

SOCIETY OF ACTUARIES
AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2008 EA-1 EXAMINATION

**Conditions Generally Applicable to
All EA-1 Examination Questions**

If applicable, the following conditions should be considered a part of the data for each question, unless otherwise stated or implied:

- (1) The normal retirement age is 65.
- (2) Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.
- (3) There are no pre-retirement death or disability benefits.
- (4) Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.
- (5) Interest rates that are compounded more frequently than annually are expressed as nominal rates.
- (6) Where multiple lives are involved, future lifetimes are assumed to be independent of each other.
- (7) The term “gross single premium” is equivalent to “contract single premium;” the term “net single premium” is equivalent to “single benefit premium;” the term “gross annual premium” is equivalent to “annual contract premium;” the term “net annual premium” is equivalent to “annual benefit premium.”
- (8) There are no policy loans in effect.
- (9) For a bond, the face amount and the redemption value are the same.
- (10) Interest rate equals yield rate.
- (11) The term “duration” means “Macaulay duration”.

2008

Data for Question 1 (3 points)

Terms of a 20-year bond:

Face amount	\$1,000
Redemption amount	\$1,050
Coupons	Payable annually with first coupon \$75 Each subsequent coupon is 3% greater than the preceding coupon
Yield	8.25% per year, compounded annually

Question 1

In what range is the purchase price of the bond?

- (A) Less than \$1,030
- (B) \$1,030 but less than \$1,065
- (C) \$1,065 but less than \$1,100
- (D) \$1,100 but less than \$1,135
- (E) \$1,135 or more

2008

Data for Question 2 (2 points)

A bond has the following payment structure:

\$1,000 payable in one year and \$1,000 payable in two years

Interest rate: 10.00%, compounded annually

Two-year spot rate: 8.00%, compounded annually

Question 2

In what range is the one-year spot rate?

- (A) Less than 13.65%
- (B) 13.65% but less than 13.75%
- (C) 13.75% but less than 13.85%
- (D) 13.85% but less than 13.95%
- (E) 13.95% or more

2008

Data for Question 3 (3 points)

The present value of an annuity is given by:

$$\frac{(S_x - S_{x+3}) - (S_{x+5} - S_{x+8})}{D_x}$$

Question 3

In what range is the sum of the annuity payments?

- (A) Less than 12
- (B) 13
- (C) 14
- (D) 15
- (E) 16 or more

2008

Data for Question 4 (5 points)

An annuity of \$10,000 is payable at the end of each year as long as at least one of three lives Smith (age 25), Jones (age 30), and Brown (age 35) is alive.

While all three are alive the \$10,000 is split as follows: Smith and Jones each receive \$2,500, and Brown receives \$5,000.

After the first death, the survivors split the share of the person dying equally. After the second death, the last survivor receives the entire annuity.

You are given the following values:

$$a_{25} = 17.95$$

$$a_{30} = 17.16$$

$$a_{35} = 16.22$$

$$a_{25:30} = 14.92$$

$$a_{25:35} = 14.48$$

$$a_{30:35} = 13.99$$

$$a_{25:30:35} = 12.68$$

X = the present value of Smith's share.

Question 4

In what range is X ?

- (A) Less than \$61,500
- (B) \$61,500 but less than \$75,000
- (C) \$75,000 but less than \$88,500
- (D) \$88,500 but less than \$102,000
- (E) \$102,000 or more

2008

Data for Question 5 (3 points)

\$3,000 is invested in a fund at a constant force of interest of δ . At time n , \$2,000 is taken out and the balance accumulates for a further n years at the same force of interest. At time $2n$, the fund is \$225,000.

X = value of fund at time $2n$ if the force of interest were halved.

Question 5

In what range is X ?

- (A) Less than \$22,000
- (B) \$22,000 but less than \$32,000
- (C) \$32,000 but less than \$42,000
- (D) \$42,000 but less than \$52,000
- (E) \$52,000 or more

Data for Question 6 (3 points)

$$s_{\overline{n}} = 10.0$$

$$\ddot{s}_{\overline{n}} = 11.0$$

Question 6

In what range is ${}_n|\ddot{a}_{\overline{2n}|}$?

- (A) Less than 3.8
- (B) 3.8 but less than 4.0
- (C) 4.0 but less than 4.2
- (D) 4.2 but less than 4.4
- (E) 4.4 or more

Data for Question 7 (3 points)

$$\ddot{a}_{10|} - \ddot{a}_{9|} = 0.50$$

$$X = \frac{(s_{19|} - s_{18|})^2}{(\ddot{s}_{9|} - \ddot{s}_{8|})^3}$$

Question 7

In what range is X ?

- (A) Less than 1.50
- (B) 1.50 but less than 1.70
- (C) 1.70 but less than 1.90
- (D) 1.90 but less than 2.10
- (E) 2.10 or more

2008

Data for Question 8 (4 points)

Type of bond:	Serial
Face amount:	\$1,000
Redemption details:	Over the next 4 years from the date of purchase, at the end of each year at par in equal installments
Coupons:	5% per annum payable annually
Yield:	4% per annum compounded annually

Question 8

In what range is the duration of the bond?

- (A) Less than 2.34
- (B) 2.34 but less than 2.38
- (C) 2.38 but less than 2.42
- (D) 2.42 but less than 2.46
- (E) 2.46 or more

2008

Data for Question 9 (3 points)

Terms of a 20-year bond issued on 1/1/2008:

Face amount	\$10,000
Redemption amount	\$10,500
Coupon rate	4.0% per year, payable semi-annually
Yield rate	5.0% per year, compounded annually

The coupons are immediately reinvested at a rate of 10% per year, compounded annually for the balance of the 20 years. Assuming the bond is held to maturity, the overall rate of return for the 20 years is ***R*** % per year, compounded annually.

Question 9

In what range is ***R*** ?

- (A) Less than 6.85%
- (B) 6.85% but less than 6.90%
- (C) 6.90% but less than 6.95%
- (D) 6.95% but less than 7.00%
- (E) 7.00% or more

2008

Data for Question 10 (4 points)

A 25-year endowment policy has annual premiums of \$272.70 payable at the beginning of the year for the first 5 years followed by annual premiums of \$746.00 payable at the beginning of the year for the next 20 years. This annual premium pattern is actuarially equivalent to an annual premium pattern of \$136.10 payable at the beginning of the year for the first 5 years followed by \$846.00 payable at the beginning of the year for the next 20 years.

P = the level annual premium payable at the beginning of the year for each of the 25 years.

Question 10

In what range is P ?

- (A) Less than \$500
- (B) \$500 but less than \$520
- (C) \$520 but less than \$540
- (D) \$540 but less than \$560
- (E) \$560 or more

2008

Data for Question 11 (4 points)

A life age 40 is scheduled to receive an annuity due of \$5,000 payable annually for a maximum of 20 years. If death occurs within the 20 years, a lump sum equal to the sum of the unpaid annuity payments is paid at the end of the year of death.

You are given:

\underline{x}	$\underline{N_x}$	$\underline{R_x}$
40	31,309	17,169
41	29,499	16,417
...
60	7,147	5,105
61	6,475	4,685

Y = the present value of the annuity, including the death benefit.

Question 11

In what range is the present value of Y ?

- (A) Less than \$72,500
- (B) \$72,500 but less than \$73,500
- (C) \$73,500 but less than \$74,500
- (D) \$74,500 but less than \$75,500
- (E) \$75,500 or more

2008

Data for Question 12 (3 points)

The following table applies to a group of 10 individuals, who are exactly age 90.

\underline{x}	$\underline{\ell}_x$
90	210
91	150
92	120
93	90
94	70
95	50
96	35
97	20
98	10
99	5
100	0

Deaths are uniformly distributed within each age.

Question 12

In what range is the total expected number of future years lived by the 10 individuals?

- (A) Less than 28.0
- (B) 28.0 but less than 35.0
- (C) 35.0 but less than 42.0
- (D) 42.0 but less than 49.0
- (E) 49.0 or more

2008

Data for Question 13 (3 points)

A loan is taken on 1/1/2008 with payments at the end of each month for 60 months. Each payment is \$100. The principal paid on the loan during the first 20 months is \$1,461. The principal paid on the loan during the next 20 months is \$1,655.

Question 13

In what range is the annual effective rate of interest on the loan?

- (A) Less than 7.56%
- (B) 7.56% but less than 7.64%
- (C) 7.64% but less than 7.72%
- (D) 7.72% but less than 7.80%
- (E) 7.80% or more

2008

Data for Question 14 (2 points)

A pension trust statement reported the following information:

Market value as of 12/31/2006	\$1,000,000
2007 contributions	\$90,000
2007 benefit payments	\$40,000
2007 plan expenses	\$10,000
Market value as of 12/31/2007	\$1,150,000

The cumulative unrealized gain/(loss) as of 12/31/2006 was (\$10,000).

The unrealized gain/(loss) during 2007 was \$5,000.

Question 14

In what range was the book value of assets as of 12/31/2007?

- (A) Less than \$1,142,000
- (B) \$1,142,000 but less than \$1,147,000
- (C) \$1,147,000 but less than \$1,152,000
- (D) \$1,152,000 but less than \$1,157,000
- (E) \$1,157,000 or more

2008

Data for Question 15 (5 points)

All employees are hired at exact age 22.

3% of the employees at each age terminate employment at the end of each year

5% of those at exact age 42 are promoted out of the business unit

All employees retire at exact age 62

There are no other decrements from the population.

Question 15

In what range is the expected length of service for a new entrant?

- (A) Less than 23.00
- (B) 23.00 but less than 23.25
- (C) 23.25 but less than 23.50
- (D) 23.50 but less than 23.75
- (E) 23.75 or more

2008

Data for Question 16 (3 points)

Smith takes out a \$200,000 mortgage at 1/1/2008 to be paid in equal monthly installments over 30 years. The first payment is due 2/1/2008. Immediately after the 60th payment is made, Smith makes an additional payment of \$50,000 and refinances the remaining balance in equal monthly installments over 10 years, with the first payment due at the beginning of the month following the refinancing.

Interest rate – original loan: 6% per year, compounded monthly

Interest rate – refinanced amount: 5% per year, compounded monthly

X = the monthly payment amount after refinancing.

Question 16

In what range is X ?

- (A) Less than \$1,250
- (B) \$1,250 but less than \$1,500
- (C) \$1,500 but less than \$1,750
- (D) \$1,750 but less than \$2,000
- (E) \$2,000 or more

2008

Data for Question 17 (3 points)

Account information:

<u>Date</u>	<u>Account Balance</u>	<u>Contribution</u>
12/31/2007	\$20,000	-
01/01/2008	-	\$3,000
03/31/2008	\$24,000	-
04/01/2008	-	\$3,000
06/30/2008	X	-
07/01/2008	-	\$3,000
09/30/2008	\$29,000	-
10/01/2008	-	\$3,000
12/31/2008	\$34,000	

The time-weighted rate of return in 2008 is 7.0%.

Question 17

In what range is **X** ?

- (A) Less than \$26,400
- (B) \$26,400 but less than \$26,500
- (C) \$26,500 but less than \$26,600
- (D) \$26,600 but less than \$26,700
- (E) \$26,700 or more

2008

Data for Question 18 (4 points)

Selected values from a multiple decrement table:

x	$q_x^{(withdrawal)}$	$q_x^{(death)}$	$q_x^{(retirement)}$
55	0.070	0.025	0.150
56	0.050	0.029	0.100
57	0.030	0.033	0.100
58	0.020	0.037	0.200
59	0.010	0.042	0.300
60	0.000	0.000	1.000

Z = the percentage of employees hired at 55 who withdraw before age 60.

Question 18

In what range is Z ?

- (A) Less than 7.0%
- (B) 7.0% but less than 9.0%
- (C) 9.0% but less than 11.0%
- (D) 11.0% but less than 13.0%
- (E) 13.0% or more

2008

Data for Question 19 (3 points)

$${}_{10}p_{40} = 0.90$$

$${}_{15}p_{50} = 0.75$$

$${}_{10}p_{55} = 0.80$$

X = the probability that two independent lives age 40 and age 50 will both die between age 55 and age 65.

Question 19

In what range is X ?

- (A) Less than 0.02900
- (B) 0.02900 but less than 0.03000
- (C) 0.03000 but less than 0.03100
- (D) 0.03100 but less than 0.03200
- (E) 0.03200 or more

2008

Data for Question 20 (3 points)

Smith retires on January 1, 2008 at age 60 and can elect one of the following actuarially equivalent annuity options:

- Option 1 A life annuity of \$1,000 per month, payable at the beginning of each month, commencing on January 1, 2008 at age 60, with the first 60 payments guaranteed
- Option 2 A deferred life annuity of X per month, payable at the beginning of each month, commencing on January 1, 2013 at age 65

Selected values:

$$\ddot{a}_{60}^{(12)} = 13.25$$

$${}_5|\ddot{a}_{60}^{(12)} = 8.88$$

Interest: 5% per year, compounded annually.

Question 20

In what range is X ?

- (A) Less than \$1,485
- (B) \$1,485 but less than \$1,495
- (C) \$1,495 but less than \$1,505
- (D) \$1,505 but less than \$1,515
- (E) \$1,515 or more

2008

Data for Question 21 (2 points)

Smith purchases a one-year term insurance policy at age 60. The benefit is payable at the end of the year of death.

Face Amount: \$1,000,000 (policy pays double if death is accidental)

$$q_{60}^{(All\ Causes\ of\ Death)} = 0.0070$$

$$q_{60}^{(Accidental\ Death)} = 0.0005$$

Interest: 6% per year, compounded annually.

Question 21

In what range is the single premium for this policy?

- (A) Less than \$6,750
- (B) \$6,750 but less than \$6,900
- (C) \$6,900 but less than \$7,050
- (D) \$7,050 but less than \$7,200
- (E) \$7,200 or more

2008

Data for Question 22 (2 points)

$$\ell_{[x]+t} = 100 - x - \frac{t}{2}, \quad 0 \leq t \leq 6$$

$$\ell_{x+t} = 103 - x - t, \quad t > 6$$

Y = the probability that a life now age 52 and who entered the selection period 2 years ago, will die between ages 54 and 58.

Question 22

In what range is Y ?

- (A) Less than 0.050
- (B) 0.050 but less than 0.060
- (C) 0.060 but less than 0.070
- (D) 0.070 but less than 0.080
- (E) 0.080 or more

2008

Data for Question 23 (2 points)

Deaths are uniformly distributed over each year of age.

For a given integer x :

$$q_x = 0.400$$

$$q_{x+1} = 0.500$$

Question 23

In what range is $q_{x+0.5}$?

- (A) Less than 0.405
- (B) 0.405 but less than 0.425
- (C) 0.425 but less than 0.445
- (D) 0.445 but less than 0.465
- (E) 0.465 or more

2008

Data for Question 24 (3 points)

$${}_{10}E_x = 0.400$$

$${}_{10|}a_x = 7.000$$

$$\ddot{s}_{x:\overline{10}|} = 15.000$$

Question 24

In what range is \ddot{a}_x ?

- (A) Less than 12.70
- (B) 12.70 but less than 12.90
- (C) 12.90 but less than 13.10
- (D) 13.10 but less than 13.30
- (E) 13.30 or more

2008

Data for Question 25 (4 points)

Smith (age 60) purchases an annuity on 1/1/2008 that has the following characteristics:

Payments \$1 annually at the beginning of each year

Term Payments are for life with the first payment on 1/1/2018

Mortality is based on the following formula: $\ell_x = 100 - x$.

Interest rate: 0%.

Y = the probability that the sum of payments made under the annuity will exceed the actuarial present value of the annuity as of 1/1/2008.

Question 25

In what range is Y ?

- (A) Less than 0.445
- (B) 0.445 but less than 0.465
- (C) 0.465 but less than 0.485
- (D) 0.485 but less than 0.505
- (E) 0.505 or more

2008

Data for Question 26 (3 points)

Retirement benefit: \$50 per month for each year of service.

Interest rate: 6.00%, compounded annually.

Pre-retirement decrements: Mortality only.

Retirement age: 65.

Data for Smith:

Date of birth 1/1/1962

Date of hire 1/1/1997

Present value as of 1/1/2007 of Smith's anticipated retirement benefit at age 65: \$26,475.

Present value as of 1/1/2008 of Smith's anticipated retirement benefit at age 65: \$28,174.

Question 26

In what range is q_{45} ?

- (A) Less than 0.00250
- (B) 0.00250 but less than 0.00300
- (C) 0.00300 but less than 0.00350
- (D) 0.00350 but less than 0.00400
- (E) 0.00400 or more

2008

Data for Question 27 (5 points)

On 1/1/2008, Smith sets up an investment under an immunization strategy to meet the obligation shown:

Obligation	A single payment of \$10,000 on 1/1/2013
Available investments	2-year and 10-year zero-coupon bonds
Yield to maturity:	5.0%, compounded annually (for both investments)

Assume that the convexity of the assets is greater than the convexity of the obligation.

X = the amount invested in 2-year zero coupon bonds.

Question 27

In what range is X ?

- (A) Less than \$2,800
- (B) \$2,800 but less than \$3,800
- (C) \$3,800 but less than \$4,800
- (D) \$4,800 but less than \$5,800
- (E) \$5,800 or more

2008

Data for Question 28 (3 points)

Terms of a bond are as follows:

Face amount	\$1,000
Redemption amount	\$1,000
Term	20 years
Coupon rate	5.0% per year, payable annually

The yield curve has the following schedule of spot rates:

<u>Maturity</u>	<u>Spot Rate</u>
1 – 5 years	5.0%
6 – 15 years	7.0%
16 – 20 years	9.0%

X = the bond's price at issue.

Question 28

In what range is X ?

- (A) Less than \$675
- (B) \$675 but less than \$725
- (C) \$725 but less than \$775
- (D) \$775 but less than \$825
- (E) \$825 or more

2008

Data for Question 29 (3 points)

Smith (age 20) purchases an insurance policy on 1/1/2008 that pays the following amounts at the end of the year of Smith's death:

<u>Year of death</u>	<u>Amount</u>
2008 – 2027	\$100,000
2028 – 2047	\$200,000
2048 and later	\$50,000

Selected actuarial values:

$$A_{20} = 0.0454$$

$$A_{20:\overline{20}|}^1 = 0.0081$$

$$A_{20:\overline{40}|} = 0.1068$$

$${}_{40}E_{20} = 0.0901$$

Question 29

In what range is the net single premium for this insurance?

- (A) Less than \$3,750
- (B) \$3,750 but less than \$3,850
- (C) \$3,850 but less than \$3,950
- (D) \$3,950 but less than \$4,050
- (E) \$4,050 or more

2008

Data for Question 30 (3 points)

Selected values from a two-decrement model:

x	$q_x^{(1)}$	$q_x^{(2)}$
46	0.0244	0.1000
47	0.0273	0.0900
48	0.0309	0.0800
49	0.0345	0.0700
50	0.0390	0.0600

$$Z = {}_{2|2}q_{46}^{(2)}$$

Question 30

In what range is Z ?

- (A) Less than 0.0905
- (B) 0.0905 but less than 0.1005
- (C) 0.1005 but less than 0.1105
- (D) 0.1105 but less than 0.1205
- (E) 0.1205 or more

2008

Data for Question 31 (4 points)

A mortality table is projected from the year 2000 to 2008 according to the following formula:

$$q_x^{[Year]} = q_x^{[2000]}(1 - AA_x)^{(Year-2000)}$$

Selected values from the basic table and the projection scale:

\underline{x}	$\underline{q_x^{[2000]}}$	$\underline{AA_x}$
35	0.000475	0.011
36	0.000514	0.012
37	0.000554	0.013
38	0.000598	0.014
39	0.000648	0.015

$$Y = {}_5q_{35}^{[2008]}$$

Question 31

In what range is Y ?

- (A) Less than 0.002430
- (B) 0.002430 but less than 0.002530
- (C) 0.002530 but less than 0.002630
- (D) 0.002630 but less than 0.002730
- (E) 0.002730 or more

2008

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**EA-1 Spring 2008
Answer Key**

Question	Answer	Points
1	D	3
2	D	2
3	D	3
4	B	5
5	A	3
6	C	3
7	D	3
8	C	4
9	B	3
10	D	4
11	D	4
12	B	3
13	D	3
14	D	2
15	B	5
16	B	3
17	C	3
18	E	4
19	D	3
20	C	3
21	D	2
22	C	2
23	C	2
24	E	3
25	C	4
26	D	3
27	D	5
28	B	3
29	D	3
30	C	3
31	B	4
Total		100

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