = \ddot{a}_a^{(12)} - \ddot{a}_{ab}^{(12)} - \ddot{a}_{ac}^{(12)} + \ddot{a}_{abc}^{(12)}$$

Now rewrite the entire PV formula with  $\ddot{a}_y^{(12)}$ ,  $\ddot{a}_{yy}^{(12)}$  and  $\ddot{a}_{yyy}^{(12)}$ :

$$\frac{140,000}{12} = 1200 \ddot{a}_{yyy}^{(12)} + 900(3)(\ddot{a}_{yy}^{(12)} - \ddot{a}_{yyy}^{(12)}) + 3 \times (\ddot{a}_y^{(12)} - \ddot{a}_{yy}^{(12)} - \ddot{a}_{yyy}^{(12)} + \ddot{a}_{yyy}^{(12)})$$

$$11,666.67 = 1200 \ddot{a}_{yyy}^{(12)} + 900(3) \ddot{a}_{yy}^{(12)} + 3 \times \ddot{a}_y^{(12)} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{group all annuities together}$$

$$- 3(900) \ddot{a}_{yyy}^{(12)} - 3 \times (2) \ddot{a}_{yy}^{(12)}$$

$$+ 3 \times \ddot{a}_{yyy}^{(12)}$$

$$= (3 \times 1500) \ddot{a}_{yyy}^{(12)} + (2700 - 6 \times) \ddot{a}_{yy}^{(12)} + 3 \times \ddot{a}_y^{(12)}$$

This is exactly the same expression produced by the first method of solution. Once you plug in the values of the annuities, you can solve for X.

The main challenge in working this problem on the exams is doing all this work and getting the correct answer without spending way too much time on one question!

Spring 1999 EA-1A

- 14 Unlike the prior problem, there is an easy way to work this one. If you don't see that approach, there is a much slower "brute force" approach. In either solution, the key is understanding the relationship between rates of decrement and probabilities.

The absolute rates of decrement are based on single decrement tables. Those rates can be used to derive the probabilities in the multiple decrement table.

I'll use  $q_x^{(d)}$  as the probability of mortality and  $q_x^{(w)}$  as the probability of withdrawal

Here is the relationship between rates and probabilities:

$$(1 - q_x^{(w)}) (1 - q_x^{(d)}) = p_x^{(w)} \cdot p_x^{(d)} = p_x^{(T)}$$

$$1 - q_x^{(w)} - q_x^{(d)} = p_x^{(T)} = p_x^{(w)} \cdot p_x^{(d)}$$

If you read the problem carefully, you'll notice that the employee becomes eligible for both death and termination benefits at age 63, after 5 years of service. Once they reach that age, they will get a benefit whether they die, or if they terminate.

The answer to the question is simply the probability that the participant survives to age 63:

$$\begin{aligned} {}^3 p_{60}^{(T)} &= p_{60}^{(T)} p_{61}^{(T)} p_{62}^{(T)} \\ &= p_{60}^{(G)} p_{61}^{(G)} p_{62}^{(G)} \\ &= [(1 - .02)(1 - .04)]^3 = (.9408)^3 = .8327 \end{aligned}$$



Solution revised  
12/09/02

Spring 1999 EA-1A

You could produce the same result by accounting for the different ways of exiting:

$$\begin{aligned} & 1 - \text{probability of exiting at } 60, 61 \text{ or } 62 \\ & = 1 - [q_{60}^{(T)} + p_{60} q_{61}^{(T)} + p_{60} p_{61} q_{62}^{(T)}] \\ & = 1 - [.0592 + .9408(.0592) + .9408(.9408)(.0592)] \\ & = .8327 \end{aligned}$$

$$\begin{aligned} q_x^{(T)} &= 1 - .9408 \\ &= .0592 \end{aligned}$$

(D)

The really long way to work the problem is to attempt to calculate the probability of survival to ages 63, 64 and 65 and allow for all cases of death, termination, or survival. To do this requires use of the probabilities of death and termination from the multiple decrement table. These are different than the single decrement rates of death and termination given in the problem.

Since death or retirement benefits are available after age 63 for this participant ( $\geq 5$  years of service, and  $\geq$  age 55), benefits will be paid for all these cases:

Death at 63 or 64, Termination at 63 or 64, Survival to 65

$$3p_{60}^{(G)} q_{63}^{(d)} + 4p_{60}^{(G)} q_{64}^{(d)} + 3p_{60}^{(G)} q_{63}^{(w)} + 4p_{60}^{(G)} q_{64}^{(w)} + 5p_{60}^{(T)}$$

Now you need to derive the values of the multiple decrement probabilities. The "best" formula to use is this one

$$p_x^{(1)} = (p_x^{(T)})^{\frac{q_x^{(T)}}{q_x^{(T)}}}$$

(14) Now take logs of both sides

$$\log(p_x^{(1)}) \doteq [q_x^{(1)} / q_x^{(T)}] \log(p_x^{(T)})$$

$$q_x^{(1)} \doteq q_x^{(T)} \frac{\log(p_x^{(1)})}{\log(q_x^{(T)})}$$

In the current problem, you can equate  $q_x^{(1)}$  to  $q_x^{(w)}$ , and  $p_x^{(1)}$  is  $q_x^{(w)} = 1 - q_x^{(w)}$ :

$$q_x^{(w)} \doteq q_x^{(T)} \frac{\log(1 - q_x^{(w)})}{\log(p_x^{(T)})}$$

$$\doteq .0592 \frac{\log(.9600)}{\log(.9408)}$$

$$= .039601$$

$$q_x^{(d)} \doteq q_x^{(T)} \frac{\log(1 - q_x^{(d)})}{\log(p_x^{(T)})}$$

$$\doteq .0592 \frac{\log(.9800)}{\log(.9408)}$$

$$= .019599$$

Since the values of  $q_x^{(w)}$  and  $q_x^{(d)}$  are constant at each age, you also have the same values of .039601 for  $q_x^{(w)}$  at each age, and .019599 for  $q_x^{(d)}$  at each age. At age 65, everyone is assumed to retire at the beginning of the year, so the values of  $q_x^{(w)}$  and  $q_x^{(d)}$  are zero at age 65.

Now you can evaluate the probability shown on the prior page

$$3p_{60}^{(T)} q_{63}^{(d)} + 4p_{60}^{(T)} q_{64}^{(d)} + 3p_{60}^{(T)} q_{63}^{(w)} + 4p_{60}^{(T)} q_{64}^{(w)} + 5p_{60}^{(T)}$$

$$(.9408)^3 (.019599) + (.9408)^4 (.019599) + (.9408)^3 (.039601) + (.9408)^4 (.039601)$$

$$+ (.9408)^5$$

$$(14) \quad 3f_{60}^{(t)} q_{63}^{(d)} + 4f_{60}^{(t)} q_{64}^{(d)} + 3f_{60}^{(t)} q_{63}^{(w)} + 4f_{60}^{(t)} q_{64}^{(w)} + 5f_{60}^{(t)} \\ .016320 + .015354 + .032976 + .031024 + .737032 \\ = .832706$$

(1).

This is exactly the same answer previously calculated  
 $(.9408)^3 = .832706$ .

We used the value of .9408 as  $f_x^{(t)}$ , which was originally calculated as  $(1 - q_x^{(w)}) (1 - q_x^{(d)})$ . In the multiple decrement table, you should have the same result using the probabilities of death and withdrawal:

$$1 - q_x^{(w)} - q_x^{(d)} = 1 - .039601 - .019599 = .940800$$

This is confirmation that the exponential approximation is a "good" approximation.

If you use one of the other approximations for the multiple decrement probabilities, you will get an answer that is close to .832706, but not the identical result - but it MUST be in the same answer range. In general, this must be the result for any reasonable method of solution.

Now I'll show the results using this formula, which is based on any number of decrements:

$$q_x^{(1)} \doteq \frac{q_x^{(1)}}{1 - \frac{1}{2} q_x^{(2)} - \frac{1}{2} q_x^{(3)} - \dots}$$

(14) Based on the two causes of decrement in this problem, we have

$$\begin{aligned} f_x^{(d)} &= \frac{f_x^{(w)}}{1 - \frac{1}{2} f_x^{(w)}} \quad \text{and} \quad f_x^{(w)} = \frac{f_x^{(d)}}{1 - \frac{1}{2} f_x^{(d)}} \\ f_x^{(d)} &\doteq f_x^{(w)} \left[ 1 - \frac{1}{2} f_x^{(w)} \right] \quad f_x^{(w)} \doteq f_x^{(d)} \left[ 1 - \frac{1}{2} f_x^{(d)} \right] \\ f_x^{(d)} &\doteq .02 \left[ 1 - \frac{1}{2} f_x^{(w)} \right] \quad f_x^{(w)} \doteq .04 \left[ 1 - \frac{1}{2} f_x^{(d)} \right] \end{aligned}$$

Now you have two equations in two unknowns. You can substitute the expression for  $f_x^{(d)}$  from the first equation into the second one, and solve for  $f_x^{(w)}$

$$\begin{aligned} f_x^{(w)} &\doteq .04 \left[ 1 - .5 (.02 (1 - .5 f_x^{(w)})) \right] \\ &= .04 - .0004 + .0002 f_x^{(w)} \\ f_x^{(w)} &= .0396 / .9998 \\ &= .039608 \end{aligned}$$

$$f_x^{(d)} = .02 (1 - .5 (.039608)) \quad \text{substituting in 1st equation} \\ = .019604$$

The values of the probabilities are slightly different than those calculated using the exponential approximation.

The value of  $P_x^{(G)}$  is not reproduced either:

$$P_x^{(G)} = 1 - f_x^{(w)} - f_x^{(d)} = 1 - .039608 - .019604 \\ = .940788$$

If you evaluate the expression shown at the top of the prior page, you should use the exact value of .940800 for  $P_x^{(G)}$ . The result will be .832726, which is quite close! (D)

Spring 1999 EA-1A

- 15 This is another "pure probability" problem. The key to working this problem is to write down the probabilities in terms of  $l_x$  values, and simplify the calculation.

First, write down the ages of everyone based on both 1-1-99 and 1-1-04:

|       | 1-1-99 Age | 1-1-04 Age |
|-------|------------|------------|
| Green | 63         | 68         |
| Smith | 60         | 65         |
| Brown | 65         | 70         |

|       |    |    |
|-------|----|----|
| Green | 63 | 68 |
| Smith | 60 | 65 |
| Brown | 65 | 70 |

The first probability can be calculated as 1 minus the probability Green does die in 2004:

$$1 - sP_{63}(q_{68}) = 1 - \frac{l_{68}}{l_{63}} \left( \frac{d_{68}}{l_{68}} \right) = 1 - \frac{d_{68}}{l_{63}} = 1 - \frac{19}{1000 - (57-1)} \\ = .979873$$

The probability that at least one dies in 2004 is equal to the sum of the individual probabilities, minus the probability that they both die in 2004. The joint death probability must be subtracted to avoid counting it twice, which would overstate the result:

$$sP_{60}q_{65} + sP_{65}q_{70} - (sP_{60}q_{65})(sP_{65}q_{70}) \\ = \frac{19}{l_{60}} + \frac{19}{l_{65}} - \left( \frac{19}{l_{60}} \right) \left( \frac{19}{l_{65}} \right) \quad \text{based on Green's calculation} \\ = \frac{(19)(1000)}{1000} + \frac{19}{1000 - (5)(19) - 1} - \left( \frac{19}{1000} \right) \left( \frac{19}{1000 - [5(19) - 1]} \right) \\ = .0190 + .02097 - .000398 \\ = .039573$$

The final joint probability is the product of the prior calculations:  $.979873 (.039573) = .038776$  D

Spring 1999 EA-1A

- 16 This question requires you to know a common identity involving successive values of  $\ddot{a}_x$ , plus you need to construct the expression for the probability.

$$2\bar{f}_{50} = 1 - 2\bar{P}_{50} = 1 - \bar{P}_{50}(\bar{P}_{51})$$

$$\sqrt{f_x \ddot{a}_{x+1}} = \ddot{a}_x = \ddot{a}_x - 1.0$$

This identity allows you to derive values for  $\bar{P}_{50}$  and  $\bar{P}_{51}$  based on  $\ddot{a}_{50}$ ,  $\ddot{a}_{51}$  and  $\ddot{a}_{52}$ :

$$f_x = \frac{(\ddot{a}_x - 1.0)(1+i)}{\ddot{a}_{x+1}}$$

$$\bar{P}_{50} = \left( \frac{11.07872}{11.92117} \right) (1.07) = .9944$$

$$\bar{P}_{51} = \left( \frac{10.92117}{11.75854} \right) (1.07) = .9938$$

$$2\bar{f}_{50} = 1 - .9944 (.9938) = .0118$$

(C)

This is another very straightforward question

Spring 1999 EA-1A

- 17 This is a typical problem involving multiple annuity contracts. The key is setting up the present value of each contract correctly. With no annuity or commutation values, the problem then is primarily algebraic manipulations.

$$1: \quad 6,000 = (a_{30:\overline{15}}) 1,000$$

$$2: \quad 75,000 = (a_{30:\overline{15}}) 5,000 + (Ia_{30:\overline{15}}) 1,000 + 5,000$$

$$3: \quad P = (a_{30:\overline{15}}) 8,000 - (Ia_{30:\overline{15}}) 500$$

Since contract 2 has payments starting at age 30, it was necessary to separate out the initial payment of 5,000. Then all of the other annuities can be written as annuities immediate, not due. You have to be careful to correctly set up the present values between the increasing annuity and the level annuity. Now you have 3 equations in 3 unknowns, and you need to determine P:

$$1: \quad a_{30:\overline{15}} = 6.0 \quad \text{substitute this into contract 2's PV}$$

$$2: \quad 75,000 = 6.0(5,000) + (Ia_{30:\overline{15}}) 1,000 + 5,000 \\ = 35,000 + (Ia_{30:\overline{15}}) 1,000$$

$$Ia_{30:\overline{15}} = 40.0$$

$$3: \quad P = 6.0(8,000) - 40.0(500) \\ = 48,000 - 20,000 \\ = 28,000$$

(D)

Spring 1999 EA-1A

- 18 This is a typical actuarial equivalence problem. The key to working this problem is understanding the  $nE_x$  symbols:

$$nE_x = \frac{D_{x+n}}{D_x}$$

When benefits are actuarially equivalent, the present values are identical. You need to set up equations for the normal retirement benefit and Option A, which will use the  $\bar{a}_x$  and  $nE_x$  factors.

1-1-99 Age 45

$$\text{PV of NRB} = \frac{D_{65}(\ddot{a}_{65})}{D_{45}} 7200 \\ = {}_{20}E_{45}(\ddot{a}_{65}) 7200$$

$$\begin{aligned}\text{PV Option A} &= \frac{D_{65}(\ddot{a}_{15.07} + {}_{15}E_{65} \ddot{a}_{70})}{D_{45}} X \\ &= {}_{10}E_{45}(\ddot{a}_{15.07} + {}_{15}E_{65} \ddot{a}_{70}) X \\ &= ({}_{10}E_{45} \ddot{a}_{15.07} + {}_{25}E_{45} \ddot{a}_{70}) X\end{aligned}$$

Now set the present values equal and solve for X:

$$\text{PV Option A} = \text{PV of NRB}$$

$$X ({}_{10}E_{45} \ddot{a}_{15.07} + {}_{25}E_{45} \ddot{a}_{70}) = {}_{20}E_{45}(\ddot{a}_{65}) 7200$$

$$\begin{aligned}X &= \frac{{}_{20}E_{45}(\ddot{a}_{65}) 7200}{{}_{10}E_{45}(\ddot{a}_{15.07}) + {}_{25}E_{45}(\ddot{a}_{70})} \\ &= \frac{.2122(9.1941) 7200}{.4808(1.07)(9.1079) + .1317(8.0605)} \\ &= 2444 \quad \text{(D)}\end{aligned}$$

Spring 1999 EA-1A

- 19 This problem tests your knowledge of various identities, which are the key to working this correctly. You are given two insurance values, which you can use to derive the values of  $\ddot{a}_{x+}$  and  $p_x$ , as well as the select  $p_{x+}$ .

The insurance values are from the standard table. You can use those to derive values for  $p_{60}$  and  $\ddot{a}_{60}$ . Then you'll be able to calculate the values of  $p_{60:11}$  and  $\ddot{a}_{60:11}$ .

First you need to express the insurance values in terms of annuities. The symbol  $A_{60:11}$  means the insurance does not pay if the person dies (that is  $A_{60:11}^+$ ). Instead,  $A_{60:11}$  is a pure endowment, which pays only if the person survives:

$$A_{60:11} = E_{60} = D_{61}/D_{60} = v p_{60} = 921/1000 \\ p_{60} = 1.07(0.921) = .9855$$

$$A_{60} = 1 - d \ddot{a}_{60}$$

In order to construct the value of  $\ddot{a}_{60:11}$ , you need to calculate the value of  $\ddot{a}_{61}$ :

$$1000 A_{60} = 1000 (1 - d \ddot{a}_{60}) = 328$$

$$1 - d \ddot{a}_{60} = .328 = 1 - iv(\ddot{a}_{60}) \\ = 1 - (0.07/1.07)(1 + v p_{60} \ddot{a}_{61})$$

$$(0.07/1.07)(1 + v p_{60} \ddot{a}_{61}) = .672$$

$$1 + v p_{60} \ddot{a}_{61} = 10.272$$

$$\ddot{a}_{61} = 9.272 (1.07) / p_{60} \\ = 10.0673$$

Spring 1999 EA-1A

- (19) Now you can use a similar formula for the select annuity:

$$\sqrt{P_x} \ddot{a}_{x+1} = a_x \Rightarrow \sqrt{P_{[x]}} \ddot{a}_{x+1} = a_{[x]}$$

True for a one year select period

$$\begin{aligned}\ddot{a}_{[x]} &= 1 + \sqrt{P_{[x]}} \ddot{a}_{x+1} \\ &= 1 + \frac{P_{[x]}}{1.07} (\ddot{a}_{x+1})\end{aligned}$$

$$\ddot{a}_{[60]} = 1 + \frac{P_{[60]}}{1.07} \ddot{a}_{61}$$

$$= 1 + \frac{(.01 + P_{60})}{1.07} \ddot{a}_{61}$$

$$= 1 + \frac{(.01 + .9855)}{1.07} (10.0673)$$

$$= 10.3661$$

Finally, you can calculate the net single premium for the annuity:

$$15,000 \ddot{a}_{[60]} = 155,491$$

D

Spring 1999 EA-1A

- 20 This is one of those relatively rare problems on the exam involving the force of mortality. The key is simply knowing how to use this to work the problem. You'll also develop the value for a select annuity using the same formula as problem 19 on this exam.

$$\text{Jordan's formula 1.18: } \ln(p_x) = -\int_0^x \mu_{x+t} dt$$

$$-\ln(p_x) = \int_0^x \mu_{x+t} dt$$

$$1.19a: \quad \frac{1}{2} \mu_{x+\frac{1}{2}}$$

This formula is interpreted to mean the average value of the force of mortality between ages  $x$  and  $x+1$ . The problem states the force of mortality is a constant. In addition, this is only true for the select period.

$$\begin{aligned} -\ln(p_{x+1}) &= \ln(1.04) \\ e^{-\ln(p_{x+1})} &= e^{\ln(1.04)} \end{aligned}$$

$$1/p_{x+1} = 1.04 \Rightarrow p_{x+1} = .9615 \Rightarrow q_{[x]} = .0385$$

$$\begin{aligned} f_x &= q_{[x]}/.6 \\ &= .0641 \end{aligned}$$

As in the prior problem, you need to derive the value of  $\ddot{a}_{41}$ , and use that to calculate  $a_{[40]}$ .

$$v p_{40} \ddot{a}_{41} = a_{40} \Rightarrow v p_{[40]} \ddot{a}_{41} = a_{[40]} \text{ with one year select period}$$

$$\ddot{a}_{41} = (1+i) a_{40} / p_{40}$$

$$\begin{aligned} v \ddot{a}_{41} &= 16/.9359 \\ &= 17.0959 \end{aligned}$$

Note-you can't calculate  $(1+i)^{-1}$

$$a_{[40]} = v p_{[40]} \ddot{a}_{41}$$

$$\begin{aligned} &= p_{[40]} (17.0959) = .9615 (17.0959) \\ &= 16.4384 \end{aligned}$$

(C)

Spring 1999 EA-1A

- 21 This is a straightforward identity problem.  
You need to calculate the expression given  
based on the various probabilities.

$$\begin{aligned} & nq_x + np_{xx} - nq_{xxx} \\ &= (1-np_x) + (1-np_{xx}) - np_{xxx}(q_{x+n:x+n:x+n}) \\ &= (1-np_x) + (1-np_{xx}) - (np_x)^3(1-p_{x+n:x+n:x+n}) \\ &= (1-np_x) + (1-np_{xx}) - (np_x)^3(1-(p_{x+n})^3) \end{aligned}$$

The only item you were not given directly is  $np_x$ ,  
but that is easily calculated

$$np_{xx} = (np_x)^2 \Rightarrow np_x = \sqrt{np_{xx}} = \sqrt{N \cdot .25} = .5$$

Now plug the values into the earlier expression

$$\begin{aligned} & (1-.5) + (1-.25) - (.5)^3(1-(.5)^3) \\ &= .5 + .75 - .125(.875) \\ &= 1.1406 \end{aligned}$$

(B)

Spring 1999 EA-1A

- 22 There are two ways to work this problem.  
The key to getting it done quickly is knowing  
an identity for calculating insurance reserves:

$$tV_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$$

Since you are given  $N_x$  values, this works well

$$\begin{aligned} 100,000 \cdot 20V_{45} &= 100,000 \left(1 - \frac{\ddot{a}_{65}}{\ddot{a}_{45}}\right) \\ &= 100,000 \left(1 - \left(\frac{N_{65}}{N_{65}-N_{66}}\right) \left(\frac{N_{45}-N_{46}}{N_{45}}\right)\right) \\ &= 100,000 \left(1 - \frac{1,060}{100} \left(\frac{242}{4450}\right)\right) \\ &= 42,355 \end{aligned}$$

(D)

The alternative "standard" solution takes much longer

$$tV_x = A_{x+t} - P_x(\ddot{a}_{x+t})$$

$$\text{Reserve} = PV(\text{Benefits}) - PV(\text{Premiums})$$

$$20V_{45} = A_{65} - P_{45} \ddot{a}_{65}$$

If you write this as  $\frac{M_{65}}{D_{65}} - \left(\frac{A_{45}}{\ddot{a}_{45}}\right) \ddot{a}_{65}$ , it will take

a long time to get all the values you need to calculate  $M_x$ .

Instead, use this approach:  $A_x = 1 - d \ddot{a}_x$

$$20V_{45} = \left(1 - d \ddot{a}_{65}\right) - \left(\frac{1 - d \ddot{a}_{45}}{\ddot{a}_{45}}\right) \ddot{a}_{65}$$

This eventually simplifies to the first identity shown above.

Spring 1999 EA-1A

- 23 The key to working this problem is knowing that the  $\bar{a}_x$  formula is DeMoivre's law, and there are formulas to quickly calculate both  $a_x$  and  $A_x$ :

$$a_x = \frac{n - \bar{a}_{n-x}}{n} \quad A_x = \frac{\bar{a}_{n-x}}{n} \quad \text{where } n = w-x$$

In this problem,  $w=110$ , which is the last age in the mortality table.

$$\begin{aligned}\ddot{a}_{65} &= 1 + a_{65} \\ &= 1 + \frac{45 - \bar{a}_{45}.07}{45(0.07)} \\ &= 1 + \frac{45 - (1.07)(\bar{a}_{45}.76)}{45(0.07)} \\ &= 1 + \frac{45 - 1.07(13.6055)}{3.15} \\ &= 10.6642\end{aligned}$$

(B)

Spring 1999 EA-1A

- 24 The key to working this problem is knowing how to read the values correctly from the table. You start in the left column with  $l[x]$ , then go across to the  $l_{x+3}$  column, then go down vertically.

The question asks for the annual payment for the annuity. Based on the information you are given, you have

$$100,000 = P \left( a_{[65]:\overline{4}1} \right)$$

Once you calculate the annuity value, you're done. Set up the annuity calculation and simplify everything into  $l_x$  values:

$$\begin{aligned} a_{[65]:\overline{4}1} &= v P_{[65]} + v^2 P_{[65]} + v^3 P_{[65]} + v^4 P_{[65]} \\ &= \frac{v}{l_{[65]}} l_{[65]+1} + \frac{v^2}{l_{[65]}} l_{[65]+2} + \frac{v^3}{l_{[65]}} l_{[65]+3} + \frac{v^4}{l_{[65]}} l_{[65]+4} \end{aligned}$$

Note that the last two  $l_x$  values are no longer select. Just plug in the  $l_x$  values:

$$\begin{aligned} a_{[65]:\overline{4}1} &= \left( \frac{1}{9,435,643} \right) \left( \frac{9,245,121}{1.07} + \frac{9,020,841}{(1.07)^2} + \frac{8,771,863}{(1.07)^3} + \frac{8,511,918}{(1.07)^4} \right) \\ &= 3.1978 \end{aligned}$$

$$P = \frac{100,000}{a_{[65]:\overline{4}1}}$$

$$= 31,271$$

(B)

Spring 1999 EA-1A

- 25 This is a modification to a problem that appears infrequently on the exam. Instead of having an annual benefit, this time you have a monthly benefit with an annual cost of living increase. The key to this problem is to write out the present value in its most basic form.

The first step is determination of the benefit payments for the first four years

| <u>Year</u> | <u>Benefit</u>         | <u>Δ Benefit</u> |
|-------------|------------------------|------------------|
| 1999        | P                      | P                |
| 2000        | 1.03P                  | .03P             |
| 2001        | $(1.03)^2 P = 1.0609P$ | .0309P           |
| 2002        | $(1.03)^3 P = 1.0927P$ | .0318P           |

The payments under the increasing annuity can be valued by multiplying each layer of benefit by a monthly life annuity. The present value of the increasing benefit is

$$12 \left[ P \ddot{a}_{65}^{(12)} + P(.03) \frac{\ddot{a}_{66}^{(12)}}{D_{65}} + P(.0309) \frac{\ddot{a}_{67}^{(12)}}{D_{65}} + P(.0318) \frac{\ddot{a}_{68}^{(12)}}{D_{65}} \right]$$

Since this is actuarially equivalent to the life annuity benefit, the present values will be equal:

$$1000 \ddot{a}_{65}^{(12)} = \frac{P}{D_{65}} \left( \ddot{a}_{65}^{(12)} + .03(D_{66}) \ddot{a}_{66}^{(12)} + .0309(D_{67}) \ddot{a}_{67}^{(12)} + .0318(D_{68}) \ddot{a}_{68}^{(12)} \right)$$

$$P = \frac{1000(8.74)94,414}{94,414(8.74) + .03(86,246)8.51 + .0309(78,601)8.29 + .0318(71,459)8.06}$$

$$= 825,178,360 / (825,178 + 22,019 + 20,135 + 18,331)$$

$$= 931.71$$

(B)