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SPRING 2004 EA-1
EXAM SOLUTIONS

Revision History:
04/12/06 Added alternate solutions for problems 2, 5, 12, 17 and 18

2004 EA-1

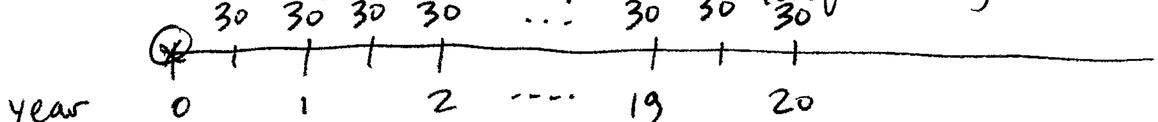
1 This is a fairly typical bond problem. With a face amount of 1,000, the bond provides coupons of 6%/2 every six months: $(.06/2)1000 = 30$.

You are told that the bond's purchase price is based on a return of 5% per annum. Since the coupons are paid semi-annually, you should convert the interest rate to match the coupon payment period: $(1+j)^2 = 1.05 \Rightarrow j = 2.47\%$

$$P = Fr\,(a_{\overline{n}|j}) + K$$
$$= 1000(.03)\,a_{\overline{40}|2.47\%} + 1000(1.0247)^{-40}$$
$$= 30(25.2322) + 1000(.3769)$$
$$= 1{,}133.85$$

Note that the purchase price is greater than the face value of 1,000. This makes sense, because the coupon rate exceeds the yield of 5%.

You are told that the coupons will be invested in a fund to earn 6% per annum, convertible quarterly:



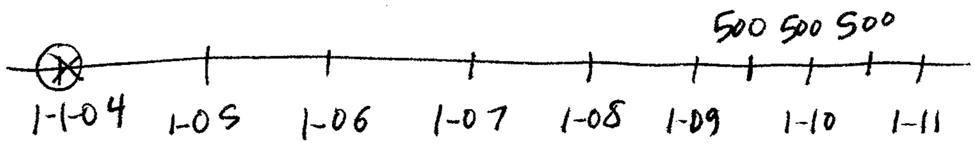
At the end of 20 years, the purchase price will be worth $1133.85(1+i)^{20}$. This should equal the accumulated value of the coupons and the face amount of the bond:

$$1133.85(1+i)^{20} = 1000 + 30\,s_{\overline{40}|k} \qquad \text{where } 1+k = (1.015)^2$$
$$K = 3.02\%$$
$$= 1000 + 30(75.79)$$
$$= 3{,}273.61 \Rightarrow 1+i = 1.0544 \quad \text{(D)}$$

2004

2 This is an unusual problem on the force of interest. Typically you think of $e^{\delta} = 1+i$, which is true for compound interest, where $\delta = \ln(1+i)$ is a constant.

In this problem, δ varies over time. The present value of the three payments on a time diagram looks like this:



If you had typical interest values to work with, the present value would be $500[v^{5.5} + v^{6.0} + v^{6.5}]$. Based on the force of interest, you have

$$PV = 500\left[e^{-\int_0^{5.5} \delta_t\,dt} + e^{-\int_0^{6.0} \delta_t\,dt} + e^{-\int_0^{6.5} \delta_t\,dt}\right]$$

You need to evaluate $e^{-\int_0^n \delta_t\,dt} = e^{-\int_0^n (50+2t)^{-1}\,dt}$

In general you know that $\int (1/x) = \ln x$

then $\int (50+2t)^{-1} = +\frac{1}{2}\ln(50+2t)$

$$e^{-\int_0^n (50+2t)^{-1}dt} = e^{-\frac{1}{2}(\ln(50+2t))\Big]_0^n}$$

$$= e^{\frac{1}{2}\ln(50) - \frac{1}{2}\ln(50+2n)}$$

$$= e^{(\ln 50)^{\frac{1}{2}}} \Big/ e^{(\ln 50+2n)^{1/2}}$$

$$= \left(\frac{50}{50+2n}\right)^{\frac{1}{2}}$$

$$PV = 500\left[\left(\frac{50}{50+11}\right)^{\frac{1}{2}} + \left(\frac{50}{50+12}\right)^{\frac{1}{2}} + \left(\frac{50}{50+13}\right)^{\frac{1}{2}}\right]$$

$$= 1347.13$$

A

(2) There is another solution technique, which attempts to avoid dealing with the force of interest.

You could try to calculate $1+i = e^{\delta}$ for each year from 2004 through 2010:

$$\left.\begin{aligned}
\delta_{2004} &= 1/(50+0) = 1.0202 \\
\delta_{2005} &= 1/(50+2) = 1.0194 \\
\delta_{2006} &= 1/(50+4) = 1.0187 \\
\delta_{2007} &= 1/(50+6) = 1.0180 \\
\delta_{2008} &= 1/(50+8) = 1.0174
\end{aligned}\right\} \text{5 years discount} = (1.0973)^{-1}$$

$$\delta_{2009} = 1/(50+10) = 1.0168$$

$$\delta_{2010} = 1/(50+12) = 1.0163$$

$$PV = \frac{500}{1.0973}\left[(1.0168)^{-\frac{1}{2}} + (1.0168)^{-1} + (1.0163)^{-\frac{1}{2}}(1.0168)^{-1}\right]$$

$$= \frac{500}{1.0973}[.9917 + .9835 + .9756]$$

$$= 1344.55 \qquad \text{A}$$

This is a VERY ROUGH approximation. If you look carefully, this result is actually outside the implied range for answer A. The more precise method of solution on the prior page is the preferred technique for this problem.

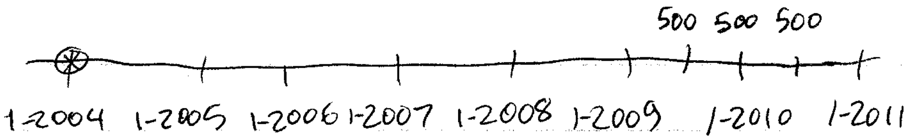
Solution added 04/12/06

(2) The problem with the prior approximation is that it does not allow for the change in the force of interest during each year. It is better to calculate the values at mid-year:

t	Date	δ_t	$v=e^{-\delta}$	$v^{\frac{1}{2}}$
.5	2004.5	1/(50+1)		
1.5	2005.5	1/53		
2.5	2006.5	1/55		
3.5	2007.5	1/57		
4.5	2008.5	1/59		
5.25	2009.25	1/(50+10.5)	.9836	.9918
5.75	2009.75	1/61.5	.9839	.9919
6.25	2010.25	1/62.5	.9841	.9920

$e^{-\delta} = (1+i)^{-1}$

$$\text{5 years of discount} = \left(e^{-\frac{1}{51}}\right)\left(e^{-\frac{1}{53}}\right)\left(e^{-\frac{1}{55}}\right)\left(e^{-\frac{1}{57}}\right)\left(e^{-\frac{1}{59}}\right)$$
$$= \left(e^{\frac{1}{51}+\frac{1}{53}+\frac{1}{55}+\frac{1}{57}+\frac{1}{59}}\right)^{-1}$$
$$= e^{-.09115}$$
$$= .9129$$



Since the payments are made at 6 month intervals, I calculated the force of interest at the mid-point of each one.

$$PV = 500(.9129)(.9918(1+.9919(1+.9920)))$$
$$= 500(.9129)(2.9514)$$
$$= 1347.14$$

A

This is clearly a better approximation, since it produces virtually the same result as the exact solution (integration).

3 On a time diagram, you have the following

	1	2	...	10	9	8	...	2	1
0	1	2	...	10	11	12	...	18	19

You can calculate the present value in several ways. You could use the formulas for increasing and decreasing annuities. You can also write the present value as the sum of 10 annuities, each with 10 level payments:

$$
\begin{aligned}
PV &= a_{\overline{10}|} + v\left(a_{\overline{10}|}\right) + \dots + v^{9}\left(a_{\overline{10}|}\right) \\
&= \ddot{a}_{\overline{10}|.05}\left(a_{\overline{10}|.05}\right) \\
&= 1.05\left(a_{\overline{10}|.05}\right)^2 \\
&= 1.05(7.7217)^2 \\
&= 62.61
\end{aligned}
$$

C

You can also calculate the present value as follows:

$$
\begin{aligned}
PV &= (Ia)_{\overline{10}|.05} + v^{10}(Da)_{\overline{9}|.05} \\
&= \frac{\ddot{a}_{\overline{10}|} - 10v^{10}}{.05} + (1.05)^{-10}\left(\frac{9 - a_{\overline{9}|.05}}{.05}\right) \\
&= \frac{1.05(7.7217) - 10v^{10}}{.05} + .6139\left(\frac{9 - 7.1078}{.05}\right) \\
&= 39.37 + 23.23 \\
&= 62.61
\end{aligned}
$$

4 This is one of the few real callable bond problems on the exam. The bond pays 6% coupons and the yield rate of 5% is lower.

The bond price formula looks like this

$P = Fr(a_{\overline{n}|i}) + Cv^n$

$= 1000(.06)a_{\overline{n}|.05} + 1000(1.05)^{-n}$

Since the 6% coupons exceed the yield rate, the worst case for the purchaser is based on the earliest call date. The reason is their yield would be lower than at any other call date, since they get the fewest coupons.

Assume 1-1-2014 call date $\Rightarrow$ fewest coupons

$P = 1000(.06)\, a_{\overline{10}|.05} + 1000(1.05)^{-10}$

$= 60(7.7217) + 1000(.6139)$

$= 1077.22$ A

 Solution added 04/12/06

5 There are 3 very different ways of working this problem. I started with a perpetuity, then subtracted payments if the annuitant had died 10 or more years earlier:

$$PV = \underset{\text{Immediate}}{\text{Perpetuity}} - v^{11}(1-{}_1p_{65}) - v^{12}(1-{}_2p_{65}) - \dots$$

$$= v^1 + v^2 + \dots + v^{10} + v^{11}\,{}_1p_{65} + v^{12}\,{}_2p_{65} + \dots$$

$$= a_{\overline{10}|.05} + v^{10}[v^1\,{}_1p_{65} + v^2\,{}_2p_{65} + \dots]$$

$$= a_{\overline{10}|.05} + (1.05)^{-10} a_{65}$$

$$= 7.7217 + .6139\,(10.1548)$$

$$= 13.9686$$

With 1,000 payments $\Rightarrow$ 13,969 (D)

Another way to work this problem is to allow for the probability that the annuitant was alive 10 years ago in order to make a payment today:

$$PV = 1000\left[\sum_{t=1}^{\infty} v^t\,{}_tp_{65} + \sum_{t=1}^{\infty} v^t({}_{t-10}p_{65} - {}_tp_{65})\right]$$

$$= 1000\left[\sum_{t=1}^{\infty} v^t\,{}_tp_{65} + \sum_{t=1}^{10} v^t(1-{}_tp_{65}) + \sum_{t=11}^{\infty} v^t({}_{t-10}p_{65} - {}_tp_{65})\right]$$

$$= 1000\left[a_{65} + a_{\overline{10}|} - a_{65:\overline{10}|} + v^{10}a_{65} - v^{10}\,{}_{10}p_{65}\,a_{75}\right]$$

$$= 1000\left[a_{\overline{10}|} - v^{10}\,{}_{10}p_{65}\,a_{75} + v^{10}a_{65} - v^{10}\,{}_{10}p_{65}\,a_{75}\right]$$

$$= 1000\left[a_{\overline{10}|.05} + (1.05)^{-10}a_{65}\right]$$

$$= 13,969$$

See next page for a "less tricky" method of solution.

Solution added 04/12/06

(5) A third solution technique is to use the idea of an insurance to provide the payments at death:

$$PV = 1000\left(a_{65} + A_{65}\left(\ddot{a}_{\overline{10}|.05}\right)\right)$$
$$= 1000\left(a_{65} + (1 - d\,\ddot{a}_{65})(\ddot{a}_{\overline{10}|.05})\right)$$

$$= 1000\left(a_{65} + \left(1 - \frac{.05}{1.05}(1 + a_{65})\right)\ddot{a}_{\overline{10}|.05}\right)$$

$$= 1000\left(10.17548 + (1 - .047619(11.17548))(8.1078)\right)$$

$$= 1000(10.17548 + 3.7931)$$

$$= 13{,}969$$

D

6 This is a straightforward reversionary annuity problem. The main thing is being careful you don't get the ages backwards for Smith or Jones

PV of normal form for Smith $= 12(1000)\,\ddot{a}_{65}^{(12)}$

PV of optional form $12P\,\ddot{a}_{65:63}^{(12)}$ both alive

$+12(P/2)(\ddot{a}_{63}^{(12)} - \ddot{a}_{65:63}^{(12)})$ Jones only

$+12(1000)(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:63}^{(12)})$ Smith only

Now set the present values equal to each other (since they are actuarially equivalent), and solve for P:

$$12\cancel{(1000)\ddot{a}_{65}^{(12)}} = 12P\,\ddot{a}_{65:63}^{(12)} + 12[P/2](\ddot{a}_{63}^{(12)} - \ddot{a}_{65:63}^{(12)}) + 12(1000)(\cancel{\ddot{a}_{65}^{(12)}} - \ddot{a}_{65:63}^{(12)})$$

$$12(1000)(\ddot{a}_{65:63}^{(12)}) = 12[P/2](\ddot{a}_{63}^{(12)} + \ddot{a}_{65:63}^{(12)})$$

$$P = \frac{2000\,\ddot{a}_{65:63}^{(12)}}{\ddot{a}_{63}^{(12)} + \ddot{a}_{65:63}^{(12)}}$$

$$= \frac{2000}{\frac{\ddot{a}_{63}^{(12)}}{\ddot{a}_{65:63}^{(12)}} + 1}$$

$$= \frac{2000}{(12.1/9.0)+1}$$

$$= 853.08$$

D

7 This is a two step problem. The first step is finding out how many loan payments will be made, ignoring the smaller final payment of X. Then you can solve for the value of X.

Original Loan		X	5,000	5,000	...	5,000	5000	5000	5000	5000
	Pmt	0	1	2	...	6	7	8	9	10
	Date	1-04	1-05	1-06	...	1-10	1-11	1-12	1-13	1-14

At 1-1-10, an extra payment of 10,000 is made. The O/S loan balance is normally the present value of the remaining payments. Reflecting the additional payment, you have

$$\text{1-10 O/S Balance} = 5000\, a_{\overline{4}|.08} - 10{,}000$$
$$= 6560.63$$

The loan is now revised to reflect annual payments of 1,000, plus a smaller final payment

$$\text{1-10 Revised Loan} = 6560.63 = 1{,}000\, a_{\overline{n}|.08} + X v^{n+1}$$

Solve for n by ignoring the final payment. Using the HP-12C calculator to solve $6.56 = a_{\overline{n}|.08}$ gives a value of 10 for n. Since the HP always rounds up to the next highest integer, this means you have 9 payments of 1,000:

$$6560.63 = 1{,}000\, a_{\overline{9}|.08} + X(1.08)^{-10}$$
$$X = (1.08)^{10}\left[6560.63 - 6246.90\right]$$
$$= 677.35 \qquad \text{E}$$

8 There are two ways to think of this series of payments. One approach is that there are 180 monthly payments of X, plus a second annuity with 96 monthly payments of 300.

The alternate view is that there is an annuity for 180 monthly payments of X+300, less an annuity for 84 payments of 300. Both approaches will give the same value of X.

The monthly interest rate is equivalent to 8% per annum:

$$(1+j)^{12} = 1.08 \Rightarrow j = .64\%$$

$$20{,}600 = (X+300)\, a_{\overline{180}|.64\%} - 300\left(a_{\overline{84}|.64\%}\right)$$

$$X = \frac{20{,}600 + 300\left(a_{\overline{84}|.64} - a_{\overline{180}|.64}\right)}{a_{\overline{180}|.64}}$$

$$= \frac{20{,}600 + 300(64.7354 - 106.4276)}{106.4276}$$

$$= 76.04$$

C

9 This is a typical EA-1 question on the amortization schedule for loans – but they changed the usual definition of the loan balance. You should write down the amortization schedule for the first few payments, and then you can add up the total interest and principal paid.

Payment Number	Principal	Interest	O/S Loan
0			10,000
1	500	.05(10,000)	9,500
2	500	.05(9,500)	9,000
⋮	⋮	⋮	⋮
20	500	.05(500)	–0–
Total	20(500)	.05(500+...+9,500+10,000)	

$$= .05(500)(1+2+\ldots+19+20)$$

$$= \frac{.05(500)(20)(21)}{2}$$

$$= 5,250$$

Total principal + interest $= 10,000 + 5,250$
$= 15,250$ (B)

10 This is a typical EA-1 question on dollar weighted and time weighted rates of return. The real key to working this problem is carefully reading the data. The market values you are given are <u>prior</u> to the contributions and withdrawals.

Now you should determine the market values after the cash flows occur:

MV Before		85,000	100,000	
Cash flow		30,000	-20,000	
Date	1/1	4/1	8/1	12/31
MV After	100,000	115,000	80,000	80,000

The time weighted rate of return measures the growth of the market value between the cash flows:

$$1+A = \left(\frac{85{,}000}{100{,}000}\right)\left(\frac{100{,}000}{115{,}000}\right)\left(\frac{80{,}000}{80{,}000}\right)$$

$$= .739$$

$$A = -26.1\%$$

To calculate the dollar weighted rate of return, you need to determine the total interest earned during the year:

$$100{,}000 + 30{,}000 - 20{,}000 + I = 80{,}000$$

$$I = -30{,}000$$

(10) The dollar weighted return is calculated based on simple interest for the year:

$$100{,}000(1+B)+30{,}000(1+\tfrac{9}{12}B)-20{,}000(1+\tfrac{5}{12}B)=80{,}000$$

You can also write a formula for the dollar weighted return based on an exposure formula. This is equivalent to the prior formula:

$$B=\frac{-30{,}000}{100{,}000(\frac{12}{12})+30{,}000(\frac{9}{12})-20{,}000(\frac{5}{12})}$$

$$= 30/\,[100+22.50-8.33]$$

$$= -26.28\%$$

$$|A+B| = |-26.10-26.28|$$ Using absolute values

$$= 52.37\%$$

D

11 The key to this problem is knowing the formula for the central death rate m_x under the assumption of Uniform Distribution of Deaths:

$$m_x = \frac{d_x}{L_x} \qquad L_x = l_x - \tfrac{1}{2}(d_x)$$
$$L_x = l_{x+1} + \tfrac{1}{2}(d_x)$$
$$975 = 960 + \tfrac{1}{2}(d_x)$$

Based on the values given, you can calculate $d_x = 30$ and then calculate $1000 m_x$:

$$1000\left(\frac{d_x}{L_x}\right) = 1000\left(\frac{30}{975}\right)$$
$$= 30.8$$

C

12 (i) $12(100)\left[\ddot{a}_x^{(12)} + 50\%\left(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}\right)\right]$

(ii) $12(110)\left[50\%\,\ddot{a}_x^{(12)} + 50\%\,\ddot{a}_y^{(12)}\right]$

(iii) $12P\left[\ddot{a}_x^{(12)} + 50\%\left(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}\right)\right]$
$+12P(10\%)\left[\ddot{a}_x^{(12)} - \ddot{a}_{xy}^{(12)}\right]$ } Tricky – must read problem carefully for this one

All three of the present values are the same, based on them being actuarially equivalent. You can simplify the expressions by getting rid of the $^{(12)}$ and the factor of 12. Also, simply set up the equation of value based on a present value of Q:

(i) $Q = 100\left[\ddot{a}_x + .5(\ddot{a}_y - \ddot{a}_{xy})\right]$

(ii) $Q = 110\left[.5\ddot{a}_x + .5\ddot{a}_y\right]$

(iii) $Q = P\left[\ddot{a}_x + .5(\ddot{a}_y - \ddot{a}_{xy}) + .1(\ddot{a}_x - \ddot{a}_{xy})\right]$

At this point, you have an algebra problem. This looks similar to prior exam questions, but it really isn't similar. You have three equations in 5 unknowns, which is not solvable. If you divided all three equations by $\ddot{a}_x$, you could get rid of one unknown, but you still could not solve the problem.

(next page)

(12) Continued

This is a 5 point question because of the time it takes you to think of an alternate method of solution. If you look at it long enough, you'll realize that you can use equations (i) and (ii) to produce equation (iii).

The first part of equation (iii) is simply (P/100) times (i). You can get the second part of (iii) by multiplying both (i) and (ii) by different ratios, and subtracting out the $\ddot{a}_y$ terms:

From (i) $Q/100 = \ddot{a}_x + .5(\ddot{a}_y - \ddot{a}_{xy})$

From (ii) $Q/110 = .5(\ddot{a}_x + \ddot{a}_y)$

Subtract $\frac{Q}{100} - \frac{Q}{110} = .5\ddot{a}_x - .5\ddot{a}_{xy}$

$\frac{Q}{500} - \frac{Q}{550} = .1(\ddot{a}_x - \ddot{a}_{xy})$

Now you can substitute the results in equation (iii)

$$Q = P[\ddot{a}_x + .5(\ddot{a}_y - \ddot{a}_{xy}) + .1(\ddot{a}_x - \ddot{a}_{xy})]$$
$$= P[\ Q/100 \quad + Q/500 - Q/550]$$
$$P = 1/[\ 1/100 + 1/500 - 1/550]$$
$$= 98.21$$

C

If you mis-read the data in the problem, you might set up equation (iii) incorrectly as

(iii) $12P[\ddot{a}_x^{(12)} + 50\%(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)})] + 12P(110\%)(\ddot{a}_x^{(12)} - \ddot{a}_{xy}^{(12)})$

But this produces a result far less than 90, outside the "implied range"

Solution added 04/12/06

(12) There is another method of solution that is less tricky. The key is still to reduce the number of variables:

(1) $12(100)(\ddot{a}_x^{(12)}+.5(\ddot{a}_y^{(12)}-\ddot{a}_{xy}^{(12)})) = 12(100)[(.5\ddot{a}_x^{(12)}+.5\ddot{a}_y^{(12)})+(.5\ddot{a}_x^{(12)}-.5\ddot{a}_{xy}^{(12)})]$

(2) $12(110)(.5\ddot{a}_x^{(12)}+.5\ddot{a}_y^{(12)})$

(3) $12P(\ddot{a}_x^{(12)}+.5(\ddot{a}_y^{(12)}-\ddot{a}_{xy}^{(12)}))+12(.1P)(\ddot{a}_x^{(12)}-\ddot{a}_{xy}^{(12)})$
$= 12P(.5(\ddot{a}_x^{(12)}+\ddot{a}_y^{(12)})+.5(\ddot{a}_x^{(12)}-\ddot{a}_{xy}^{(12)}))+12(.1P)(\ddot{a}_x^{(12)}-\ddot{a}_{xy}^{(12)})$

Now replace $(\ddot{a}_x^{(12)}+\ddot{a}_y^{(12)})$ with Z, and $(\ddot{a}_x^{(12)}-\ddot{a}_{xy}^{(12)})$ with W:

(1) $12(100)(.5Z+.5W)$
(2) $12(110)(.5Z)$
(3) $12P(.5Z+.5W)+12(.1P)W$

Set (1)=(2), and get expressions for W and $(Z+W)$:

$12(100)(.5Z+.5W) = 12(110)(.5Z)$
$100Z+100W = 110Z$
$W = .1Z$

$12(100)(.5)(Z+W) = 12(110).5Z$
$100Z+100W = 110Z$
$Z+W = 1.1Z$

Now substitute those values into the third expression:

$12(110)(.5)Z = 12P(.5)(1.1Z)+12(.1P)(.1Z)$
$55 = P[.55+.01]$
$P = 98.21$

C

13 This is a typical EA-1 question on actuarial equivalence. When two benefits are actuarially equivalent, they have the same present value:

$$30{,}000\,\ddot{a}_{62} = 50{,}000 + v^3\,{}_3p_{62}\,(\ddot{a}_{\overline{5}|.07})\,X$$

$$X = \frac{30{,}000\,\ddot{a}_{62} - 50{,}000}{v^3\,{}_3p_{62}\,(\ddot{a}_{\overline{5}|.07})}$$

$$= \frac{30{,}000(12.67977) - 50{,}000}{(1.07)^{-3}(.99)^3(1.07)(a_{\overline{5}|.07})}$$

$$= \frac{330{,}393}{3.4749}$$

$$= 95{,}080$$

E

Note:

I write $\ddot{a}_{\overline{5}|.07}$, but actually calculated the value as $1.07(a_{\overline{5}|.07})$. For the EA-1 exam problems, I use the HP-12C calculator. I always leave it set to calculate annuity immediate values, instead of annuity due. This helps avoid arithmetic errors that can occur if I forget to set the calculator to do an annuity due instead of an annuity immediate.

14 This is another typical problem on actuarial equivalence. One key is to read the problem carefully to see that you are given the temporary life annuity for 10 years. The present value of the normal form benefit is based on a different annuity:

$$\text{PV of normal form} = 12(3000)\ddot{a}^{(12)}_{\overline{62:\overline{10}|}}$$

$$\ddot{a}^{(12)}_{\overline{62:\overline{10}|}} = \ddot{a}^{(12)}_{\overline{10}|.07} + {}_{10|}\ddot{a}^{(12)}_{62}$$

$$= \ddot{a}^{(12)}_{\overline{10}|.07} + \ddot{a}^{(12)}_{62} - \ddot{a}^{(12)}_{62:\overline{10}|}$$

$$\ddot{a}^{(12)}_{\overline{10}|.07} = \frac{\ddot{a}_{\overline{120}|j}}{12} \quad \text{where } 1+j = (1.07)^{\frac{1}{12}} = 1.0057$$

$$= \frac{1.0057}{12}\left(a_{\overline{120}|.57\%}\right)$$

$$= 7.2871$$

$$\text{PV of normal form} = 12(3000)[7.2871 + 9.61521 - 6.94029]$$

$$= 358,634$$

$$\text{PV of optional form} = 12X\ddot{a}^{(12)}_{62}$$

Since the benefits are actuarially equivalent, you can equate the present values, and solve for X:

$$12X\ddot{a}^{(12)}_{62} = 358,634$$

$$X = \frac{358,634}{12\ddot{a}^{(12)}_{62}}$$

$$= 3,108.22$$

C

15 This is another problem on actuarially equivalent benefits, but with a slight twist. Instead of different benefits for the same participant, you are given different benefits for Smith and Jones. And you are told that Jones' present value is 4X that of Smith.

Smith's present value $X(\ddot{a}_{61:\overline{5|}})$

Jones' present value $20{,}000(\ddot{a}_{\overline{10|}.07} + {}_{10|}\ddot{a}_{60})$

$$4X(\ddot{a}_{61:\overline{5|}}) = 20{,}000(\ddot{a}_{\overline{10|}.07} + {}_{10|}\ddot{a}_{60})$$

$$X = \frac{5{,}000(\ddot{a}_{\overline{10|}.07} + \ddot{a}_{60} - \ddot{a}_{60:\overline{10|}})}{\ddot{a}_{61:\overline{5|}}}$$

The only factor you aren't given is $\ddot{a}_{61:\overline{5|}}$. You can derive it based on some typical relationships:

$$\ddot{a}_x = 1 + vp_x(\ddot{a}_{x+1})$$

$$\ddot{a}_{x:\overline{n|}} = 1 + vp_x(\ddot{a}_{x+1:\overline{n-1|}})$$

$$\ddot{a}_{61:\overline{5|}} = 1 + vp_{61}(\ddot{a}_{62:\overline{4|}})$$

$$= 1 + \frac{.99394}{1.07}(3.58056)$$

$$= 4.3260$$

$$X = \frac{5{,}000[1.07(a_{\overline{10|}.07}) + 11.53496 - 7.26514]}{4.3260}$$

$$= 13{,}621$$

C

16 Unlike the prior few problems, this is a difficult and messy problem on actuarial equivalence. The key to the solution is carefully writing down the payments on a time line diagram, and calculating the present value of remaining payments at 1-1-2006.

Annuity #1 : 40 payments over 10 years

		1000	$1000(1.01)^1$	$1000(1.01)^2$	$1000(1.01)^3$	...	$1000(1.01)^7$	$1000(1.01)^8$	...	$1000(1.01)^{39}$
Payment	0	1	2	3	4	...	8	9	...	40
Date	1-1-04				12-31-04		12-31-05			

The idea of writing things down on the diagram allows you to check that the exponents make sense. Here, the exponent for 1.01 is one less than the payment number.

The present value of o/s payments at 1-1-06 requires a quarterly rate of interest : $(1+j)^4 = 1.07 \Rightarrow j = 1.71\%$

$$\text{1-1-06 PV of \#1} = 1000\left[\frac{(1.01)^8}{(1+j)^1} + \frac{(1.01)^9}{(1+j)^2} + \cdots + \frac{(1.01)^{39}}{(1+j)^{32}}\right]$$

$$= 1000(1.01)^7\left[\left(\frac{1.01}{1+j}\right)^1 + \left(\frac{1.01}{1+j}\right)^2 + \cdots + \left(\frac{1.01}{1+j}\right)^{32}\right]$$

$$= 1072.14\, a_{\overline{32}|k} \quad \text{where } 1+k = \frac{1+j}{1.01} = 1.0070$$

$$= 1072.14\, a_{\overline{32}|.70\%}$$

$$= 30{,}647$$

(next page)

(16) Annuity #2 : 15 payments over 15 years

		X	$X-25$	$X-25(2)$	...	$X-25(13)$	$X-25(14)$
payment	0	1	2	3	...	14	15
Date	1-1-04	12-31-04	12-31-05	12-31-06			

1-1-06 PV of #2 $= (X-25)\, a_{\overline{13}|.07} - 25\,(Ia_{\overline{13}|.07})$

The key to this present value calculation is knowing the formula for $Ia_{\overline{n}|i} = a_{\overline{n}|i} + \dfrac{a_{\overline{n}|i} - nv^n}{i}$

$$30{,}647 = \text{PV of \#2} = (X-25)\, a_{\overline{13}|.07} - 25\left(a_{\overline{13}|.07} + \frac{a_{\overline{13}|.07} - 13(1.07)^{-13}}{.07}\right)$$

$$= X a_{\overline{13}|.07} - 50\, a_{\overline{13}|.07} - 25\left(a_{\overline{13}|.07} - 13(1.07)^{-13}\right)/.07$$

$$X = \frac{30{,}647 + 50 a_{\overline{13}|.07} + 25\left(a_{\overline{13}|.07} - 13(1.07)^{-13}\right)/.07}{a_{\overline{13}|.07}}$$

$$= 3{,}666.97 + 50 + 126.62$$

$$= 3{,}843.59 \qquad \text{(B)}$$

There is another method of solution for Annuity #2, which is to write down the present value without the $Ia_{\overline{13}|}$, and solve for X by algebraic manipulation:

$$30{,}647 = PV2 = v^1(X-50) + v^2(X-75) + \dots + v^{12}(X-325) + v^{13}(X-350)$$

$$30{,}647v = v^2(X-50) + \dots + v^{12}(X-300) + v^{13}(X-325) + v^{14}(X-350)$$

$$30{,}647(1-v) = v^1(X-50) - 25(v^2 + \dots + v^{12} + v^{13}) - v^{14}(X-350)$$

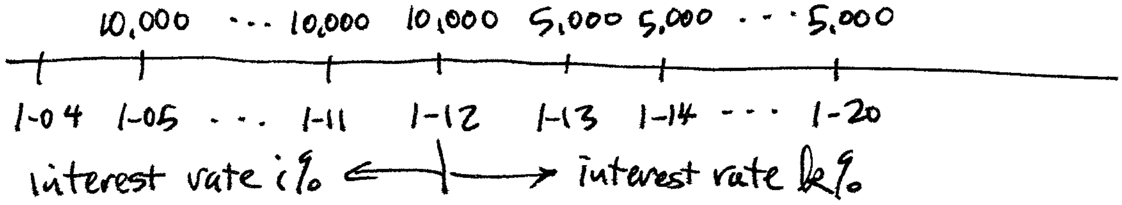
$$v(X-50) - v^{14}(X-350) = 30{,}647(1-v) + 25v\,(a_{\overline{12}|.07})$$

$$X = 3843.59$$

17 This is a good test of your knowledge of how sinking funds work. There are no payments made on the initial loan, which will be paid off by the sinking fund balance at 12/31/19.

1-1-2020 accumulated loan balance $= 100{,}000(1.05)^{16}$

1-1-2020 Accum S.F. balance $= (10{,}000\, s_{\overline{8}|i})(1+k)^8 + 5{,}000\, s_{\overline{8}|k}$



You are told that in 2011, the loan interest and the interest earned in the Sinking Fund are equal. This allows you to solve for the value of i:

O/S Loan at 1-1-05 $= 100{,}000(1.05)$
O/S Loan at 1-1-11 $= 100{,}000(1.05)^7$
2011 Loan interest $= .05(100{,}000)(1.05)^7$

Accum SF at 1-1-05 $= 10{,}000$
Accum SF at 1-1-11 $= 10{,}000\, s_{\overline{7}|i}$
2011 interest on S.F. $= i(10{,}000)\, s_{\overline{7}|i}$

$$= i(10{,}000)\frac{(1+i)^7 - 1}{i}$$

$$= 10{,}000\left((1+i)^7 - 1\right)$$

(next page)

(17) Now set the values equal, and solve for i

$$.05(100,000)(1.05)^7 = 10,000[(1+i)^7 - 1]$$
$$.05(10)(1.05)^7 = (1+i)^7 - 1$$
$$(1+i)^7 = 1.7036$$
$$i = 7.91\%$$

To solve for k, equate the value of the accumulated loan and accumulated S.F. at 1-1-2020:

$$100,000(1.05)^{16} = (10,000\, s_{\overline{8}|7.91\%})(1+k)^8 + 5,000\, s_{\overline{8}|k}$$
$$218,287 = 106,011(1+k)^8 + 5,000\, s_{\overline{8}|k}$$

At this point, you can simply test the answer ranges to see which one is correct. Start with 6.60%, and calculate the resulting value:

$$\text{if } k = 6.60\% \Rightarrow 106,011(1.066)^8 + 5,000\, s_{\overline{8}|6.6\%} = 227,336$$

This is too large, so go down to the next lower range

$$\text{if } k = 6.10\% \Rightarrow 106,011(1.061)^8 + 5,000\, s_{\overline{8}|6.1\%} = 219,910$$

Clearly, k must be less than 6.1% ⇒ A

You can solve for the exact value of k by using iteration. Rearrange the formula above to produce one with k by itself on one side:

$$218,287 - 5,000\, s_{\overline{8}|k} = 106,011(1+k)^8$$
$$\left[\frac{218,287 - 5,000\, s_{\overline{8}|k}}{106,011}\right]^{1/8} = 1+k$$

(next page)

 Solution added 04/12/06

(17) $$k = \left[\frac{218{,}287 - 5{,}000\, s_{\overline{8}|k}}{106{,}011}\right]^{1/8} - 1$$

Now you can start with 6.6% as the initial guess, and generate a new guess for the value of k:

k_0 : 6.60%
k_1 : 5.90%
k_2 : 6.00%
k_3 : 5.99%

At this point, you can stop. The value of k has converged on answer range (A). One disadvantage of using iteration is there is NO guarantee that the values will converge. It is entirely dependent on the formula you set up for iteration.

A third approach is to use your calculator to solve for k. These instructions are for the HP-12C (similar for other calculators):

$$\underbrace{218{,}287}_{FV} = \underbrace{106{,}011}_{PV=-106{,}011}(1+k)^8 + \underbrace{5{,}000}_{PMT=-5{,}000}\, s_{\overline{\underbrace{8}_{N}}|k}$$

Press the "i" button, and the calculator does the iteration. After about 15 seconds, you have k = 5.9886%. If you enter the PV/FV/PMT with the wrong signs, you will get (i) Error 5, or (ii) bogus results, such as 11.8972% or 13.0141%

18 This is a strange spin on a life insurance problem. As usual, there are no commutation functions. You get to accumulate the premiums, pay out the death claims, and see how much money remains after 3 years.

The insurance premium is calculated based on $l_x = 100 - x$:

$$A^1_{50:\overline{3}|} = v\,q_{50} + v^2\,p_{50}\,q_{51} + v^3\,{}_2p_{50}\,q_{52}$$

$$= (1.05)^{-1}\frac{d_{50}}{l_{50}} + (1.05)^{-2}\frac{l_{51}}{l_{50}}\left(\frac{d_{51}}{l_{51}}\right) + (1.05)^{-3}\frac{l_{52}}{l_{50}}\left(\frac{d_{52}}{l_{52}}\right)$$

$$= \frac{1}{50}\left[(1.05)^{-1} + (1.05)^{-2} + (1.05)^{-3}\right]$$

$$= a_{\overline{3}|.05}/50$$

$$= .0545$$

Total premium $= 1300\,(10{,}000)(.0545)$
$= 708{,}044$

The fund earns 5.25% each year, and the actual claims are based on $l_x = 102 - x$:

Year	Age	Fund at BOY	i earned	q_x	Claims paid
1	50	708,044	5.25%	1/52	$1300\left(\frac{1}{52}\right)10{,}000 = 250{,}000$
2	51		"	1/51	$1300\left(\frac{51}{52}\right)\left(\frac{1}{51}\right)10{,}000 = 250{,}000$
3	52		"	1/50	$1300\left(\frac{50}{52}\right)\left(\frac{1}{50}\right)10{,}000 = 250{,}000$

(next page)

Solution added 04/12/06

(18) Now you can write down the fund value at ~~Thes~~ end of each year. You don't actually have to calculate any value except the final one:

Year	Fund at BOY	Claims Paid	Fund at EOY
1	708,044	250,000	$1.0525(708{,}044) - 250{,}000$
2			$(1.0525)^2(708{,}044) - 250{,}000\,s_{\overline{2}\rceil}$
3			$(1.0525)^3(708{,}044) - 250{,}000\,s_{\overline{3}\rceil}$
			$= 35{,}455$

E

Another method of solution is to calculate the insurance premium based on the actual experience (5.25% and $l_x = 102-x$). If you accumulate the difference in the two premiums at 5.25% for three years, you get exactly the same answer. The reason is that this revised premium would produce a zero fund balance after three years, since the assumptions exactly match the experience:

A_x based on 5.25% and $l_x = 102 - x$

$$A^1_{50:\overline{3}\rceil} = \frac{(1.0525)^{-1} + (1.0525)^{-2} + (1.0525)^{-3}}{52}$$

$$= a_{\overline{3}\rceil .0525}/52$$

$$= .0521$$

Difference in total premiums $= 1300(10{,}000)(.0545 - .0521) = 30{,}409$

Accumulated value at 5.25% for 3 years 35,455

19 The keys to working this problem are
(1) you must remember the formula for this type of Joint and Survivors benefit, and
(2) carefully read the data - you are not given annuities due, but annuities immediate

When you have a J+S annuity that reduces upon either the death of the participant or the spouse, the formula with a K% continuation is as follows:

$$K\ddot{a}_x + K\ddot{a}_y + (1-2K)\ddot{a}_{xy}$$

In this problem, the benefit is 10,000 and the continuation fraction is 60%. Based on the ages of the participant and their spouse, you can write the present value:

$$PV = 10{,}000\left[.60(\ddot{a}_{65}) + .60(\ddot{a}_{64}) + [1-2(.60)]\ddot{a}_{65:64}\right]$$

You can easily write the values for $\ddot{a}_{65}$ and $\ddot{a}_{65:64}$. For $\ddot{a}_{64}$, use the typical formula

$$\ddot{a}_x = v p_x \ddot{a}_{x+1} \qquad \ddot{a}_{64} = v p_{64} \ddot{a}_{65}$$

$$= \frac{(1-q_{64})(1+a_{65})}{1.07}$$

$$\ddot{a}_{64} = 11.0250 = (1-8.685/1000)(1+9.8207)/1.07$$

$$PV = 10{,}000\left[.6(1+9.8207) + .60(11.0250) - .20(1+8.4129)\right]$$
$$= 112{,}248$$

C

20 A service table is constructed from several single decrement tables. In this problem, the service table has two decrements: death and withdrawal. At any age x, the probability of survival is written as

$$p_x^{(T)} = 1 - q_x^{(w)} - q_x^{(d)}$$

The term cost for a benefit is the present value of the expected exits during the current year:

2004 withdrawal term cost $= 93{,}084 = (1{,}000)\,v\,q_{40}^{(w)}(1{,}000)$
2004 death term cost $= \quad X = (1{,}000)\,v\,q_{40}^{(d)}(2{,}000)$

You are not given values for either decrement at age 40. But you are told the total present value of bonus payments at 1/1/2004:

PV of bonuses $= 2{,}167{,}971 = 1{,}000\,v^2(1-q_{40}^{(d)}-q_{40}^{(w)})(1-q_{41}^{(d)}-q_{41}^{(w)})(3{,}000)$

Step 1 - calculate $q_{40}^{(w)}$

$93{,}084(1.07)/1{,}000{,}000 = q_{40}^{(w)} = .0996$

Step 2 - plug values into bonus formula

$2{,}167{,}971\,(1.07)^2/(3{,}000{,}000) = (1-q_{40}^{(d)}-.0996)(1-.0007-200q_{40}^{(d)})$

$.8274 = [.9004-q_{40}^{(d)}][.9993-200q_{40}^{(d)}]$

(next page)

(20) For simplicity, I'll replace $q_{40}^{(d)}$ with Y. You must know the formula to solve a quadratic formula* — this is the key to working the problem

$$(.9004 - Y)(.9993 - 200Y) = .8274$$
$$200Y^2 - 181.0793Y + .0724 = 0$$

$$Y = \frac{181.08 \pm \sqrt{(181.08)^2 - 4(200)(.0724)}}{2(200)}$$
$$= \frac{181.08 \pm 180.92}{400}$$
$$= .0004 \text{ or } .905$$

Based on the value given for $q_{40}^{(w)}$, the larger value is not valid for $q_{40}^{(d)}$. As a result, $q_{40}^{(d)} = .0004$. Now you can calculate the term cost for the death benefit:

$$X = (1000)v\, q_{40}^{(d)}\,(2{,}000)$$
$$= 1000(.0004)(2{,}000)/1.07$$
$$= 747.66$$

A

*Quadratic formula

if $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

21 This is a short question, if you use the right identity formula:

$A_x = v\,q_{76} + v^2{}_{1|}q_{76} + \ldots = (v d_x + v^2 d_{x+1} + \ldots)/\ell_x$

$A_{x+1} = \qquad\qquad (\quad v d_{x+1} + v^2 d_{x+2} + \ldots)/\ell_{x+1}$

$A_x = v q_x + v p_x A_{x+1}$

$A_{76} = v q_{76} + v p_{76} A_{77}$

$$A_{77} = \frac{A_{76} - v q_{76}}{v p_{76}}$$

${}_1E_{76} = v p_{76} = .90$

$p_{76} = .90(1.03)$

$= .9270$

$q_{76} = 1 - .9270$

$= .0730$

$v q_{76} = .0730/1.03$

$= .0709$

$$A_{77} = \frac{.80 - .0709}{.90}$$

$= .8101$ (C)

If you start out using this identity, you will also get to the correct answer:

$A_x = 1 - d\,\ddot{a}_x$

$A_{x+1} = 1 - d\,\ddot{a}_{x+1}$

$\ddot{a}_x = 1 + v p_x \ddot{a}_{x+1}$

22 In general, this formula relates the probability of survival to the force of mortality:

$$_n p_x = e^{-\int_x^{x+n} \mu_y \, dy} \qquad \text{Bowers 3.2.14}$$

The problem does not state anything specific about the force of mortality, but it seems reasonable to assume that it is constant for each one year interval:

$\mu_{x+t} = \mu_x$ for $0 \le t < 1$

$$_t p_x = e^{-\int_0^t \mu_x \, dy}$$
$$= e^{-t\mu_x}$$

Once you double the force of mortality, then the probability of survival is squared:

$$\text{new } {_t p_x} = e^{-t(2\mu_x)}$$
$$= \left(e^{-t\mu_x}\right)^2$$
$$= \left(\text{old } {_t p_x}\right)^2$$

Based on the values given for e_x, you can solve for the old p_x values at ages 107 and 108. Then you can calculate the new value for e_{107}:

$$e_x = p_x + {_2 p_x} + {_3 p_x} + \ldots$$

old $e_{108} = p_{108}(1 - p_{109}) \qquad e_{109} = 0 \Rightarrow p_{109} = 0$

$.20 = p_{108}$

(next page)

(22) old $e_{107} = p_{107} + {}_2p_{107} + \cdots$

$= p_{107}\,(1 + p_{108} + 0)$

$.6 = p_{107}(1 + .2)$

old $p_{107} = .60/1.2$

$= .50$

New $p_{108} = (\text{old } p_{108})^2$

$= (.2)^2$

$= .04$

new $p_{107} = (\text{old } p_{107})^2$

$= (.5)^2$

$= .25$

new $e_{107} = p_{107}(1 + p_{108} + 0)$

$= .25(1 + .04)$

$= .26$

B

23 The key to working this problem is recognizing that you have five separate bonds, each of which is redeemed over a five year period. The face amount of each bond is \$200.

The preferred technique is to use the serial bond formula

$$P = K + \frac{g}{i}(C-K)$$

$$= Cv^n + \frac{Fr}{Ci}(C - Cv^n)$$

Cv^n = present value of redemption amounts

One key point of the problem is that Fr and Ci must be calculated using the semiannual interest rates. This matches the coupon payment period:

$(1+i)^2 = 1.05 \Rightarrow i = 2.47\%$

$Cv^n = 200(v^{20} + v^{22} + \ldots + v^{28})$

This is easier to calculate by using the 5% annual rate:

$Cv^n = 200((1.05)^{-10} + \ldots + (1.05)^{-14})$

$= 200(a_{\overline{14}|.05} - a_{\overline{9}|.05})$

$= 558.16$

$P = Cv^n + (Fr/Ci)(C - Cv^n)$

$= 558.16 + [(200(.03))/(200(.0247))](5(200) - 558.16)$

$= 558.16 + 536.75$

$= 1094.91$ C

(next page)

(23) You can also calculate the price of the series of bonds using any other bond formula:

$$P = Fr\,a_{\overline{n}|i} + K \quad \text{for each individual bond}$$

$$P_1 = 200(.03)\,a_{\overline{20}|.0247} + 200(1.0247)^{-20}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$P_5 = 200(.03)\,a_{\overline{28}|.0247} + 200((1.0247)^{-28}$$

$$\Sigma P = 6\left[a_{\overline{20}|2.47\%} + a_{\overline{22}|2.47\%} + \dots + a_{\overline{28}|2.47\%}\right] + 200\left[(1.05)^{-10} + \dots + (1.05)^{-14}\right]$$

$$= 1094.91$$

Compared to using Makeham's formula, you need to calculate the five annuities at 2.47%. This is not a big deal, since you can use the calculator!

24 You can work this problem using the same identities as in problem 21:

$$A_x = vq_x + v^2{}_{1|}q_x + v^3{}_{2|}q_x + \ldots$$
$$= v\left[q_x + p_x A_{x+1}\right]$$
$$A_{x+1} = v\left[q_{x+1} + p_{x+1} A_{x+2}\right]$$

$$\frac{A_x}{A_{x+1}} = \frac{q_x + (1-q_x)A_{x+1}}{q_{x+1} + (1-q_{x+1})A_{x+2}}$$

$$q_x + A_{x+1} - q_x A_{x+1} = \frac{A_x}{A_{x+1}}\left(q_{x+1} + (1-q_{x+1})A_{x+2}\right)$$

$$q_x = \frac{A_x\left[q_{x+1} + (1-q_{x+1})A_{x+2}\right]}{A_{x+1}\,(1-A_{x+1})} - \frac{A_{x+1}}{(1-A_{x+1})}$$

$$= \frac{.18\left[.01125 + .98875(.20)\right]}{.19(.81)} - \frac{.19}{.81}$$

$$= \frac{.037620}{.1539} - \frac{.19}{.81}$$

$$= .0098765$$

$$1000q_x = 9.8765$$ A

NOTE: This is one of those rare problems where the data is "fishy". Several of the data items only have 2 significant digits. In general, that means the answer ranges should only have 2 significant digits – but they have 4 significant digits instead!

2004

25 The key to working this problem is writing down the series of payments on a time-line diagram:

Decreasing Annuity		10	9	...	2	1	0	0	...
Increasing Perpetuity		1	2	...	9	10	11	11	...
Payment *	0	1	2	...	9	10	11	12	...

Each of the annuities has the same present value. If you add the two payment streams together, it will be easy to calculate the present value

PV of Sum $= 2 * (\text{PV of either annuity})$

$2X =$ PV of perpetuity immediate of 11

$= 11/i$

$X = 5.5/i$

PV of decreasing annuity $= X = (Da)_{\overline{10}|i}$

$$= \frac{10 - a_{\overline{10}|i}}{i}$$

$$\frac{5.5}{i} = \frac{10 - a_{\overline{10}|i}}{i}$$

$4.5 = a_{\overline{10}|i}$

$i = 17.963\%$ from the calculator

$X = 5.5/i$

$= 30.62$ D

26 This is another problem based on London's Survival Models book.

I bought the book, but don't know the solution yet.

27 This is a typical exam question on loan amortization schedules. The key to the problem is noticing that you are given the nominal annual rate, instead of the effective annual rate.

Original loan $= M a_{\overline{120}|j}$ $\quad j = \frac{.075}{12} = .625\%$

Loan amortization schedule

Payment number	Principal Paid	Interest Paid	O/S Loan
0			$M a_{\overline{120}\|.625\%}$
1	Mv^{120}	$M(1-v^{120})$	$M a_{\overline{119}\|}$
2	Mv^{119}	$M(1-v^{119})$	$M a_{\overline{118}\|}$
⋮	⋮	⋮	⋮
54	Mv^{67}	$M(1-v^{67})$	$M a_{\overline{66}\|}$
⋮	⋮	⋮	⋮
90	Mv^{31}	$M(1-v^{31})$	$M a_{\overline{30}\|}$

Interest paid in 54th pmt $\quad 100 = M(1-v^{67})$

Principal O/S after 90th pmt $\quad P = M a_{\overline{30}|}$

$$= M\,\frac{(1-v^{30})}{i}$$

$$= \frac{100}{1-v^{67}}\left(\frac{1-v^{30}}{.625\%}\right)$$

$$= 7992.38$$

A

28 This is a typical bond problem. Write down the formulas for the price of the bond:

$$P = 1000\left(\frac{r}{2}\right) a_{\overline{20}|j} + 1{,}100(1.04)^{-10} \quad \text{where } (1+j) = (1.04)^{\frac{1}{2}} \Rightarrow j = 1.98\%$$

$$P - 95.50 = 1000\left(\frac{r}{2}\right) a_{\overline{20}|k} + 1{,}100(1.05)^{-10} \quad \text{where } 1+k = (1.05)^{\frac{1}{2}} \Rightarrow k = 2.47\%$$

You now have two equations in two unknowns, so you can solve for the value of r. If you subtract the two bond prices, you have one equation for r:

$$95.50 = 500r\left(a_{\overline{20}|1.98\%} - a_{\overline{20}|2.47\%}\right) + 1{,}100\left((1.04)^{-10} - (1.05)^{-10}\right)$$
$$= 374.13r + 67.82$$
$$r = 7.400\%$$

B

29) The key to this problem is knowing all the formulas from Bowers table 3.6.1, or being able to derive the results for the various mortality table assumption.

You are asked the value of Z under the constant force of mortality assumption:

$${}_tp_x = (p_x)^t \quad \text{under constant force}$$

$${}_yq_{x+t} = 1-(p_x)^y$$

$$Z = {}_yq_{98.15}$$

$$= 1-(p_{98})^y$$

You are also given values for $\mu_{98.55}$ and ${}_yq_{98.35}$ under the uniform distribution of death assumption. You need to know the corresponding formulas to solve for q_{98} and y:

$$\mu_{x+t} = \frac{q_x}{1-t(q_x)} \quad \text{under U.D.D.}$$

$$\mu_{98.55} = \frac{q_{98}}{1-.55(q_{98})} = .5980 \Rightarrow q_{98} = .5980 - .55(.5980)q_{98}$$

$$q_{98} = .5980/1.3289$$

$$= .4500$$

$${}_yq_{x+t} = \frac{y(q_x)}{1-t(q_x)} \quad \text{under U.D.D.}$$

$${}_yq_{98.35} = \frac{y(q_{98})}{1-.35(q_{98})} = .2486 \qquad y = \frac{.2486(1-.35(q_{98}))}{q_{98}}$$

$$= .2486(1-.35(.45))/.45$$

$$= .4654$$

(next page)

(29) Now you can solve for the value of Z

$Z = 1 - (p_{98})^{y}$

$= 1 - (1 - .45)^{.4654}$

$= .2429$

A

30 This is one of the standard questions asked each year on stationary population theory. The key is knowing the formula for the average age at death of those who die between age z and age $z+n$:

$$\text{Avg age at death} = z + \frac{T_z - T_{z+n} - n\ell_{z+n}}{\ell_z - \ell_{z+n}} = 50 \quad \text{(given)}$$

In the context of this problem, you are solving for X, the number of employees who retire each year; this is ℓ_{z+n}, which is ℓ_{55}.

$$50 = 24 + \frac{T_{24} - T_{55} - 31\ell_{55}}{\ell_{24} - \ell_{55}}$$

T_{55} is zero, since the stationary population only includes people between ages 24 and 55. T_{24} is 15,000, or the total number in the population. ℓ_{24} is 510, or the number of new lives each year.

$$50 = 24 + \frac{15{,}000 - 0 - 31X}{510 - X}$$

$$26(510 - X) = 15{,}000 - 31X$$

$$5X = 1{,}740$$

$$X = 348$$

A

31 This problem mainly consists of knowing the definitions for multiple decrement tables. Z is the change in $d_{26}^{(1)}$ $(= l_{26}^{(T)} q_{26}^{(1)})$ due to the change in value of $q_{25}^{(2)}$. You must calculate $l_{25}^{(T)}$.

Original $q_{25}^{(2)} = .30$

$$l_{26}^{(T)} = 6500$$
$$q_{26}^{(1)} = .05$$
$$d_{26}^{(1)} = l_{26}^{(T)} q_{26}^{(1)} = 6500(.05)$$
$$= 325$$
$$p_{25}^{(T)} = \frac{l_{26}^{(T)}}{l_{25}^{(T)}} = 1 - q_{25}^{(1)} - q_{25}^{(2)}$$
$$\frac{6500}{l_{25}^{(T)}} = 1 - .05 - .30$$
$$l_{25}^{(T)} = 10{,}000$$

New $q_{25}^{(2)} = .25$

$$p_{25}^{(T)} = 1 - q_{25}^{(1)} - q_{25}^{(2)}$$
$$= 1 - .05 - .25$$
$$= .70$$
$$l_{26}^{(T)} = l_{25}^{(T)} p_{25}^{(T)}$$
$$= 7{,}000$$
$$d_{26}^{(1)} = l_{26}^{(T)} q_{26}^{(1)}$$
$$= 7{,}000(.05)$$
$$= 350$$
$$\Delta d_{26}^{(1)} = 350 - 325$$
$$= 25.0$$

E

32 This is another typical bond problem. You need to write the formulas for the price of each bond. You will have two equations in two unknowns, and then you can solve for the yield rate.

Bond A: $P = 100(6\%/2)\, a_{\overline{40}|j} + 100v^{40}$

NOTE: j is the semi-annual yield rate

Bond B: $P = 100(5\%/2)\, a_{\overline{40}|j} + 125v^{40}$

$$3a_{\overline{40}|j} + 100v^{40} = 2.5a_{\overline{40}|} + 125v^{40}$$
$$.5a_{\overline{40}|j} = 25v^{40}$$

you could try iteration here, but it is simpler to multiply both sides by $(1+j)^{40}$

$$.5s_{\overline{40}|j} = 25$$
$$s_{\overline{40}|j} = 50$$
$$j = 1.109\%$$

As noted above, this is the semi-annual yield rate. The question asks for the annual effective yield:

Annual $(1+j)^2 - 1 = 2.239\%$ D

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