

SOCIETY OF ACTUARIES
AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2003 EA-1 EXAMINATION

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2003

Data for Question 1 (4 points)

Over a 3-year period, a series of deposits are made to a savings account. All deposits within a given year are equal in size and are made at the beginning of each relevant period. Deposits for each year total \$1,200. The following chart shows the frequency of deposits and the interest rate credited for each year:

| <u>Year</u> | <u>Frequency of deposits</u> | <u>Interest rate credited during year</u> |
|-------------|------------------------------|---|
| 1 | Semi-annually | $d^{(12)} = 6\%$ |
| 2 | Quarterly | $i^{(3)} = 8\%$ |
| 3 | Every 2 months | $\delta = 7\%$ |

Question 1

In what range is the value of the account at the end of the 3rd year?

- [A] Less than \$3,989
- [B] \$3,989 but less than \$4,017
- [C] \$4,017 but less than \$4,045
- [D] \$4,045 but less than \$4,073
- [E] \$4,073 or more

2003

Data For Question 2 (3 points)

A perpetuity provides continuous payments at an annual rate of \$10,000. Three charities are to share in the perpetuity. Charity A is to receive the payments for the first x years. Charity B is to receive the payments for the next y years. Charity C is to receive all subsequent payments.

B = present value of payments to Charity B as of the commencement of the perpetuity.

C = present value of payments to Charity C as of the commencement of the perpetuity.

$$B = C/2$$

Question 2

In what range is v ?

- [A] Less than 0.640
- [B] 0.640 but less than 0.650
- [C] 0.650 but less than 0.660
- [D] 0.660 but less than 0.670
- [E] 0.670 or more

2003

Data For Question 3 (4 points)

Date of loan: 1/1/2003.

Amount of loan: \$10,000.

Term of loan: 30 years.

Interest rate: 7.5% compounded annually.

Payments: Annual payments with the first payment at 12/31/2003; each subsequent payment is 3% larger than the immediately preceding payment.

X = the total interest paid through the 20th payment.

Question 3

In what range is X ?

- [A] Less than \$14,750
- [B] \$14,750 but less than \$15,250
- [C] \$15,250 but less than \$15,750
- [D] \$15,750 but less than \$16,250
- [E] \$16,250 or more

2003

Data For Question 4 (3 points)

Smith obtains a \$300,000, 30-year mortgage with level end of month payments. The interest rate is 7.5%, compounded annually. With the 120th payment, Smith makes an additional payment of \$2,000 and immediately refinances the outstanding balance with a 20-year, 7.0% compounded annually, mortgage with level end of month payments of P.

Question 4

In what range is P?

- [A] Less than \$1,900
- [B] \$1,900 but less than \$1,950
- [C] \$1,950 but less than \$2,000
- [D] \$2,000 but less than \$2,050
- [E] \$2,050 or more

2003

Data For Question 5 (4 points)

All assets of a pension plan are invested by manager Smith and manager Jones. There are no other plan assets. The following chart shows the market values of the plan's assets with each manager:

| | <u>Date</u> | <u>Smith</u> | <u>Jones</u> |
|--------------|-------------|--------------|--------------|
| Balance | 12/31/2001 | \$ 2,500,000 | \$ 2,500,000 |
| Contribution | 1/1/2002 | 0 | 1,500,000 |
| Balance | 6/30/2002 | 2,800,000 | 4,500,000 |
| Transfer | 7/1/2002 | 1,000,000 | (1,000,000) |
| Balance | 12/31/2002 | 4,180,000 | 3,500,000 |

X = one-half of the sum of both managers' time-weighted percentage returns for 2002.

Y = dollar-weighted percentage return for 2002 for the entire pension plan.

$$Z = Y - X$$

Question 5

In what range is Z?

- [A] Less than 0.09%
- [B] 0.09% but less than 0.18%
- [C] 0.18% but less than 0.27%
- [D] 0.27% but less than 0.36%
- [E] 0.36% or more

2003

Data For Question 6 (3 points)

Selected entries in a mortality table are:

$$l_{60} = 10,000$$

$$l_{61} = 7,500$$

$$l_{62} = 5,625$$

X = number of deaths between $60\frac{3}{4}$ and $61\frac{3}{4}$ under the Balducci assumption between integral ages.

Question 6

In what range is X ?

- [A] Less than 1,970
- [B] 1,970 but less than 2,020
- [C] 2,020 but less than 2,070
- [D] 2,070 but less than 2,120
- [E] 2,120 or more

2003

Data For Question 7 (4 points)

There are 9,800 members in a stationary population. Each year, there are four times as many deaths at age 25 and over as there are deaths at ages under 25. The average age at death for the former group is 66. The average age at death for the latter group is 16.

X = the number of members who die younger than age 25 each year.

Question 7

In what range is X ?

- [A] Less than 25
- [B] 25 but less than 28
- [C] 28 but less than 31
- [D] 31 but less than 34
- [E] 34 or more

2003

Data For Question 8 (4 points)

At normal retirement age 60, a pension plan provides three actuarially equivalent optional forms of payment:

- A. A lifetime annuity paying \$1,000 per month.
- B. A 5-year certain and life annuity paying X per month.
- C. A 5-year certain and life annuity paying (X + 300) per month for the first 36 months and Y per month thereafter.

All payments are made at the beginning of the month.

Selected actuarial factors:

$$\ddot{a}_{60}^{(12)} = 11.53$$

$${}_3|\ddot{a}_{60}^{(12)} = 8.83$$

$${}_5|\ddot{a}_{60}^{(12)} = 7.35$$

$i = 7\%$, compounded annually.

Question 8

In what range is Y?

- [A] Less than \$873.00
- [B] \$873.00 but less than \$883.00
- [C] \$883.00 but less than \$893.00
- [D] \$893.00 but less than \$903.00
- [E] \$903.00 or more

2003

Data For Question 9 (4 points)

A pension plan has 1,000 active participants all age 60 as of January 1, 2003. The plan provides a pre-retirement death benefit to active participants of an annuity certain of \$1,000 per month, the first payment payable on the January 1 next following the year of death and the final payment payable on December 1, 2007.

A two decrement service table is used in the actuarial valuation of this plan: mortality ($q^{(\text{mort})}$) and disability ($q^{(\text{dis})}$). Selected values from the service table are provided below:

$$p_{60}^{(\text{total})} = 0.990 \qquad q_{60}^{(\text{mort})} = 0.003$$

$$p_{61}^{(\text{total})} = 0.985 \qquad q_{61}^{(\text{dis})} = 0.011$$

The actual number of deaths and disabilities were as expected.

There were no new entrants or other withdrawals during 2003.

$i = 7\%$, compounded annually

X = One-year term cost for the death benefits for 2004, valued as of January 1, 2004.

Question 9

In what range is X ?

- [A] Less than \$116,000
- [B] \$116,000 but less than \$118,000
- [C] \$118,000 but less than \$120,000
- [D] \$120,000 but less than \$122,000
- [E] \$122,000 or more

2003

Data For Question 10 (3 points)

X is deposited at the beginning of years 1-14.

No deposits are made during years 15-32.

(X-100) is deposited at the beginning of years 33-42.

The annual effective rate of interest is 7%.

The accumulated value of the deposits at the end of 42 years is \$100,000.

Question 10

In what range is X?

- [A] Less than \$555
- [B] \$555 but less than \$575
- [C] \$575 but less than \$595
- [D] \$595 but less than \$615
- [E] \$615 or more

2003

Data For Question 11 (3 points)

Smith makes a single deposit into a fund. The fund pays interest at the end of each six-month period at a nominal annual rate of i compounded semi-annually.

Smith reinvests the interest payments in a separate fund earning interest at an annual effective rate of 6%.

After a 15-year period, Smith has earned an effective annual rate of return of 7.56%.

Question 11

In what range is i ?

- [A] Less than 8.31%
- [B] 8.31% but less than 8.37%
- [C] 8.37% but less than 8.43%
- [D] 8.43% but less than 8.49%
- [E] 8.49% or more

2003

Data For Question 12 (3 points)

A ten-year loan of \$10,000 at an 8% annual effective rate can be repaid using any of the four following methods:

- I. Amortization method, with level annual payments at the end of each year.
- II. Repay the principal at the end of ten years while paying 8% annual effective interest on the loan at the end of each year. The principal is repaid by making equal annual deposits at the end of each year into a sinking fund earning interest at 6% annual effective so that the sinking fund accumulates to \$10,000 at the end of the 10th year.
- III. Same as II, except the sinking fund earns 8% annual effective.
- IV. Same as II, except the sinking fund earns 12% annual effective.

Question 12

Rank the annual payment amounts of each method.

- [A] $I < II < III < IV$
- [B] $II < I = III < IV$
- [C] $I < IV < III < II$
- [D] $IV < I < III < II$
- [E] The correct answer is not given by [A], [B], [C], or [D] above

2003

Data For Question 13 (3 points)

Date of loan: 1/1/2003.

Payments: Level payments at the end of each year.

First payment: 12/31/03.

Payment period: 20 years.

The principal portion of the 13th payment is \$54.40.

The principal portion of the 18th payment is \$70.09.

Question 13

In what range is the total amount of interest paid on the loan?

- [A] Less than \$625
- [B] \$625 but less than \$640
- [C] \$640 but less than \$655
- [D] \$655 but less than \$670
- [E] \$670 or more

2003

Data For Question 14 (4 points)

You are given the following information:

- i) The force of mortality is constant for all ages.
- ii) The force of mortality equals the force of interest.
- iii) $\bar{a}_{20} = 1.4\bar{a}_{10}$

X = the probability that someone age 20 will die between the ages of 40 and 50.

Question 14

In what range is X?

- [A] Less than 0.085
- [B] 0.085 but less than 0.090
- [C] 0.090 but less than 0.095
- [D] 0.095 but less than 0.100
- [E] 0.100 or more

2003

Data For Question 15 (5 points)

Annuity A: Ten payments payable every other year with the first payment on 12/31/2003. The first payment is \$1,000 and each subsequent payment increases by \$100.

Annuity B: Ten payments payable every other year with the first payment on 12/31/2003. The first payment is \$2,000 and each subsequent payment is 95% of the immediately preceding payment.

Annuity C: A perpetuity with annual payments of X. The first payment is on 1/1/2003.

As of 1/1/2003, the present value of Annuity C equals the present value of Annuity A plus the present value of Annuity B.

Interest rate: 7.00%, compounded annually.

Question 15

In what range is X?

- [A] Less than \$1,050
- [B] \$1,050 but less than \$1,150
- [C] \$1,150 but less than \$1,250
- [D] \$1,250 but less than \$1,350
- [E] \$1,350 or greater

2003

Data For Question 16 (2 points)

Face value of a bond: \$1,000.

Bond price: \$980.

Coupon rate: 10.00% annual, payable at end of year.

Risk-free yield rate: 7.00%, compounded annually.

Term of bond: One year.

Implicit probability of default: q .

Question 16

In what range is q ?

- [A] Less than 0.0450
- [B] 0.0450 but less than 0.0455
- [C] 0.0455 but less than 0.0460
- [D] 0.0460 but less than 0.0465
- [E] 0.0465 or greater

2003

Data For Question 17 (4 points)

Participant age on 1/1/2003: 65.

Joint annuitant age on 1/1/2003: 62.

Interest rate: 7%, compounded annually.

Joint annuity terms:

2-year annuity with annual payments made on 12/31/2003 and 12/31/2004.

Annual payment amounts are based on survivorship as of each payment date:

| | |
|--|----------|
| If both participant and joint annuitant are alive | \$10,000 |
| If either participant or joint annuitant is alive but not both | \$5,000 |
| If neither participant nor joint annuitant is alive | \$0 |

| <u>Age (x)</u> | <u>q_x</u> |
|----------------|-------------------------|
| 62 | 0.0200 |
| 63 | 0.0250 |
| 64 | 0.0300 |
| 65 | 0.0350 |
| 66 | 0.0400 |
| 67 | 0.0450 |

Question 17

In what range is the present value of the joint annuity on 1/1/2003?

- [A] Less than \$17,000
- [B] \$17,000 but less than \$17,750
- [C] \$17,750 but less than \$18,500
- [D] \$18,500 but less than \$19,250
- [E] \$19,250 or greater

2003

Data For Question 18 (4 points)

Participant data as of 1/1/2003: 1,000 active participants, all exact age 54.

Decrements: Withdrawals and deaths are uniformly distributed between consecutive integral ages in their respective single decrement table. Retirements occur at the beginning of the year. Decrements are set forth in the table below:

| Age (x) | Absolute Rate of Decrement | | |
|---------|----------------------------|--------------------------|--------------------------|
| | $q_x^{\text{wl}\dagger}$ | $q_x^{\text{dl}\dagger}$ | $q_x^{\text{rl}\dagger}$ |
| 54 | 0.04 | 0.0044 | 0.00 |
| 55 | 0.00 | 0.0049 | 0.15 |
| 56 | 0.00 | 0.0055 | 0.10 |

There are no new entrants after 1/1/2003.

Question 18

In what range is the expected number of deaths from active service during 2003 and 2004?

- [A] Less than 8.00
- [B] 8.00 but less than 8.25
- [C] 8.25 but less than 8.50
- [D] 8.50 but less than 8.75
- [E] 8.75 or greater

2003

Data For Question 19 (3 points)

Participant age on 1/1/2003: 65.

Spouse age on 1/1/2003: 62.

Annuity commencement date: 1/1/2003.

Interest rate: 7.00%, compounded annually.

Actuarially Equivalent Annuity Forms

- (1) Single Life: \$10,000 per year, payable at the beginning of each year.
- (2) Joint and 75% Survivor: Y per year, payable at the beginning of each year that both participant and spouse are alive, decreasing to 75% of Y upon the first death. All payments cease upon the second death.

| Annuity factors: | Age (x) | \ddot{a}_x | $\ddot{a}_{x:x-3}$ |
|------------------|---------|--------------|--------------------|
| | 62 | 10.6974 | 9.4604 |
| | 63 | 10.4827 | 9.2108 |
| | 64 | 10.2642 | 8.9590 |
| | 65 | 10.0426 | 8.7060 |

Question 19

In what range is Y?

- [A] Less than \$8,700
- [B] \$8,700 but less than \$8,800
- [C] \$8,800 but less than \$8,900
- [D] \$8,900 but less than \$9,000
- [E] \$9,000 or greater

2003

Data For Question 20 (5 points)

Date of loan: 1/1/2003.

Payments: Annual payments of \$1,000 at the end of each year.

First payment: 12/31/2003.

Payment period: 25 years.

Loan interest rate: 8%, compounded annually.

X is the present value of interest payments at 1/1/2003 over the life of the loan, using an interest rate of 12% compounded annually.

Question 20

In what range is X?

- [A] Less than \$4,600
- [B] \$4,600 but less than \$5,600
- [C] \$5,600 but less than \$6,600
- [D] \$6,600 but less than \$7,600
- [E] \$7,600 or more

2003

Data For Question 21 (3 points)

Smith buys a bond with the following characteristics:

Face amount: \$1,000.

15 year maturity.

Coupons of 8% payable semi-annually.

If the bond were held to maturity, its yield would be 9.0% compounded annually.

With 10 years remaining to maturity and immediately after payment of the tenth coupon, Smith sells the bond at a price that yields 8.25% compounded annually to the buyer.

Smith's annual effective rate of return on his investment in the bond is X.

Question 21

In what range is X?

- [A] Less than 8.7%
- [B] 8.7% but less than 9.1%
- [C] 9.1% but less than 9.5%
- [D] 9.5% but less than 9.9%
- [E] 9.9% or more

2003

Data For Question 22 (4 points)

Two decrements are considered: mortality and withdrawal.

$$q_x^{(d)} = 0.03$$

$$\mu_x^{(w)} = 0.20$$

In the respective associated single-decrement tables, mortality is uniformly distributed between consecutive integral ages and withdrawal has a constant force of decrement between consecutive integral ages.

Question 22

In what range is $q_x^{(d)}$?

- [A] Less than 0.0270
- [B] 0.0270 but less than 0.0275
- [C] 0.0275 but less than 0.0280
- [D] 0.0280 but less than 0.0285
- [E] 0.0285 or more

2003

Data For Question 23 (5 points)

A pension plan pays annuity benefits annually at the beginning of the year.

Smith is currently age x .

Smith could retire with a \$100 benefit payable for life, with the first 10 payments guaranteed.

Instead, Smith elects the “refund” annuity option, which provides annual payments of Z for life, and a death benefit. The death benefit is paid at the end of the year of death, and equals $(12 * Z)$ minus the sum of the annual payments already made.

The two annuity payment forms are actuarially equivalent.

$$i = 0.07$$

$$\ddot{a}_x = 9.3$$

$$q_{x+k} = 0.02, \text{ for } 0 \leq k \leq 10$$

Question 23

In what range is Z ?

- [A] Less than \$90.00
- [B] \$90.00 but less than \$93.00
- [C] \$93.00 but less than \$96.00
- [D] \$96.00 but less than \$99.00
- [E] \$99.00 or more

2003

Data For Question 24 (4 points)

You are given a two-decrement model.

| x | $q_x^{(1)}$ | $q_x^{(2)}$ | $q_x^{(\tau)}$ | $l_x^{(\tau)}$ | $d_x^{(1)}$ | $d_x^{(2)}$ |
|-----|-------------|-------------|----------------|----------------|-------------|-------------|
| 60 | | | 0.08 | | 120 | |
| 61 | 0.2 | 0.1 | | 1,725 | | |
| 62 | | | | | | 77 |

Both decrements are uniformly distributed over $[x, x + 1]$ in their associated single-decrement models.

Question 24

In what range is ${}_3q_{60}^{(2)}$?

- [A] Less than 0.1375
- [B] 0.1375 but less than 0.1385
- [C] 0.1385 but less than 0.1395
- [D] 0.1395 but less than 0.1405
- [E] 0.1405 or more

2003

Data For Question 25 (3 points)

A life insurance benefit of \$10,000 is provided to Smith while actively employed. The death benefit is payable at the end of the year of death.

Retirements are assumed to occur in the middle of the year.

Date of birth: January 1, 1938.

Absolute rate of retirement: $q'_{65}^{(r)} = 0.50$

Absolute rate of mortality: $q'_{65}^{(d)} = 0.04$

In the associated single-decrement tables, the force of mortality is constant within the year of age.

$i = 7\%$, compounded annually.

Y = the one-year term cost for Smith's death benefit as of 1/1/2003.

Question 25

In what range is Y ?

- [A] Less than \$290
- [B] \$290 but less than \$300
- [C] \$300 but less than \$310
- [D] \$310 but less than \$320
- [E] \$320 or more

2003

Data For Question 26 (3 points)

An annual annuity due is provided to independent lives X and Y.

During the first 20 years, the annuity pays \$1.00 if at least one of X and Y is alive.

After 20 years, the annuity pays \$1.00 only if both X and Y are alive.

$$\ddot{a}_X = 10.0 \qquad {}_{20|}\ddot{a}_X = 3.0$$

$$\ddot{a}_Y = 8.4 \qquad {}_{20|}\ddot{a}_Y = 2.5$$

$$\ddot{a}_{XY} = 6.2 \qquad {}_{20|}\ddot{a}_{XY} = 1.9$$

Question 26

In what range is the present value of this annuity?

- [A] Less than \$8.00
- [B] \$8.00 but less than \$9.00
- [C] \$9.00 but less than \$10.00
- [D] \$10.00 but less than \$11.00
- [E] \$11.00 or more

2003

Data For Question 27 (2 points)

\$100 discounted for 5 months is \$99.

Question 27

Which of the following is an equivalent expression for $d^{(4)}$?

[A] $4(1 - (99/100)^{\frac{3}{5}})$

[B] $4(1 - (100/99)^{\frac{3}{5}})$

[C] $4(1 - (99/100)^{\frac{5}{4}})$

[D] $4(1 - (99/100)^{\frac{5}{3}})$

[E] The correct answer is not given by [A], [B], [C], or [D] above

2003

Data For Question 28 (4 points)

An employee terminates employment at age 45.

Normal form of benefit: Annuity due with an annual benefit of \$5,000, commencing at age 65 and payable for ten years certain and life thereafter. No benefit is provided if the employee does not survive to age 65.

Alternative form of benefit: Life annuity due with an annual benefit of Y commencing at age 55. No benefit is provided if the employee does not survive to age 55.

The two forms are actuarially equivalent.

$i = 5\%$, compounded annually

| <u>Age x</u> | <u>\ddot{a}_x</u> | <u>${}_{10}p_x$</u> |
|---------------------------|--------------------------------|--------------------------------|
| 45 | 18.0 | 0.95 |
| 55 | 17.2 | 0.91 |
| 65 | 16.7 | 0.88 |
| 75 | 16.5 | 0.85 |

Question 28

In what range is Y?

- [A] Less than \$2,750
- [B] \$2,750 but less than \$2,850
- [C] \$2,850 but less than \$2,950
- [D] \$2,950 but less than \$3,050
- [E] \$3,050 or more

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2003 EA-1
Answer Key

| Question Number | Answer | Question Number | Answer |
|------------------------|---------------|------------------------|---------------|
| 1 | D | 16 | E |
| 2 | D | 17 | B |
| 3 | C | 18 | C |
| 4 | C | 19 | D |
| 5 | D | 20 | C |
| 6 | B | 21 | D |
| 7 | E | 22 | B |
| 8 | D | 23 | D |
| 9 | D | 24 | D |
| 10 | C | 25 | A |
| 11 | C | 26 | D |
| 12 | E | 27 | A |
| 13 | B | 28 | B |
| 14 | D | | |
| 15 | B | | |