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SPRING 2007 EA-1
EXAM SOLUTIONS

Revision History:
02/25/11 Revised solutions for problems 10, 27 and 30
02/01/10 Revised solutions for problems 2 and 10

2007 EA-1 Solutions

1 This is a seemingly simple question, but you need to read it carefully. The last payment is made at the end of the year of death, so it is not really like a_{30}. That annuity pays a dollar at the end of the year if the annuitant is still alive.

It may be helpful to think about payments made under various annuities:

a_{30} pays \$1 if Smith is alive at 12/31
va_{30} pays v if Smith is alive at 12/31
$v\ddot{a}_{30}$ pays v at 01/01 if Smith is alive at 01/01, which is equal to \$1 at 12/31 if Smith is alive at 01/01

What the problem is asking for is $v\ddot{a}_{30}$

$$v\ddot{a}_{30} = v(1+a_{30})$$

$$= \frac{1+17.57}{1.05}$$

$$= 17.69$$

B

2 The original version of this exam problem was slightly different. It showed both summations using v^t instead of v^{t+1}. They revised the problem in June of 2008, but they forgot to change the correct answer range.

If you look at the summations, the first one is $1000 a_{xy}$. The second summation provides an annuity to someone, currently at age y, after the death of someone currently at age x.

$$
\begin{aligned}
Z &= 1000\, a_{xy} + 500\,(a_y - a_{xy}) \\
&= 500(a_{xy} + a_y) \\
&= 500(8+9) \\
&= \$500
\end{aligned}
$$

A

You can derive the formula $a_y - a_{xy}$, but this is NOT the way to work the problem on the exam. It takes far too long to simplify all the results.

$$\sum_{t=0} v^{t+1}\ {}_{t|}q_x\,({}_{t+1}p_y)\,\ddot{a}_{y+t+1}$$

$$= v^1 q_x p_y \ddot{a}_{y+1} + v^2 p_x q_{x+1}\ {}_2p_y \ddot{a}_{y+2} + v^3\ {}_2p_x q_{x+2}\ {}_3p_y \ddot{a}_{y+3} + \ldots$$

Now you need to expand the annuity, and write each term entirely based on ${}_np_x$ and ${}_np_y$ values.

(next page)

(2) continued

$$v^1 q_x p_y \ddot{a}_{y+1} = v^1 (1-p_x) p_y (1 + v p_{y+1} + v^2 {}_2p_{y+1} + \dots)$$
$$= v p_y + v^2 {}_2p_y + v^3 {}_3p_y + \dots - p_x (v p_y + v^2 {}_2p_y + v^3 {}_3p_y + \dots)$$

$$v^2 p_x q_{x+1} {}_2p_y \ddot{a}_{y+2} = v^2 (p_x - {}_2p_x) {}_2p_y (1 + v p_{y+2} + v^2 {}_2p_{y+2} + \dots)$$
$$= p_x (v^2 {}_2p_y + v^3 {}_3p_y + v^4 {}_4p_y + \dots) - {}_2p_x (v^2 {}_2p_y + v^3 {}_3p_y + v^4 {}_4p_y + \dots)$$

$$v^3 {}_2p_x q_{x+2} {}_3p_y \ddot{a}_{y+3} = v^3 ({}_2p_x - {}_3p_x) {}_3p_y (1 + v p_{y+3} + v^2 {}_2p_{y+3} + \dots)$$
$$= {}_2p_x (v^3 {}_3p_y + v^4 {}_4p_y + v^5 {}_5p_y + \dots) - {}_3p_x (v^3 {}_3p_y + v^4 {}_4p_y + v^5 {}_5p_y + \dots)$$

If you look carefully at the formulas above, you almost have a telescoping sum. Most of the terms that are multiplied by ${}_np_x$ in one formula are cancelled out in the next one.

Here is what you are left with for the second summation:

$$\sum_{t=0} v^{t+1} {}_{t|}q_x ({}_{t+1}p_y) \ddot{a}_{y+t+1} = v p_y + v^2 {}_2p_y + v^3 {}_3p_y + \dots$$
$$- p_x (v p_y) - v^2 {}_2p_x {}_2p_y - v^3 {}_3p_x {}_3p_y - \dots$$
$$= a_y - a_{xy}$$

This matches what I had originally written down for the value of that summation.

3 This is a fairly typical exam question on probability calculations. One key to the problem is simply knowing the definition of the ${}_nE_x$ symbol, which is $v^n \, {}_np_x$. This is also called a pure endowment, since it equals the present value of a dollar payable to a life age x if they survive to age x+n.

$$p = 1 - {}_{30}q_{10} = {}_{30}p_{10} = l_{40}/l_{10}$$

$${}_{30}E_{10} = v^{30} \, {}_{30}p_{10} = v^{30} \, l_{40}/l_{10}$$
$${}_{10}E_{30} = v^{10} \, {}_{10}p_{30} = v^{10} \, l_{40}/l_{30}$$

$${}_{20}p_{10} = .960 = l_{30}/l_{10}$$

$$({}_{30}E_{10})({}_{10}E_{30}) = .125 = v^{30}\left(\frac{l_{40}}{l_{10}}\right) v^{10}\left(\frac{l_{40}}{l_{30}}\right)$$

Somehow you need to use l_{30}/l_{10} in this final formula. One way to do ~~this~~ is to add it as a factor, and also use the inverse as a factor:

these cancel out

$$\begin{aligned}
.125 &= v^{40}(l_{40}/l_{10})(l_{40}/l_{30})(l_{30}/l_{10})(l_{10}/l_{30}) \\
&= v^{40}(l_{40}/l_{10})(l_{40}/l_{10})(l_{10}/l_{30}) \\
&= (1.05)^{-40}(l_{40}/l_{10})^2(1/.960)
\end{aligned}$$

$$.125(1.05)^{40}(.960) = (l_{40}/l_{10})^2$$

$$(l_{40}/l_{10})^2 = .8448$$

$$p = l_{40}/l_{10} = .9191$$

E

4 This is a typical exam question on De Moivre's law. This is the name for any mortality table definition similar to $l_x = \omega - x$, where ω is the last age in the mortality table.

It can be shown that these formulas are correct if the mortality follows De Moivre's law:

$$a_x = \frac{n - \ddot{a}_{\overline{n}|i}}{n(i)} \qquad A_x = \frac{a_{\overline{n}|i}}{n}$$

Y is described as a 1000 annual life annuity due starting at age 60. Z is a lump sum payment of 15,000 if Smith survives to age 65.

$$Y = 1000\,\ddot{a}_{60}$$
$$= 1000(1 + a_{60})$$

$a_{60} \Rightarrow \omega = 100 \quad x = 60 \quad \omega - x = 40$

$$a_{60} = \frac{40 - \ddot{a}_{\overline{40}|.06}}{40(.06)} = \frac{40 - 1.06(15.0463)}{2.40}$$
$$= 10.0212$$

$$Z = 15{,}000\,v^5\,{}_5p_{60}$$
$$= 15{,}000(1.06)^{-5}\frac{l_{65}}{l_{60}}$$
$$= 15{,}000(.7473)\frac{35}{40}$$
$$Z = 9{,}808$$

$$Y = 1000(1 + 10.0212)$$
$$= 11{,}021$$

$$|Y - Z| = 11{,}021 - 9{,}808$$
$$= 1{,}213$$

D

5 This problem tests your understanding of the relationship between monthly and annual annuities. You need to solve for the annual effective rate of interest.

In general, when the payment period is different than the period for compounding interest, I convert the annuity so these items match. The easiest approach is to convert the interest rate so the compounding period is the same as the annuity payment period.

$$\ddot{a}^{(12)}_{\overline{2}|i} = \ddot{a}_{\overline{24}|j} \quad \text{where } (1+j)^{12} = 1+i$$

$$a^{(12)}_{\overline{2}|i} = a_{\overline{24}|j}$$

$$\ddot{a}_{\overline{24}|j} = (1+j)\, a_{\overline{24}|j}$$

$$1+j = \frac{1.892447}{1.883280} = 1.004868$$

$$1+i = (1.004868)^{12}$$

$$= 1.060$$

$$i = 6.0\%$$

C

6 This is a typical question on modified duration. It is interesting to do this for a perpetuity!

The first step is to convert the interest rate so it is annual, to match the payment frequency. The nominal interest rate is 7% compounded semi-annually. The annual interest rate is $7.1225\% = (1.035)^2 - 1$.

X is the present value of the perpetuity. With end of year payments, the present value is $1/i$

$$X = \frac{1}{.071225} = 14.04$$

The modified duration is equal to the regular duration divided by $(1+i)$. The regular duration is the weighted average of the time each payment is made, where the weight is the present value of the payment:

$$Y = \left(\frac{1}{1+i}\right) \frac{\sum t \cdot v^t R_t}{\sum v^t R_t} \quad \text{where } R_t \text{ is payment at time } t$$

$$= \frac{1}{1.071225}\left(\frac{v + 2v^2 + 3v^3 + \ldots}{v + v^2 + v^3 + \ldots}\right)$$

Let $P = v + 2v^2 + 3v^3 + \ldots$

$v \cdot P = \quad v^2 + 2v^3 + \ldots$

$(1-v)P = v + v^2 + v^3 + \ldots$

Now substitute these into the equation for Y

$$Y = (1/1+i)(P/(1-v)P)$$

$$= \frac{v}{1-v} = \frac{1}{1+i-1} = \frac{1}{i} \qquad \therefore Y = X$$

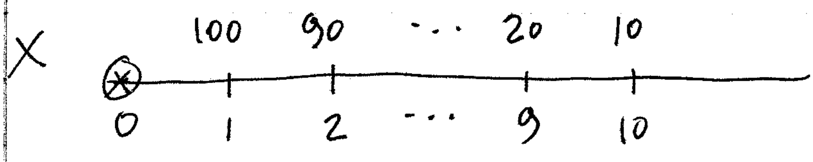
$$Y - X = 0$$

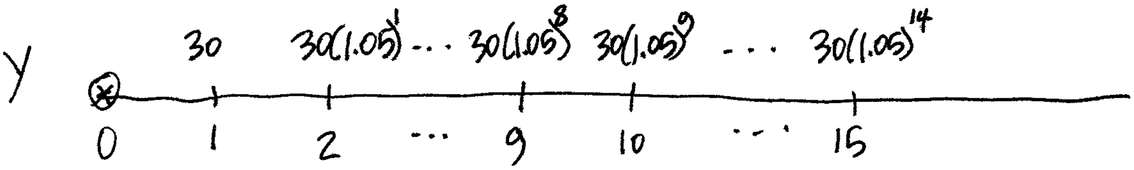
A

7 This is a typical question on increasing / decreasing annuities. The key to working the problem quickly is to know the formulas for both:

$$(Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i} \qquad (Da)_{\overline{n}|i} = \frac{n - a_{\overline{n}|i}}{i}$$

You should start by writing the series of payments on a time-line diagram





You need to convert Y to a simple annuity formula. Simply write out the present value

$$Y = 30(1.065)^{-1} + \frac{30(1.05)}{(1.065)^2} + \cdots \frac{30(1.05)^{14}}{(1.065)^{15}}$$

In general, you can factor out the first term and then you'll be able to simplify the result

$$Y = 30(1.065)^{-1}\left[1 + \frac{1.05}{1.065} + \ldots + \left(\frac{1.05}{1.065}\right)^{14}\right]$$

$$= 30(1.065)^{-1}\, \ddot{a}_{\overline{15}|j} \quad \text{where } 1+j = \frac{1.065}{1.05} = 1.0143$$

2007

(7) $Y = \frac{30}{1.065}(1.0143)(a_{\overline{15}|.0143})$

I use the HP-12C calculator, which is why I always solve problems using immediate annuity factors. It avoids errors caused by switching between annuities due and annuities immediate (for me, anyway!)

$$Y = (28.169)(1.0143)(13.4160)$$
$$= 383.32$$

$$X = 10(Da)_{\overline{10}|}$$
$$= 10\left(\frac{10 - a_{\overline{10}|.065}}{.065}\right)$$
$$= 10\left(\frac{10 - 7.1888}{.065}\right)$$
$$= 432.49$$

$$|X - Y| = |432.49 - 383.32|$$
$$= 49.17$$

E

8 This is a typical question on select and ultimate mortality tables and probabilities. One point of the problem is simply interpreting the symbol. With the 1/3 in front of the q, the individual must live one year, then die in the next 3 years:

$${}_{1|3}q_{[66]+1} = \left(p_{[66]+1}\right)\left(1 - {}_{3}p_{[66]+2}\right)$$

$$= \left(\frac{l_{[66]+2}}{l_{[66]+1}}\right)\left(1 - \frac{l_{[66]+5}}{l_{[66]+2}}\right)$$

Based on the table given in the problem, there is a 3 year select period. That means $l_{[66]+5} = l_{71}$. Now you read the $l_{[x]}$ values from the table to calculate the probability

$${}_{1|3}q_{[66]+1} = \left(\frac{947}{963}\right)\left(1 - \frac{883}{947}\right)$$

$$= .9834(.0676)$$

$$= .0665$$

D

9 This is a matter of interpreting the symbol given, which is a joint life annuity, for a temporary period of 3 years. You need to express this in terms of probabilities to calculate the value:

$$\ddot{a}_{60:62:\overline{3}|} = 1 + v\, p_{60}\, p_{62} + v^2\, {}_2p_{60}\, {}_2p_{62}$$

$$= 1 + \frac{(1-q_{60})(1-q_{62})}{1.05} + \frac{(1-q_{60})(1-q_{61})(1-q_{62})(1-q_{63})}{(1.05)^2}$$

$$= 1 + \frac{(1-.0156)(1-.0184)}{1.05} + \frac{(1-.0156)(1-.0184)(1-.0169)(1-.0201)}{(1.05)^2}$$

$$= 1 + .9203 + .8443$$

$$= 2.7646$$

E

10 This is a tricky death benefit calculation. The problem gives you some annuity values, but it isn't clear how to use those directly.

You need to write down a formula for the present value of the death benefit:

$$X = 100{,}000 \left[v\, q_{65}\, p_{65} + v^2\, p_{65}\, q_{66}\, {}_2p_{65} + v^3\, {}_2p_{65}\, q_{67}\, {}_3p_{65} + \ldots \right]$$

$$= 100{,}000 \sum_{t=0} v^{t+1} \left({}_{t|}q_{65} \right)\left({}_{t+1}p_{65} \right)$$

$$= 100{,}000 \sum_{t=0} v^{t+1} \left({}_tp_{65} - {}_{t+1}p_{65} \right)\left({}_{t+1}p_{65} \right)$$

You can evaluate the part of the summation based on ${}_{t+1}p_{65}$, since that equals $a_{65:65}$. You need to manipulate ${}_tp_{65}$ to produce another probability of survival based on $t+1$ years:

$${}_tp_{65} = \frac{l_{65+t}}{l_{65}} = \frac{l_{65+t}}{l_{64}}\left(\frac{l_{64}}{l_{65}}\right) = \frac{{}_{t+1}p_{64}}{p_{64}}$$

$$X = 100{,}000 \sum_{t=0} v^{t+1} \left[\frac{\left({}_{t+1}p_{64}\right)\left({}_{t+1}p_{65}\right)}{p_{64}} - \left({}_{t+1}p_{65}\right)\left({}_{t+1}p_{65}\right) \right]$$

$$= 100{,}000 \left[\frac{a_{64:65}}{p_{64}} - a_{65:65} \right]$$

$$= 100{,}000 \left(10.0/(1-.002) - 9.9 \right)$$

$$= 12{,}004$$

D

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11 The key to this problem is knowing the definitions for nominal versus effective rate of interest (and for rate of discount):

$$\left(1+\frac{i^{(m)}}{m}\right)^m = 1+i = \left(1-\frac{d^{(m)}}{m}\right)^{-m}$$

$$1+\frac{i^{(12)}}{12} = (1+i)^{\frac{1}{12}} \qquad 1-\frac{d^{(12)}}{12} = (1+i)^{-\frac{1}{12}}$$

$$i^{(12)} = 12\left[(1+i)^{\frac{1}{12}}-1\right] \qquad d^{(12)} = 12\left[1-(1+i)^{-\frac{1}{12}}\right]$$

$$\frac{i^{(12)}}{d^{(12)}} = \frac{(1+i)^{\frac{1}{12}}-1}{1-(1+i)^{-\frac{1}{12}}} = 1.01 = \frac{(1+i)^{\frac{2}{12}}-(1+i)^{\frac{1}{12}}}{(1+i)^{\frac{1}{12}}-1}$$

Now replace $(1+i)^{\frac{1}{12}}$ with x, and you will use the quadratic formula to solve for the value of x

$$1.01(x-1) = x^2-x \qquad ax^2+bx+c=0$$

$$x^2-x-1.01x+1.01=0 \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x^2-2.01x+1.01=0$$

$$x=\frac{2.01\pm\sqrt{(2.01)^2-4(1)(1.01)}}{2}$$

$$=\frac{2.01\pm\sqrt{4.0401-4.04}}{2}$$

$$=(2.01\pm.01)/2$$

$$=1.0100 \text{ or } 1.0000 \quad \Rightarrow (1+i)^{\frac{1}{12}}=1.01 \Rightarrow i=(1.01)^{12}-1$$

$i=$ 12.68%

D

12 This is a typical loan problem involving the loan amortization schedule. There is one wrinkle, which is the zero interest rate for the first two years:

Date	Loan amount
1-1-07	1,500
1-1-08	1,500
1-1-09	1,500
1-1-10	(1,500)1.05 − P

The first payment is made at 12-31-09, so there are 9 payments after 1-1-10. The easiest way to set up the amortization schedule is to change the interest rate to match the payment frequency:

$1+j = (1.05)^3 \Rightarrow j = 15.76\%$

$$(1{,}500)1.05 - P = P\, a_{\overline{9}|15.76\%}$$

$$1{,}575 = P\, \ddot{a}_{\overline{10}|15.76\%}$$

$$P = 1575 / [1.1576\, a_{\overline{10}|15.76\%}]$$

$$= 279.01$$

Payment	Principal	Interest	O/S Loan after pmt
0	–	–	1,500
1			$1{,}575 - 279.01 = 279.01\, a_{\overline{9}|}$
2	$279.01 v^9$	$279.01(1-v^9)$	
⋮			
6	$279.01 v^5 = 279.01(1.1576)^{-5} = 134.21$		

C

13 This is a confusing loan problem. The key is reading the problem carefully, and figuring out the amount of loan payment in each scenario.

Loan I

Time	O/S Loan amount
1	$100{,}000(1.06) - .03(100{,}000) = 100{,}000(1.03)$
2	$[100{,}000(1.03)][1.06 - .03] = 100{,}000(1.03)^2$
$\vdots$	$\vdots$
5	$100{,}000(1.03)^5$

$$X = 100{,}000(1.03)^5 / a_{\overline{25}|.06}$$
$$= 115{,}927 / 12.7834$$
$$= 9{,}069$$

Loan II

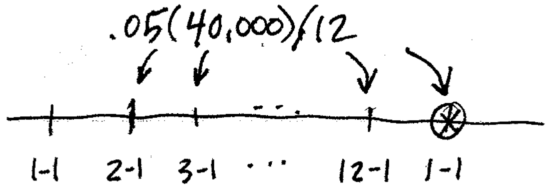
Time	O/S Loan amount
1	$100{,}000(1.06) - .03(100{,}000) = 100{,}000(1.03)$
2	$(100{,}000(1.06) - 3{,}000)1.06 - 3{,}000$
	$= 100{,}000(1.06)^2 - 3{,}000\, s_{\overline{2}|.06}$
$\vdots$	$\vdots$
5	$100{,}000(1.06)^5 - 3{,}000\, s_{\overline{5}|.06}$
	$= 133{,}823 - 16{,}911 = 116{,}911$

$$Y = 116{,}911 / a_{\overline{25}|.06}$$
$$= 9{,}146$$

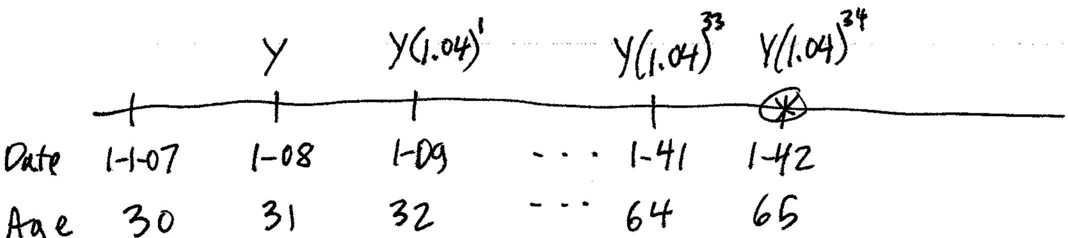
$$|X - Y| = |9{,}069 - 9{,}146| = 77.00$$

B

14 Need to get the monthly contributions converted to annual basis first. Then can easily calculate accumulated value based on annual increase of 4%.



$$Y = 2{,}000\left(s^{(12)}_{\overline{1}|.05}\right) = 166.67\, s_{\overline{12}|j} \quad \text{where } (1+j)^{12} = 1.05 \Rightarrow j = .4074\%$$
$$= 166.67(12.2726) = 2{,}045.43$$



As a check, the exponents on Y indicate that you have 35 annual payments. This corresponds to the annual salary for ages 30 through 64.

$$X = Y(1.04)^{34} + Y(1.04)^{33}(1.05)^{1} + \ldots + Y(1.04)^{1}(1.05)^{33} + Y(1.05)^{34}$$
$$= Y(1.04)^{34}\left[1 + \left(\frac{1.05}{1.04}\right)^{1} + \ldots + \left(\frac{1.05}{1.04}\right)^{33} + \left(\frac{1.05}{1.04}\right)^{34}\right]$$
$$= 2045.43(3.7943)\, s_{\overline{35}|k} \quad \text{where } 1+k = (1.05/1.04) \Rightarrow k = .9615\%$$
$$= 2045.43(3.7943)(41.3757)$$
$$= 321{,}117$$

C

15 This is a typical problem involving life insurance identities. Depending on which ones you know, there are at least 2 approaches. I'll start with a life insurance idea:

$$A_{76} = vq_{76} + vp_{76}A_{77}$$

$$\frac{A_{76} - vq_{76}}{vp_{76}} = A_{77}$$

$$A_{77} = (1.03A_{76} - q_{76})/(1-q_{76})$$

You can use the D_x commutation function to solve for the value of p_{76}, and then q_{76}. By the way this is the first problem with commutation functions since the 2000 exam!

$$D_x = v^x l_x \qquad \frac{D_{76}}{D_{77}} = \frac{v^{76} l_{76}}{v^{77} l_{77}} = \frac{1+i}{p_{76}} \Rightarrow p_{76} = (1+i)\frac{D_{77}}{D_{76}}$$

$$= .927$$

$$q_{76} = .073$$

$$A_{77} = (1.03(.80) - .073)/.927$$

$$= .8101$$

B

You can also work the problem using annuity formulas

$$A_x = 1 - d\ddot{a}_x \qquad A_{76} = 1 - d\ddot{a}_{76} \qquad A_{77} = 1 - d\ddot{a}_{77}$$

$$A_{77} = 1 - iv(\ddot{a}_{77})$$

$$= 1 - iv(\ddot{a}_{76} - 1.0)(D_{77}/D_{76})$$

$$= 1 - d\left(\frac{1 - A_{76} - d}{d}\right)\frac{D_{77}}{D_{76}}$$

$$= 1 - \left(1 - .80 - \frac{.03}{1.03}\right)\left(\frac{400}{360}\right)$$

$$= .8101$$

$$\ddot{a}_{76} = 1 + vp_{76}\ddot{a}_{77}$$

$$\ddot{a}_{76} = (1 - A_{76})/d$$

$$\ddot{a}_{77} = (\ddot{a}_{76} - 1.0)(1.03/p_{76})$$

$$= (\ddot{a}_{76} - 1.0)(D_{77}/D_{76})$$

16 The key to this question is knowing how to calculate the value of $P^1_{30:\overline{10}|}$, and also being able to interpret the actuarial symbols you are given in the problem.

$${}_{10}E_{30} = v^{10}\,{}_{10}p_{30}$$

This is a "pure endowment", which is the value of \$1 paid if a life age 30 survives to the end of 10 years

$$P^1_{30:\overline{10}|} = \frac{A^1_{30:\overline{10}|}}{\ddot{a}_{30:\overline{10}|}}$$

This is a term life insurance premium, where the insurance covers the ten year period starting at age 30

$A_x = 1 - d\ddot{a}_x$ is the identity you must remember

$$A^1_{30:\overline{10}|} = A_{30} - v^{10}\,{}_{10}p_{30}\,A_{40}$$

$$= (1 - d\ddot{a}_{30}) - {}_{10}E_{30}(1 - d\ddot{a}_{40})$$

$$= 1 - d\ddot{a}_{30} - .35 + v^{10}\,{}_{10}p_{30}\,(d\ddot{a}_{40})$$

$$= 1 - .35 - d\,(\ddot{a}_{30} - v^{10}\,{}_{10}p_{30}\,\ddot{a}_{40})$$

$$= .65 - iv(\ddot{a}_{30:\overline{10}|})$$

$$= .65 - \frac{.10}{1.10}(1.0 + 5.60)$$

$$= .05$$

$$1000\,P^1_{30:\overline{10}|} = 1000\left(\frac{A^1_{30:\overline{10}|}}{\ddot{a}_{30:\overline{10}|}}\right)$$

$$= 1000(.05/6.60)$$

$$= 7.58$$

A

17 This is a typical problem on Joint and Survivor annuities. It is pretty easy if you are already familiar with the idea of a reversionary annuity:

$\ddot{a}_{y|x} = \ddot{a}_x - \ddot{a}_{xy}$

This is an annuity payable to a life age x, after the death of a life age y.

Based on the description of the annuity, you can write the present value in three pieces:

(1) $500\,\ddot{a}_{xy}$

(2) $300\,(\ddot{a}_y - \ddot{a}_{xy})$

(3) $B\,(\ddot{a}_x - \ddot{a}_{xy})$

Now you can add the pieces together, and set it equal to the annuity of B payable to Smith:

$$B\,\ddot{a}_x = 500\,\ddot{a}_{xy} + 300\,\ddot{a}_y - 300\,\ddot{a}_{xy} + B\ddot{a}_x - B\,\ddot{a}_{xy}$$

$$B\,\ddot{a}_{xy} = 300\,\ddot{a}_y + 200\,\ddot{a}_{xy}$$

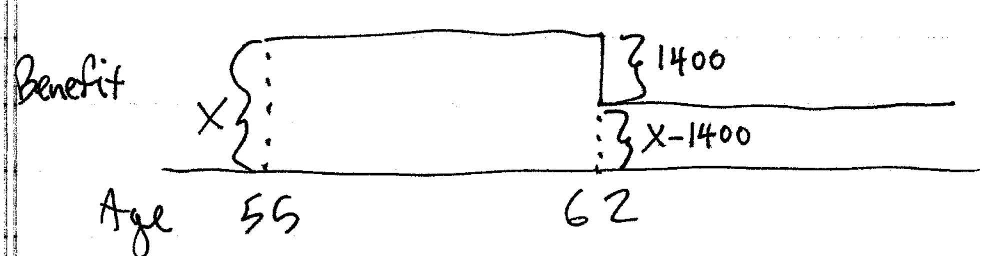
$$B = \frac{300(14) + 200(8)}{8}$$

$$= 725$$

D

18 This is a typical exam question on actuarial equivalence. The key point of this question is if you know how to set up the present value of the Level Income option:

$$12(750)\,\ddot{a}_{55}^{(12)} = 12X\,\ddot{a}_{55:\overline{7|}}^{(12)} + 12(X-1400)\,v^7\,{}_7p_{55}\,\ddot{a}_{62}^{(12)}$$



As shown above, the benefit starts out at X, then decreases by 1400 at age 62. You need to know that the ${}_7E_{55}$ factor is the same as $v^7\,{}_7p_{55}$:

$$750\,\ddot{a}_{55}^{(12)} = X\left(\ddot{a}_{55}^{(12)} - v^7\,{}_7p_{55}\,\ddot{a}_{62}^{(12)}\right) + (X-1400)\,v^7\,{}_7p_{55}\,\ddot{a}_{62}^{(12)}$$

$$= X\,\ddot{a}_{55}^{(12)} - 1400\,{}_7E_{55}\,\ddot{a}_{62}^{(12)}$$

$$750(13.728) = X(13.728) - 1400(.656)(12.218)$$

$$X = 750 + 1400(.656)(12.218)/(13.728)$$

$$= 1567.38$$

C

This seems like a fairly short 4 point problem.

19 This problem is similar to problems 21 and 22 on the 2006 exam. The key concept in the problem is whether you understand the idea of a yield curve, and what a "spot rate" means.

A "spot rate" is the rate of return calculated for a zero coupon bond. This matches the technique for calculating present values under a yield curve. Assuming you have cash flows of C_t in year t, the present value uses the spot rate for each year t (different rate each year):

$$PV = C_1(1+i_1)^{-1} + C_2(1+i_2)^{-2} + C_3(1+i_3)^{-3} + \dots$$
$$= \sum C_t(1+i_t)^{-t}$$

In this problem, the bond price can be calculated based on its yield of 10%:

$$P = 1000(1.10)^{-1} + 1{,}000(1.10)^{-2}$$
$$= 1735.54$$

This price must be the same when using the spot rates to discount the cash flows:

$$1735.54 = (1+i_1)^{-1}(1000) + (1+i_2)^{-2}(1000)$$
$$= 1000(1+i_1)^{-1} + 1{,}000(1.08)^{-2}$$
$$1.736 = (1+i_1)^{-1} + .8573$$
$$i_1 = 13.87\%$$

D

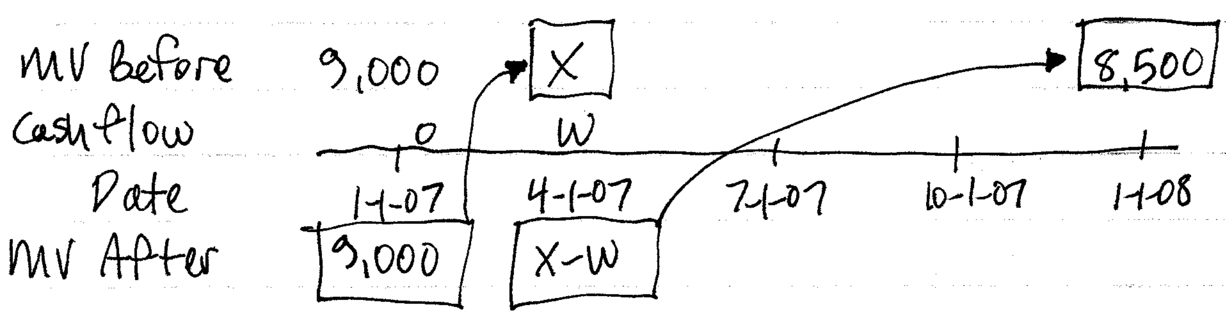
2007

20 This is a typical exam question on calculation of time weighted return and dollar weighted return. One key to working these problems is to read the data very carefully. But this problem's data is much simpler than prior similar exam questions

The next step is to write down the problem's data on a time line diagram:

MVA Before	9,000	X			8,500
cash flow	0	W			
Date	1-1-07	4-1-07	7-1-07	10-1-07	1-1-08
MVA After Cash flow	9,000	X−W			8,500

The time weighted rate of return is calculated using product of several ratios of market values. The ratios measure the increase in market value up to the point of a cash flow. I'll re-write the diagram to show how the ratios are constructed



The ratios represent the growth in the market value from one cash flow date to the next cash flow.

(20) (Continued)

Time weighted rate of Return $= 16\% \Rightarrow 1.16 = \left(\frac{X}{9000}\right)\left(\frac{8500}{X-W}\right)$

Next, set up the formula for the dollar weighted rate of return. The beginning market value and the cash flow earn interest to the end of the year. For simplicity, I will use simple interest:

$$8{,}500 = 1.20(9{,}000) - \left[1+\tfrac{9}{12}(.20)\right]W$$

$$W = \frac{1.20(9{,}000) - 8{,}500}{1+\frac{9}{12}(.20)}$$

$$= 2{,}000$$

Now you can substitute this in the time weighted return formula to solve for X:

$$1.16 = \frac{X}{9{,}000}\left(\frac{8{,}500}{X-2{,}000}\right)$$

$$1.16(X-2{,}000) = X(8{,}500/9{,}000)$$

$$1.16X - .9444X = 1.16(2{,}000)$$

$$X = \frac{2{,}320}{.21556}$$

$$= 10{,}763$$

C

21 This is a fairly short problem on increasing annuities. The first step is to write down the series of payments on a time line diagram:

"Payment"		1	2	...	9	10	10	10 ...
Time	0	1	2	...	9	10	11	12 ...
Date	1-07	1-08	1-09	...				

Rather than put the actual amount of scholarships on the time line diagram, I am only showing the number of scholarships at each point in time.

The easiest way to value this perpetuity is to break it into 2 pieces. You have an increasing annuity for 10 years, plus a perpetuity at time 11:

$$PV = 25{,}000\left[(Ia)_{\overline{10}|.05} + v^{10}\left(\frac{10}{.05}\right)\right]$$

The key to working this problem is correctly valuing the perpetuity (immediate versus due) and knowing the formula for $(Ia)_{\overline{n}|i} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

$$PV = 25{,}000\left[\frac{\ddot{a}_{\overline{10}|} - 10v^{10}}{.05} + (1.05)^{-10}\frac{(10)}{.05}\right]$$

$$= 25{,}000\left[1.05(a_{\overline{10}|.05}) - 10(1.05)^{-10} + 10(1.05)^{-10}\right]/.05$$

$$= 25{,}000\,(8.1078)/.05$$

$$= 4{,}053{,}911$$

B

22 This problem has appeared several times in various exams recently. The key to working the problem is knowing the various mortality ~~table~~ definitions. That allows you to use the information given to construct the various probabilities of survival based on either UDD or constant force of mortality.

One multiple decrement definition you need to know is this one:

$$ {}_tp_x^{(T)} = [{}_tp_x^{\prime(1)}][{}_tp_x^{\prime(2)}][{}_tp_x^{\prime(3)}] \cdots $$

$$ p_x^{(T)} = p_x^{\prime(1)} \cdot p_x^{\prime(2)} $$

$$ {}_{.75}p_x^{(T)} = {}_{.75}p_x^{\prime(1)} \cdot {}_{.75}p_x^{\prime(2)} $$

Decrement 1 has U.D.D, so you know that ${}_tq_x^{\prime(1)} = t(q_x^{\prime(1)})$

$$ {}_{.75}p_x^{\prime(1)} = 1 - {}_{.75}q_x^{\prime(1)} $$
$$ = 1 - .75(.075) $$
$$ = .9438 $$

Decrement 2 has a constant force of "mortality" during each year of age, so you know that ${}_tp_x^{\prime(2)} = [p_x^{\prime(2)}]^t$

$$ {}_{.75}p_x^{\prime(2)} = [p_x^{\prime(2)}]^{.75} $$
$$ = [1 - .095]^{.75} $$
$$ = .9279 $$

$$ {}_{.75}p_x^{(T)} = {}_{.75}p_x^{\prime(1)} \cdot {}_{.75}p_x^{\prime(2)} = .9438(.9279) $$
$$ = .8757 $$

B

23 This is a typical question on stationary population theory. The key definition you need to know is the average age at death for those who die between ages x and x+n:

$$x + \frac{T_x - T_{x+n} - n(l_{x+n})}{l_x - l_{x+n}}$$

T_x represents the members in the stationary population at and above age x, in one year

l_x represents the members at age x in the population, in one year

$$55 = 25 + \frac{T_{25} - T_{65} - 40 l_{65}}{l_{25} - l_{65}}$$

You are told the size of the stationary population is S. This is equal to $T_{25} - T_{65}$, since all members enter at age 25, and exit at age 65. You can use the probability to express the value of l_{65}:

$${}_{40}p_{25} = .8 = \frac{l_{65}}{l_{25}} \quad \therefore\ l_{65} = .8(l_{25})$$

$$55 = 25 + \frac{S - 40(.8 l_{25})}{l_{25} - (.8 l_{25})}$$

Since 1000 new entrants come into the population at age 25, you know that $l_{25} = 1000$:

$$55 = 25 + \frac{S - 40(.8)(1000)}{(.2)(1000)} \Rightarrow 30 = \frac{S - 32{,}000}{200}$$

$S = 38{,}000$

B

24 This question has appeared several times on the exam. One key point of the question is interpreting the data you are given, as well as the question asked. The problem asks for the value of $_{2|}q_{68}^{(T)}$, which is the probability of living from age 68 to age 70, then exiting prior to age 71:

$$_{2|}q_{68}^{(T)} = {}_2p_{68}^{(T)}(1-p_{70}^{(T)})$$
$$= p_{68}^{(T)}p_{69}^{(T)}(1-p_{70}^{(T)})$$

The only multiple decrement formula you need for this problem is the definition of $_tp_x^{(T)} = {}_tp_x'^{(1)}\,{}_tp_x'^{(2)}\,{}_tp_x'^{(3)}\cdots$

$$p_{68}^{(T)} = p_{68}'^{(1)}p_{68}'^{(2)} = (1-q_{68}'^{(1)})(1-q_{68}'^{(2)})$$
$$= (1-.05)(1-.35)$$
$$= .6175$$
$$p_{69}^{(T)} = 1-q_{69}^{(T)} = 1-.80$$
$$= .20$$
$$p_{70}^{(T)} = p_{70}'^{(1)}p_{70}'^{(2)} = (1-q_{70}'^{(1)})(1-q_{70}'^{(2)})$$
$$= (1-.06)(1-.94)$$
$$= .00564$$

$$_{2|}q_{68}^{(T)} = .6175(.20)(1-.00564)$$
$$= .116535$$

A

This is way too close to the end of the answer range! The data given only has 2 significant digits of accuracy but the answer ranges have 4 significant digits – fishy!

25 This question tests your knowledge of several definitions related to bonds.

I. TRUE

The definition of the duration for a series of cash flows is that it is a weighted average of the number of years (duration) for each cash flow. The weights are the present values of each cash flow:

$$\bar{d} = \frac{\sum t \cdot v^t R_t}{\sum v^t R_t}$$

A zero coupon bond only has a single cash flow; which is the bond's redemption amount:

$$\bar{d} = \frac{t \cdot v^t R}{v^t R} = t$$

II. TRUE

The concept of a callable bond is that the issuer can decide when the bond is redeemed (or called).

III. FALSE

Serial bonds are issued at the same point in time, but have differing redemption dates.

I and II are true

A

26 This is a fairly typical bond problem. One key idea is simply how to interpret the question. The total investment return is typically <u>not</u> the sum of the coupon payments. It would also include the difference between the redemption value of the bond and the price paid.

The sum of the 20 semi-annual coupons is $20(.02)(1000) = 400$.

$$P = (Fr)a_{\overline{n}|i} + K$$
$$= 1000(.02)\,a_{\overline{20}|.025} + 1000(1.025)^{-20}$$
$$= 311.78 + 610.27$$
$$= 922.05$$

Total return $= 400 + 1000 - 922.05$
$= 477.95$

D

27 A realized gain is the result of selling an investment. It increases both the book value of assets and the market value of assets.

An unrealized gain is the excess of the market value over the book value of assets. A reconciliation of the book value of assets between years includes the realized gain (or loss). A reconciliation of the market value of assets between years includes both the realized gain (or loss) and <u>the change</u> in the unrealized gain (or loss)

You can use the book values given to determine the realized gain:

1-1-2007 BV 1-1-2006 BV

$$1{,}450{,}000 = 1{,}325{,}000 + (250{,}000 + 80{,}000 + RG) - (55{,}000 + 25{,}000)$$

$$= 1{,}325{,}000 + 250{,}000 + RG$$

$$RG = -125{,}000$$ actually a realized loss

You can now determine the 1-1-2006 market value:

1-1-06 $UG = X - 1{,}325{,}000$

1-1-07 $UG = 1{,}650{,}000 - 1{,}450{,}000 = 200{,}000$

given $\Delta UG = 3(RG) = 3(-125{,}000) = -375{,}000$

$$-375{,}000 = 200{,}000 - (X - 1{,}325{,}000)$$

$$X = 1{,}900{,}000$$

D

28 The key to working this problem is knowing <u>all</u> the relationships for the three mortality assumptions. The question asks for the value of $\frac{1}{3}p_{100}$.

I. UDD

Under UDD, the typical definition that is tested is

$${}_tq_x = t(q_x)$$

$$\tfrac{1}{3}q_{100} = \tfrac{1}{3}(q_{100}) = \tfrac{1}{3}\left(\frac{l_{100}-l_{101}}{l_{100}}\right)$$

$$= \tfrac{1}{3}(28{,}500/95{,}000) = .10$$

$$\tfrac{1}{3}p_{100} = 1-.10 = .90$$

II. Constant Force

With constant force, the typical definition that is tested is

$${}_tp_x = (p_x)^t$$

$$\tfrac{1}{3}p_{100} = (p_{100})^{\frac{1}{3}} = (l_{100}/l_{101})^{\frac{1}{3}}$$

$$= (66{,}500/95{,}000)^{1/3}$$

$$= .8879$$

III. Balducci

The typical definition under Balducci that is tested is

$${}_{1-t}q_{x+t} = (1-t)q_x$$

This time, you need to know another definition:

$${}_tq_x = \frac{t(q_x)}{1-(1-t)q_x} \qquad {}_tp_x = \frac{p_x}{1-(1-t)q_x}$$

$$\tfrac{1}{3}p_{100} = p_{100}/(1-\tfrac{2}{3}(q_{100}))$$

$$= \frac{(66{,}500/95{,}000)}{1-\frac{2}{3}(1-66{,}500/95{,}000)}$$

$$= .70/(1-\tfrac{2}{3}(1-.70))$$

$$= .875$$

III < II < I

C

29 This is a typical exam question that tests your probability ability. The main point of the problem is finding a way to use the information given to calculate the value of P. The first step is to write an expression for P:

$$P = \left[{}_{25}p_{30}\,(1-{}_{20}p_{55})\right]\left[{}_{5}p_{50}\,(1-{}_{20}p_{55})\right]$$

You don't have values for these two probabilities, but you have something relatively close. Now you should re-write the expression using ${}_{20}p_{30}$:

$$P = {}_{20}p_{30}\,({}_{5}p_{50})(1-{}_{20}p_{55})({}_{5}p_{50})(1-{}_{20}p_{55})$$

The last step is figuring out that ${}_{5}p_{50} = \dfrac{{}_{25}p_{50}}{{}_{20}p_{55}}$

$$P = {}_{20}p_{30}\left(\frac{{}_{25}p_{50}}{{}_{20}p_{55}}\right)(1-{}_{20}p_{55})\left(\frac{{}_{25}p_{50}}{{}_{20}p_{55}}\right)(1-{}_{20}p_{55})$$

$$= .75\left(\frac{.55}{.60}\right)^2(1-.60)^2$$

$$= .1008$$

B

This was fairly short in terms of the work required for a 4 point problem.

30 This is a typical exam question on actuarial equivalence and Joint and Survivor annuities. For clarity in writing the present value definitions, let Smith's age be w and the spouse's age be y.

PV of normal form $\quad 1000\, a_w$

PV of 2nd annuity $\quad X\left[a_w + 50\%(a_y - a_{wy})\right]$

PV of 3rd annuity $\quad 875\left[a_{\overline{wy}}\right]$

Since these are all actuarially equivalent, the present values of each annuity are equal. Since the problem gives you no present values or annuity factors, it becomes an algebra problem to determine the value of X.

$$1000\, a_w = X\left[a_w + .50(a_y - a_{wy})\right]$$
$$1000\, a_w = 875\left[a_w + 1.0(a_y - a_{wy})\right]$$

My standard technique in these problems is to replace the reversionary annuity with a symbol. But first, divide through both equations by a_w:

$$1000 = X\left[1 + .50(R)\right]$$
$$1000 = 875\left[1 + 1.0(R)\right] \Rightarrow \quad R = 125/875 = .1429$$
$$X = 1000/(1 + .5(.1429))$$
$$= 933.33 \qquad \text{D}$$

31 This question is straightforward, if you know the various definitions for the mortality assumption of uniform distribution of decrements. The key definition is that ${}_tp_x \mu_{x+t} = q_x$:

$$\mu_{x+t} = \frac{q_x}{{}_tp_x}$$

$${}_tp_x = 1 - {}_tq_x$$

${}_tq_x = t(q_x)$ under UDD $\Rightarrow \mu_{x+t} = \dfrac{q_x}{1-t(q_x)}$

Another key definition is $L_x \doteq \dfrac{l_x + l_{x+1}}{2}$

$$930 = \frac{l_x + 910}{2}$$

$$l_x = 950$$

Now you can calculate the force of mortality at $x+.2$

$$\mu_{x+.2} = \frac{q_x}{1-.2(q_x)} \qquad q_x = \frac{l_x - l_{x+1}}{l_x} = \frac{950-910}{950} = 40/950$$

$$\mu_{x+.2} = \frac{40/950}{1-.2(40/950)} = .0425$$

C