SoftwarePolish
Rick Groszkiewicz
2964 Nestle Creek Drive
Marietta, GA 30062-4857
Voice/fax (770) 971-8913
email: rickg@softwarepolish.com

SPRING 2009 EA-1
EXAM SOLUTIONS

Revision History:
03/11/13 Revised solution for problems 8 and 20
02/25/11 Revised solutions for problems 7 and 35

2009 EA-1 Solutions

1 This is a typical exam question on conversion of interest rates. There are two ways of working the problem, which are quite similar. You need to calculate the present value of 60 monthly payments, and you are given a quarterly interest rate:

$$P = 1200\, \ddot{a}^{(12)}_{\overline{5}|j}$$ where j would be the nominal interest, convertible monthly

I prefer to always convert the interest rate so the compounding frequency matches the payment frequency. An alternate expression for P is:

$$P = 100\, \ddot{a}_{\overline{60}|j}$$ where j is the monthly interest rate

$$(1+j)^{12} = 1+i = \left(1 + \frac{.05}{4}\right)^4$$

$$j = (1.050945)^{1/12} - 1$$
$$= .4149\% \text{ per month}$$

$$P = 100(1.004149)(a_{\overline{60}|.4149\%})$$
$$= 100(1.004149)(53.0173)$$
$$= 5324$$

C

I use the HP-12C calculator, and I always calculate the annuities as immediate, not due. This avoids arithmetic errors caused by forgetting how I set the calculator!

2 This is a basic question on mortality table definitions. The key idea is whether you can interpret the symbols given in the problem.

${}_{1|}q_{45}$ is the one year deferred probability of death. The participant must survive from 45 to 46, then die in the next year:

$${}_{1|}q_{45} = p_{45}(q_{46}) = d_{46}/l_{45} \Rightarrow d_{46} = .006141(l_{45})$$
$$= .006141$$

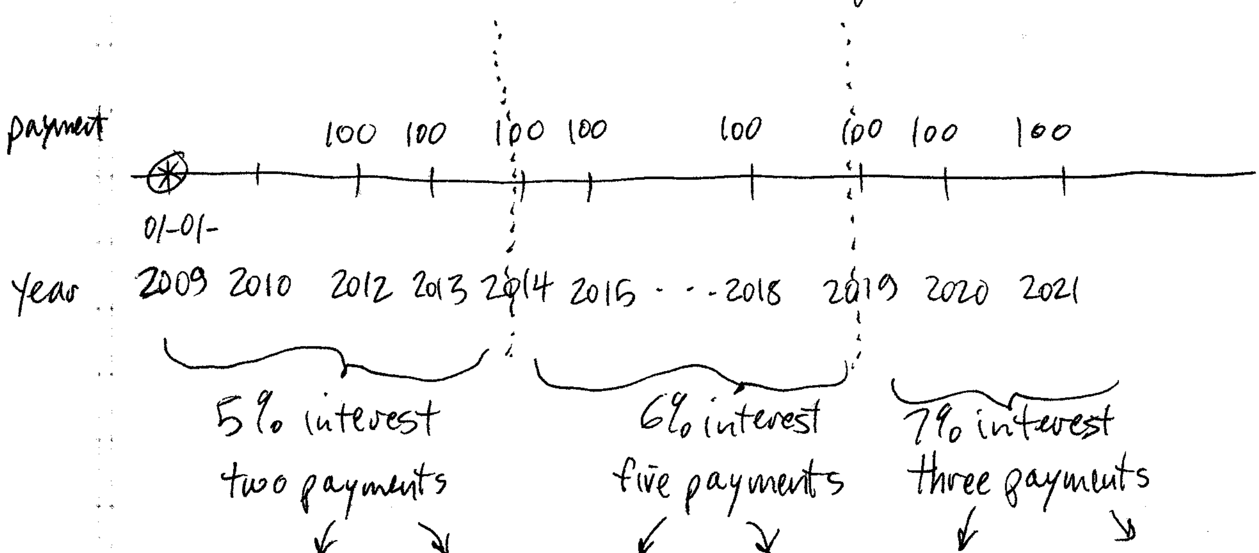
$$l_{45} - d_{45} - d_{46} = l_{47}$$
$$l_{45} = l_{47} + d_{45} + d_{46}$$
$$= 89{,}472 + 502 + .006141\, l_{45}$$

$$l_{45} = 90{,}530$$
$$d_{46} = .006141(90{,}530)$$
$$= 555.94$$

D

3 This is a basic question on present value calculations using a yield curve. The spot rates correspond to the yield to maturity for a zero coupon bond.

You should write down all the payments on a time line diagram. The payments for years 2009 - 2013 must be discounted using 5% for each year. For years 2014 through 2018, all the payments must be discounted back to 01-01-2009 using 6% for each year. Payments after 2018 must be discounted using 7% for each year.



$$X = 100\left[(1.05)^{-3} + (1.05)^{-4} + (1.06)^{-5} + \dots + (1.06)^{-9} + (1.07)^{-10} + (1.07)^{-11} + (1.07)^{-12}\right]$$

$$= 100\left[a_{\overline{4}|.05} - a_{\overline{2}|.05} + \left(a_{\overline{9}|.06} - a_{\overline{4}|.06}\right) + \left(a_{\overline{12}|.07} - a_{\overline{9}|.07}\right)\right]$$

$$= 100\,(1.6865 + 3.3366 + 1.4275)$$

$$= 645.06$$

A

4 This is a basic question that tests whether you understand the terminology regarding set back and set forwards for the mortality rates. The other point is whether you know how to calculate the probability ${}_tp_{\overline{xy}}$ (joint and last survivor):

$$
\begin{aligned}
{}_tp_{\overline{xy}} &= {}_tp_x + {}_tp_y - {}_tp_{xy} \\
&= {}_tp_x + {}_tp_y - ({}_tp_x)({}_tp_y) \\
{}_3p_{\overline{xy}} &= {}_3p_{\overline{85:86}}
\end{aligned}
$$

You need to be very careful in how you calculate these probabilities. It is very easy to get confused about the correct ages!

${}_3p_x \Rightarrow$ male age 85 $\Rightarrow$ use q_x values for ages 86 to 88

$$
\begin{aligned}
{}_3p_x &= (1-q_{86})(1-q_{87})(1-q_{88}) \\
&= (1-.0953)(1-.1075)(1-.1204) \\
&= .7102
\end{aligned}
$$

${}_3p_y \Rightarrow$ female age 86 $\Rightarrow$ use q_y values for ages 84 to age 86

$$
\begin{aligned}
{}_3p_y &= (1-q_{84})(1-q_{85})(1-q_{86}) \\
&= (1-.0762)(1-.0852)(1-.0953) \\
&= .7646
\end{aligned}
$$

$$
\begin{aligned}
{}_3p_{\overline{xy}} &= {}_3p_x + {}_3p_y - ({}_3p_x)({}_3p_y) \\
&= .7102 + .7646 - (.7102)(.7646) = .9318
\end{aligned}
$$

E

5 This is a straightforward question on annuity calculations. It seems fairly short for a 4 point question.

$$\ddot{a}_{20} = 1 + v\,p_{20} + v^2\,{}_2p_{20} + \ldots$$

$$= 1 + \frac{.90}{1.04} + \frac{.90(.95)}{(1.04)^2} + \frac{.90(.95)^2}{(1.04)^3} + \ldots$$

When evaluating an infinite series, the first step is factoring out the first term:

$$(\ddot{a}_{20} - 1.0) = .90(1.04)^{-1} + .90(.95)(1.04)^{-2} + \ldots$$

$$(1.04)(.90)^{-1}(\ddot{a}_{20} - 1.0) = 1 + .95/1.04 + (.95/1.04)^2 + \ldots$$

$$(\ddot{a}_{20} - 1.0)\frac{1.04}{.90} - 1.0 = \frac{.95}{1.04} + \left(\frac{.95}{1.04}\right)^2 + \ldots$$

The right side is a perpetuity immediate, which has value of $1/j$ where the interest rate is j:

$$1 + j = 1.04/.95 = 1.09474$$

$$(\ddot{a}_{20} - 1.0)(1.04/.90) - 1.0 = 1/.09474 = 10.5556$$

$$\ddot{a}_{20} - 1.0 = (.90/1.04)(11.5556)$$

$$\ddot{a}_{20} = 11.0$$

D

6 This question has appeared on the exam several times. The easy way to work it is simply knowing that the mortality table satisfies De Moivre's law:

$l_x = \omega - x$ where ω is the last age in the mortality table

$$\mu_x = -\frac{1}{l_x}\left[\frac{d}{dx} l_x\right]$$
$$= \left(\frac{-1}{\omega - x}\right)(-1)$$
$$= \frac{1}{\omega - x}$$

The hard way to work the problem is to "derive" a formula for ${}_np_x$ based on using integrals:

$${}_np_x = e^{-\int_x^{x+n} \mu_y \, dy}$$

It is much easier if you know the l_x formula instead:

$${}_np_x = \frac{l_{x+n}}{l_x} = \frac{100-x-n}{100-x}$$

The problem asks for the value of $\ddot{a}_{40}$:

$$\ddot{a}_{40} = 1 + v\,p_{40} + v^2\,{}_2p_{40} + \ldots$$

(6) $\ddot{a}_{40} = 1 + v\left(\frac{l_{41}}{l_{40}}\right) + v^2\left(\frac{l_{42}}{l_{40}}\right) + \cdots$

You are told that the interest rate is zero, which further simplifies the calculation:

$$\ddot{a}_{40} \text{ at zero } i = 1 + \frac{59}{60} + \frac{58}{60} + \cdots + \frac{1}{60}$$
$$= \frac{1}{60}[60 + 59 + \cdots + 1]$$

The key is knowing the formula for the sum of the first n integers:

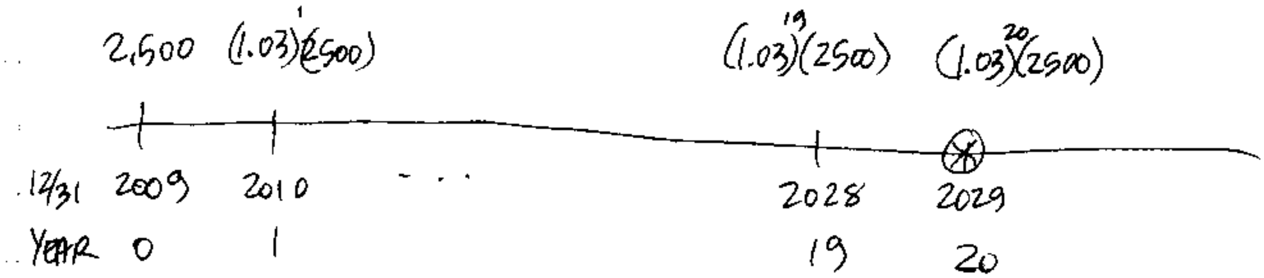
$$\sum_{t=1}^{n} t = \frac{t(t+1)}{2}$$

$$\ddot{a}_{40} \text{ at zero } i = \frac{1}{60}\left[\frac{60(61)}{2}\right]$$
$$= 30.50$$

C

2009

7.



$.05(50{,}000)=2500$

$$X = 2{,}500(1.06)^{20} + (1.03)^{1}(2500)(1.06)^{19} + \dots + (1.03)^{19}(2500)(1.06)^{1} + (1.03)^{20}(2500)$$

Standard technique is to factor out the first term. Resulting series can be evaluated as an annuity:

$$X = 2500(1.06)^{20}\left[1 + \frac{1.03}{1.06} + \dots + \left(\frac{1.03}{1.06}\right)^{20}\right]$$

$$= 2500(1.06)^{20}\,\ddot{a}_{\overline{21|}j} \qquad \text{where } 1+j = \frac{1.06}{1.03} = 1.02913$$

$$= 2500(3.2071)(1.02913)\,a_{\overline{21|}2.913\%}$$

$$= 128{,}272$$

C

Key to working this problem is to write down series of payments carefully, and count the number of years correctly. Notice that the sum of the exponents in the accumulated value formula is 20 for each term.

8 Typical identity question on this exam.

$(1+i)(\ddot{a}_{\overline{n+1}|} - 1.0) = \ddot{a}_{\overline{n}|}$ based on definition of $\ddot{a}_{\overline{n}|}$

$$1+i = \frac{\ddot{a}_{\overline{n}|}}{\ddot{a}_{\overline{n+1}|} - 1.0}$$

$$= \frac{6.091836}{5.381005}$$

$$= 1.132100$$

$$i = 13.21\%$$

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$$

$$\left(\frac{i}{1+i}\right)\ddot{a}_{\overline{n}|} = 1-v^n$$

$$v^n = 1 - \left(\frac{i}{1+i}\right)\ddot{a}_{\overline{n}|}$$

$$= 1 - \left(\frac{.1321}{1.1321}\right)(6.091836)$$

$$= .2892$$

$$\ddot{s}_{\overline{n}|} = (1+i)^n \ddot{a}_{\overline{n}|}$$

$$= \frac{\ddot{a}_{\overline{n}|}}{v^n}$$

$$\ddot{s}_{\overline{n}|} = \ddot{a}_{\overline{n}|} / v^n$$

$$= 6.091836 / .2892$$

$$= 21.07$$

D

9 One key point is terminology – the problem asks for the duration, which is not the same as the modified duration.

$$\text{Duration} = \bar{d} = \frac{\sum t v^t R_t}{\sum v^t R_t}$$

$R_t \Rightarrow$ payments received from investment, which include annual coupons and the final redemption value

$$X = \bar{d} = \frac{1000(1v^1 + 2v^2 + 3v^3 + 4v^4 + 5v^5)(.05) + 10005v^5}{1000(v^1 + v^2 + v^3 + v^4 + v^5)(.05) + 1000v^5}$$

The face and redemption values are the same, so they cancel out entirely

$$X = \bar{d} = \frac{.05(Ia_{\overline{5}|.05}) + 5v^5}{.05(a_{\overline{5}|.05}) + v^5}$$

The main point to the solution is whether you know the formula for an increasing annuity:

$$Ia_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$X = \frac{.05(\ddot{a}_{\overline{5}|} - 5v^5)/.05 + 5(1.05)^{-5}}{.05(1 - v^5)/.05 + v^5}$$

$$= \frac{1.05\, a_{\overline{5}|.05} - 5v^5 + 5v^5}{1 - v^5 + v^5} = 1.05(a_{\overline{5}|.05}) = 4.55 \quad \text{E}$$

10 This seems fairly simple for a 4 point question!

This is a typical EA-1 question on the relationship between single and multiple decrement tables.

$$p_x^{(T)} = p_x'^{(1)} \cdot p_x'^{(2)}$$
$$1-q_x^{(T)} = (1-q_x'^{(1)})(1-q_x'^{(2)})$$

$$.76 = (1-4q_x'^{(2)})(1-q_x'^{(2)})$$
$$= (1-q_x'^{(1)})(1-.25q_x'^{(1)})$$

Now you have a typical quadratic equation to solve, which is something that is tested frequently on EA-1.

Let $x = q_x'^{(1)}$

$$.76 = (1-x)(1-.25x)$$
$$= 1-1.25x+.25x^2$$
$$0 = .25x^2-1.25x+.24$$

$$0 = ax^2+bx+c$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{1.25 \pm \sqrt{(1.25)^2-4(.25)(.24)}}{2(.25)}$$
$$= (1.25 \pm \sqrt{1.5625-.24})/.5$$
$$= 2(1.25 \pm 1.15)$$
$$= 4.80 \text{ or } .20$$

C

ignore 4.80 → invalid for q_x

2009

11 This is a question on calculation of life insurance premiums. These are not very common questions on the EA-1 exam.

The key to working this problem is knowing several identity formulas:

$$A_x = 1 - d\ddot{a}_x \qquad \frac{A_x}{\ddot{a}_x} = P_x = \frac{1}{\ddot{a}_x} - d \qquad P_x + d = \frac{1}{\ddot{a}_x}$$

$$P_{50} + d = 1/\ddot{a}_{50} \qquad P_{51} + d = 1/\ddot{a}_{51} \Rightarrow \ddot{a}_{51} = 1/(P_{51}+d)$$

$$\ddot{a}_x = 1 + vp_x(\ddot{a}_{x+1}) \qquad = [.03712 + .04/1.04]^{-1}$$

$$\ddot{a}_{50} = 1/(P_{50}+d) = 1 + vp_{50}\ddot{a}_{51} \qquad = 13.2307$$

$$[.03550 + .04/1.04]^{-1} = 1 + (1.04)^{-1}(l_{51}/l_{50})(13.2307)$$

$$13.5205 = 1 + (l_{51}/100{,}000)(13.2307/1.04)$$

$$(1.04/13.2307)(100{,}000)(12.5205) = l_{51} = 98{,}417$$

E

2009

12 First step is to identify where the l_x values are to produce the value of $d_{[53]+1}$

X	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	l_{x+3}	$x+3$
50					
51					
52					55
53		✓	✓	✓	56
54					

Problem gives you the value of l_{55}. You can use the value of q_{55} to generate l_{56}. Then you need to use the select q's to generate the value of $l_{[53]+1}$. Finally, you can calculate $d_{[53]+1}$ as the product of $q_{[53]+1}$ and $l_{[53]+1}$.

$l_{56} = l_{55}\, p_{55} = 13{,}200(1-q_{55}) = 11{,}484$

$l_{56} = (l_{[53]+2})(p_{[53]+2})$

$l_{[53]+2} = l_{56}/(1-q_{[53]+2})$

$= 11{,}484/(1-.120)$

$= 13{,}050$

$l_{[53]+1} = l_{[53]+2}/(1-q_{[53]+1})$ because $l_{[53]+2} = (l_{[53]+1})(p_{[53]+1})$

$= 13{,}050/(1-.100)$

$= 14{,}500$

$d_{[53]+1} = (l_{[53]+1})(q_{[53]+1})$

$= 14{,}500(.100)$

$= 1{,}450$ A

13 This is a confusing question on multiple decrement calculations. The key point of the problem is correctly interpreting the information that you are given.

You are told there are multiple decrements, and that all decrements other than mortality occur 1/3 of the way between consecutive ages. You are also given data for the number of lives and exits between ages 50 and 51:

exits		57	
lives	1000		$933 = 1000 - 57 - 10$ deaths
age	50	$50\frac{1}{3}$	51

The problem tells you nothing about the distribution of the mortality decrement. You could use any reasonable assumption: Balducci, constant force, or UDD (uniform distribution of decrements).

Since all other decrements occur only at one point in time (at age $50\frac{1}{3}$), there is no difference between the multiple decrement table probability of death, and the single decrement table rate of death. The reason is that there is no period where multiple decrements are competing to force an employee to exit.

(13) For simplicity, I will assume the mortality decrement follows U.D.D. Based on the data given, you have this relationship:

$$933 \text{ survivors} = \left[1000\left({}_{\frac{1}{3}}p'^{(d)}_{50}\right) - 57 \text{ exits}\right]\left({}_{\frac{2}{3}}p'^{(d)}_{50\frac{1}{3}}\right)$$

Under UDD, you have these results

$${}_tq_x = t(q_x) \qquad {}_{1-t}q_{x+t} = \frac{(1-t)q_x}{1-t(q_x)}$$

$${}_{\frac{1}{3}}q'^{(d)}_{50} = \left(\tfrac{1}{3}\right)\left(q'^{(d)}_{50}\right) \qquad {}_{\frac{2}{3}}q'^{(d)}_{50\frac{1}{3}} = \frac{\frac{2}{3}\left(q'^{(d)}_{50}\right)}{1-\frac{1}{3}\left(q'^{(d)}_{50}\right)}$$

I'll rewrite the original formula using q_x instead of p_x, then substitute the UDD relationships:

$$933 = \left[1000\left(1-{}_{\frac{1}{3}}q'^{(d)}_{50}\right)-57\right]\left[1-{}_{\frac{2}{3}}q'^{(d)}_{50\frac{1}{3}}\right]$$

$$933 = \left[1000\left(1-\tfrac{1}{3}\left(q'^{(d)}_{50}\right)\right)-57\right]\left[1-\frac{\frac{2}{3}\left(q'^{(d)}_{50}\right)}{1-\frac{1}{3}\left(q'^{(d)}_{50}\right)}\right]$$

For simplicity, I'll replace $q'^{(d)}_{50}$ with Q:

$$933 = \left[1000\left(1-\tfrac{1}{3}Q\right)-57\right]\left[1-\frac{\frac{2}{3}Q}{1-\frac{1}{3}Q}\right]$$

$$933\left(1-\tfrac{1}{3}Q\right) = \left[1000\left(1-\tfrac{1}{3}Q\right)-57\right]\left[1-\tfrac{1}{3}Q-\tfrac{2}{3}Q\right]$$

$$933-311Q = \left[943-333.33Q\right]\left[1-Q\right]$$
$$= 943-943Q-333.33Q+333.33Q^2$$

(13) Now re-write the terms with zero on the right:

$$333.33Q^2 - 965.33Q + 10 = \text{zero}$$

You can solve this using the formula for a quadratic equation – this is tested at least once per year on the EA-1 exam!

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Q = \frac{965.33 \pm \sqrt{(965.33)^2 - 4(333.33)10}}{2(333.33)}$$

$$= \frac{965.33 \pm 958.40}{666.67}$$

$$= .010396 \text{ or (a bogus result – ignore)}$$

C

This is much easier to work if you assume that the mortality corresponds to Balducci instead of U.D.D

$$933 \text{ survivors} = \left[1000\left({}_{\frac{1}{3}}p'^{(d)}_{50}\right) - 57 \text{ exits}\right]\left({}_{\frac{2}{3}}p'^{(d)}_{50\frac{1}{3}}\right)$$

$$933 = 1000\, p'^{(d)}_{50} - 57\left({}_{\frac{2}{3}}p'^{(d)}_{50\frac{1}{3}}\right)$$

Under Balducci

$${}_{1-t}q_{x+t} = (1-t)q_x$$

$${}_{2/3}q'^{(d)}_{50\frac{1}{3}} = \frac{2}{3}q'^{(d)}_{50}$$

Substituting in the prior formula, you have

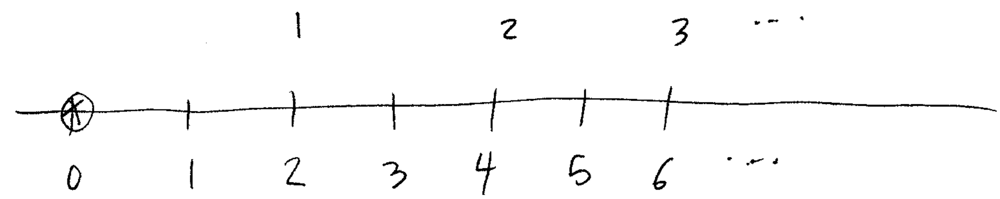
$$933 = 1000\left(1 - q'^{(d)}_{50}\right) - 57\left(1 - \frac{2}{3}\left(q'^{(d)}_{50}\right)\right)$$

$$= 1000 - 1000q'^{(d)}_{50} - 57 + 38q'^{(d)}_{50}$$

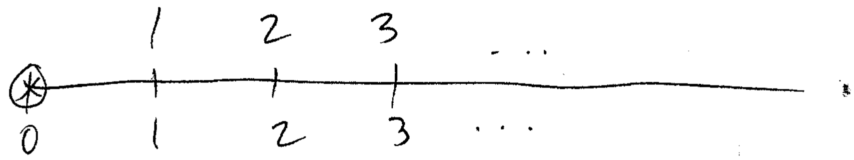
$$q'^{(d)}_{50} = .010395$$

C

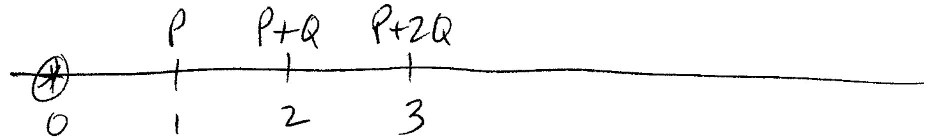
14



This is the same thing as a perpetuity which uses a biennial interest rate $j = (1.06)^2 - 1 = 12.36\%$, and which has annual payments



In general, the formula for an increasing perpetuity is $\frac{P}{i} + \frac{Q}{i^2}$, where the payments are as follows:



In this problem, P and Q are both equal to 1:

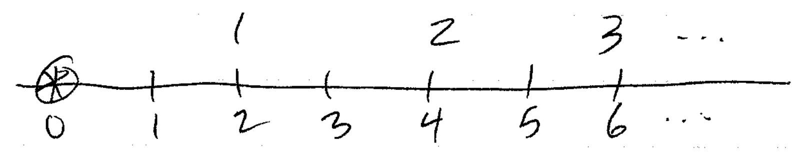
$$X = \frac{1}{i} + \frac{1}{i^2} = \frac{1}{.1236} + \frac{1}{(.1236)^2} = 73.55$$

B

You can also work the problem by writing down the formula for the present value of the series of payments, and performing algebraic manipulations. See the next page for an example of this solution.

2009

(14) If you don't know the formula for an increasing perpetuity immediate, you could use this approach



$$X = 1(1.06)^{-2} + 2(1.06)^{-4} + 3(1.06)^{-6} + \dots$$

$$X(1.06)^{-2} = \qquad 1(1.06)^{-4} + 2(1.06)^{-6} + \dots$$

$$X[1-(1.06)^{-2}] = (1.06)^{-2} + (1.06)^{-4} + (1.06)^{-6} + \dots$$

The right hand side is a perpetuity immediate using an interest rate $j = (1.06)^2 - 1 = 12.36\%$

$$X\,[1-(1.06)^{-2}] = 1/(.1236)$$

$$X = \frac{(.1236)^{-1}}{[1-(1.06)^{-2}]}$$

$$= 73.55$$

B

2009

15 For serial bond problems, the key is using Makeham's formula to write down the price of a single bond. Under the standard formula, you have

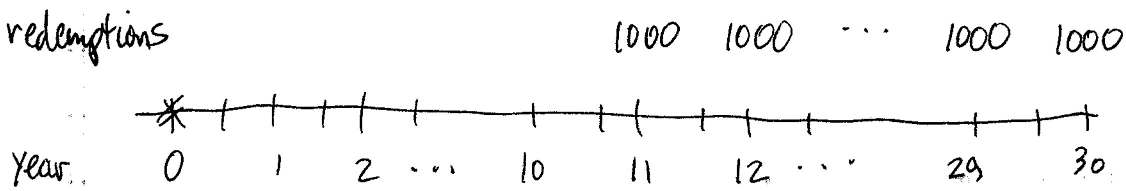
$P = Fr\,a_{\overline{n}|i} + K \qquad \text{where } K = Cv^n$

Under Makeham's formula, you have

$P = K + (g/i)(C-K) \quad \text{where } K = Cv^n \text{ and } g = Fr/C$

The formulas are actually equivalent, but there is no annuity in Makeham's formula. This helps simplify the calculation, since you will have 20 terms to add up for the serial bond.

The bond itself is a bit unusual based on the interest rates you are given. The coupon rate is 4% every six months, and there are 20 redemptions of 1,000. The yield rate is given as 4.5% every six months:



$$
\begin{aligned}
P_{11} &= K + (g/i)(C-K) \\
&= 1000(1.045)^{-22} + (.04/.045)\left(1000 - 1000(1.045)^{-22}\right) \\
&= 1000(1.045)^{-22} + (.8889)\left(1000 - 1000(1.045)^{-22}\right) \\
&= .8889(1000) + .1111(1000)(1.045)^{-22}
\end{aligned}
$$

(15) You can write down formulas for the set of 20 serial bonds:

$P_{11} = .8889(1000) + .1111(1000)(1.045)^{-22}$
$P_{12} = .8889(1000) + .1111(1000)(1.045)^{-24}$
$\vdots \qquad \vdots \qquad \vdots$
$P_{30} = .8889(1000) + .1111(1000)(1.045)^{-60}$

One simplification is to use an annual rate of interest instead of the semi-annual rate:

$(1.045)^2 = 1.092025$

$P_{11} = .8889(1000) + .1111(1000)(1.092025)^{-11}$
$P_{12} = .8889(1000) + .1111(1000)(1.092025)^{-12}$
$\vdots \qquad \vdots \qquad \vdots$
$P_{30} = .8889(1000) + .1111(1000)(1.092025)^{-30}$

$\Sigma P = .8889(1000)(20) + .1111(1000)[(1.092)^{-11} + \ldots (1.092)^{-30}]$
$= 17,778 + .1111(1000)(a_{\overline{30}|.0920} - a_{\overline{10}|.0920})$
$= 17,778 + 414$
$= 18,192$

C

16 This is one of several recent questions on spot rates and yield curves. A spot rate is one rate from a yield curve. It is the yield to maturity for a zero coupon bond.

To calculate the present value of the four payments, you need to discount each one using the spot rate that matches the time until the payment is made:

$$X = 1000\left[(1.04)^{-1} + (1.05)^{-2} + (1.0575)^{-3} + (1.0625)^{-4}\right]$$
$$= 1000\left[.9615 + .9070 + .8456 + .7847\right]$$
$$= 3,499$$

B

17 This is something that has not been asked before! In the real world, there are many different methods to use to calculate the weighted average retirement age.

In this problem, you are given no assumptions for interest and mortality. So you can't calculate a present value weighted average retirement age.

The only calculation you can make is a simple average, based on the retirement decrements given. The key idea is to determine what percentage of someone (who is under age 62) will retire at each future age:

Age	Probability of reaching age, then retiring
62	$100\%(40\%) = .40$
63	$(1-.40)(25\%) = .15$
64	$(1-.40)(1-.25)(.25) = .1125$
65	$(1-.40)(1-.25)(1-.25)(1.00) = .3375$

need to check $\Sigma = 100\%$

Now that you have the probabilities, you can calculate the weighted retirement age:

$X = 62(40\%) + 63(15\%) + 64(11.25\%) + 65(33.75\%)$
$= 63.39$

B

18 This is a basic question on present value calculations. The key point of the problem is calculating the value of the 120 guaranteed payments.

$$100{,}000 = 12Z\left(\ddot{a}^{(12)}_{\overline{10}|} + {}_{10|}\ddot{a}^{(12)}_{45}\right)$$

$${}_{10|}\ddot{a}^{(12)}_{45} = N^{(12)}_{55} / D_{45}$$

$$\ddot{a}^{(12)}_{x} \doteq \ddot{a}_x - 11/24$$

$$N^{(12)}_{x} \doteq N_x - (11/24)D_x$$

$$N^{(12)}_{55} \doteq N_{55} - (11/24)D_{55}$$

$${}_{10|}\ddot{a}^{(12)}_{45} = \left(N_{55} - \left(\tfrac{11}{24}\right) D_{55}\right)/D_{45}$$

$$= \left[24{,}032{,}177 - \left(\tfrac{11}{24}\right)(1{,}639{,}330)\right]/2{,}392{,}905$$

$$= 9.7291$$

To calculate $\ddot{a}^{(12)}_{\overline{10}|3\%}$, I prefer to convert the interest rate from 3% per annum to a monthly rate:

$$\ddot{a}^{(12)}_{\overline{10}|.03} = \frac{\ddot{a}_{\overline{120}|j}}{12} \quad \text{where } (1+j)^{12} = 1.03 \Rightarrow j = .2466\%$$

$$\ddot{a}_{\overline{120}|.2466\%} = (1.002466)\, a_{\overline{120}|.2466\%}$$

$$= 104.02$$

Now you can solve for the value of Z

$$Z = 100{,}000/\left[12\left(\ddot{a}^{(12)}_{\overline{10}|} + {}_{10|}\ddot{a}^{(12)}_{45}\right)\right]$$

$$= 100{,}000/\left[104.02 + 12(9.7291)\right]$$

$$= 452.97$$

D

19 The key to working this problem is knowing the relationship between the single decrement tables and the multiple decrement table. The question asks for the probability that a participant remains active from age 53 to age 62:

$$X = {}_{9}p_{53}^{(T)}$$

The probability that someone survives one year in the multiple decrement table is the same as the product of the probabilities of surviving one year in each of the single decrement tables:

$$\begin{aligned} p_z^{(T)} &= \left[p_z'^{(1)}\right]\left[p_z'^{(2)}\right] \\ &= (1-.015)(1-.050) \quad \text{for ages } 50 \le z \le 70 \\ &= .985(.95) \\ &= .93575 \end{aligned}$$

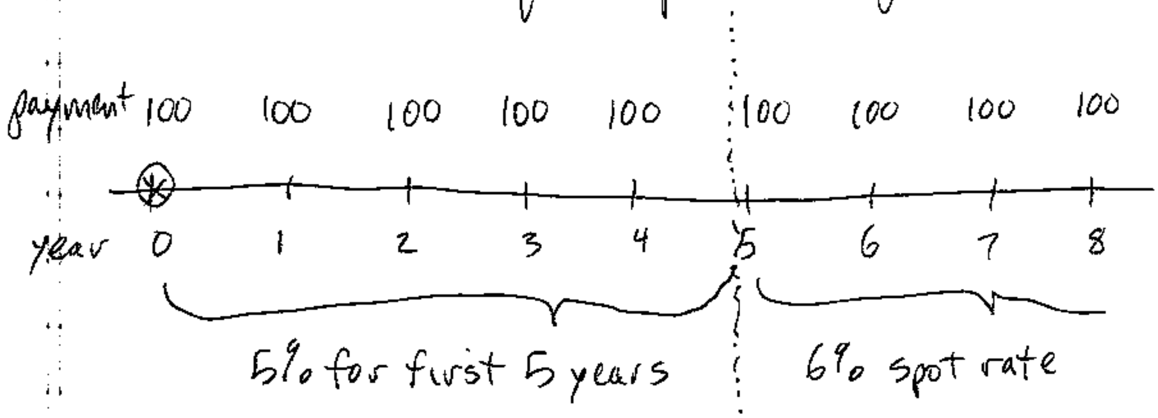
$$\begin{aligned} X &= {}_{9}p_{53}^{(T)} = (.93575)^9 \\ &= .5501 \end{aligned}$$

E

2009

20. This is a basic question on calculating present values with spot rates. You must discount payments made during the first 5 years with 5%.

The problem gives you an annuity due, with the first payment immediately. The next 4 payments also fall within the first five years, and will be discounted at the first spot rate of 5%:



$$X = 100\left[\ddot{a}_{\overline{5}|.05} + \left(\ddot{a}_{\overline{9}|.06} - \ddot{a}_{\overline{5}|.06}\right)\right]$$

I use the HP-12C calculator, and leave it set on calculating immediate annuities. This tends to avoid arithmetic errors (caused in the past, when I would change the calculator setting incorrectly, or simply forgot)!

$$X = 100\left[1.05\,a_{\overline{5}|.05} + 1.06\left(a_{\overline{9}|.06} - a_{\overline{5}|.06}\right)\right]$$
$$= 100\left[1.05(4.3295) + 1.06(6.8017 - 4.2124)\right]$$
$$= 729.06$$

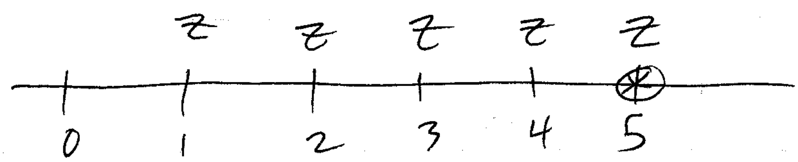
C

2009

21 The key point to this question is knowing how to calculate the payments using a sinking fund. It is described clearly in the problem

<u>Sinking Fund</u>

Under the sinking fund approach, there are annual payments that accumulate at 5.0% to the original value of the loan:



$$Z\, s_{\overline{5}|.05} = 100{,}000$$

In addition, the loan interest is paid directly to the lender each year. Y is the sum of the loan interest payments and the sinking fund payments:

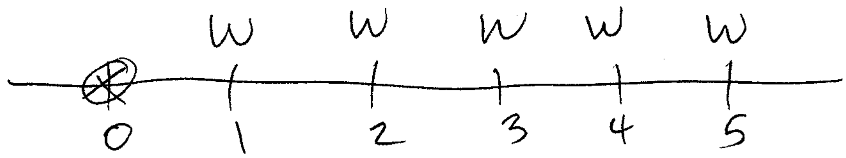
$$
\begin{aligned}
Y &= 5Z + 5(6\%)(100{,}000) \\
&= 5\left[Z + 6000\right] \\
&= 5\left[\frac{100{,}000}{s_{\overline{5}|.05}} + 6{,}000\right] \\
&= 120{,}487
\end{aligned}
$$

(next page)

(21) continued

<u>Level annual payments</u>

The normal approach is to make five payments which have a present value of the loan amount:



$$W(a_{\overline{5}|.06}) = 100{,}000$$

X is the sum of the five loan payments:

$$\begin{aligned} X &= 5W \\ &= 5\,(100{,}000 / a_{\overline{5}|.06}) \\ &= 118{,}698 \end{aligned}$$

The problem asks for the difference between X and Y:

$$\begin{aligned} |X - Y| &= |\,118{,}698 - 120{,}487| \\ &= 1{,}789 \end{aligned}$$

D

22 One point of this question is interpretation of the statement about mortality improvement. In 2010, the mortality rate at each age will be 99% of the 2009 mortality rate. For each subsequent year, the mortality rate at each age will be 99% of the prior year's mortality rate at that age.

The problem asks for the present value at 1-1-09 of the third payment:

		100,000	100,000	100,000	. . .
Date	1-1-09	1-1-10	1-1-11	1-1-12	. . .
Age	x	$x+1$	$x+2$	$x+3$	

One trick is that this is an immediate annuity, so the third payment occurs at 1-1-12.

With no mortality improvement, this is the value of Z:

$$Z = 100{,}000\,(p_x)(p_{x+1})(p_{x+2})v^3$$
$$= 100{,}000\,(1-q_x)(1-q_{x+1})(1-q_{x+2})v^3 \text{ with no improvement}$$

Now you can modify this to reflect the 1% per year improvement, starting with q_{x+1} at 1-1-10:

$$Z = 100{,}000\,(1-q_x)(1-.99q_{x+1})(1-(.99)^2q_{x+2})v^3$$
$$= 100{,}000\,(1-.051)(1-.99(.057))(1-(.99)^2(.063))(1.05)^{-3}$$
$$= 72{,}516$$

(B)

23 This is a typical exam question involving joint and survivor probabilities. The key point of the question is that payments are made as long as either Smith or Jones is alive. This is a joint and last survivor probability:

$$p_{\overline{xy}} = p_x + p_y - p_{xy}$$

$$X = 1000\, v^4\, {}_4p_{\overline{60:61}} = 1000(1.06)^{-4}\left({}_4p_{60} + {}_4p_{61} - ({}_4p_{60})({}_4p_{61})\right)$$

The fifth payment is made at the beginning of the fifth year, if either participant is alive. To calculate the value of X, you must derive the probability values using the D_x commutation functions:

$$D_y = v^y l_y \qquad {}_np_y = \frac{l_{y+n}}{l_y} = (1+i)^n \frac{D_{y+n}}{D_y} \qquad \frac{D_{y+n}}{D_y} = \frac{v^{y+n} l_{y+n}}{v^y l_y} = v^n \frac{l_{y+n}}{l_y}$$

$$\begin{aligned} {}_4p_{60} &= (1.06)^4 (D_{64}/D_{60}) \\ &= (1.06)^4 (219/285) \\ &= .9701 \end{aligned} \qquad \begin{aligned} {}_4p_{61} &= (1.06)^4 (D_{65}/D_{61}) \\ &= (1.06)^4 (205/267) \\ &= .9693 \end{aligned}$$

$$\begin{aligned} X &= 1000(1.06)^{-4}(.9701 + .9693 - (.9701)(.9693)) \\ &= 791.37 \end{aligned}$$

D

24 This is a typical actuarial equivalence question on Joint and Survivor annuities. If two forms of payment are actuarially equivalent, then they have the same present value.

This problem is a simplified version of others on the prior EA-1 exams. There are no factors of any kind given in the problem. Once you write down the initial formulas for the present values, this is solely an algebra problem.

PV annuity 1 = $12(100)\ddot{a}_z^{(12)}$
PV annuity 2 = $12(94)(\ddot{a}_z^{(12)} + 50\%(\ddot{a}_y^{(12)} - \ddot{a}_{zy}^{(12)}))$
PV annuity 3 = $12(X)(\ddot{a}_z^{(12)} + 75\%(\ddot{a}_y^{(12)} - \ddot{a}_{zy}^{(12)}))$

I assumed Smith is age z, the spouse is age y for the above formulas. Next, set the present values equal, which gives two equations to solve

$$12(100)\ddot{a}_z^{(12)} = 12(94)(\ddot{a}_z^{(12)} + .50(\ddot{a}_y^{(12)} - \ddot{a}_{zy}^{(12)}))$$
$$12(100)\ddot{a}_z^{(12)} = 12(X)(\ddot{a}_z^{(12)} + .75(\ddot{a}_y^{(12)} - \ddot{a}_{zy}^{(12)}))$$

There appear to be too many unknowns to solve! The trick is to divide through by $12\ddot{a}_z^{(12)}$, then see if you can simplify the equations further.

(next page)

(24) $$100 = 94\left(1 + .50\left(\frac{\ddot{a}_y^{(12)} - \ddot{a}_{zy}^{(12)}}{\ddot{a}_z^{(12)}}\right)\right)$$

$$100 = X\left(1 + .75\left(\frac{\ddot{a}_y^{(12)} - \ddot{a}_{zy}^{(12)}}{\ddot{a}_z^{(12)}}\right)\right)$$

The key idea is that you don't care what the values are for each annuity. Simply replace the ratio of the annuities with R, and now you have two equations in two unknowns:

$$100 = 94(1 + .50R)$$
$$100 = X(1 + .75R)$$

Solve for the value of R from the first equation, and then you can calculate the value of X:

$$(100/94) = 1 + .5R$$
$$2((100/94) - 1) = R = .12766$$

$$X = 100/(1 + .75R)$$
$$= 91.26$$

B

2009

25 This is a potentially confusing problem on select and ultimate tables. The first step is to write down a formula for ${}_{1|}q_{[46]+1}$

$$
\begin{aligned}
{}_{1|}q_{[46]+1} &= (p_{[46]+1})(q_{[46]+2}) \\
&= (1-q_{[46]+1})(q_{[46]+2}) \\
&= (1-q_{[46]+1})(q_{48})
\end{aligned}
$$

Since there is only a 2 year select period, the final q_x above is based on the ultimate table.

$$
\begin{aligned}
{}_{1|}q_{[46]+1} &= (1-1.3(q_{47}))(q_{48}) \\
&= (1-1.3(.065))(.060) \\
&= .05493
\end{aligned}
$$

C

26 The main key to this identity problem is knowing the definition for $_nE_x$. This is a typical EA-1 problem!

$$_{10}E_{60} = D_{70}/D_{60}$$

$$\ddot{a}_{60:\overline{5|}} = \ddot{a}_{60} - (D_{65}/D_{60})\ddot{a}_{65}$$

$$\ddot{a}_{65:\overline{5|}} = \ddot{a}_{65} - (D_{70}/D_{65})\ddot{a}_{70}$$

$$D_{70}/D_{65} = (\ddot{a}_{65} - \ddot{a}_{65:\overline{5|}})/\ddot{a}_{70}$$

$$D_{65}/D_{60} = (\ddot{a}_{60} - \ddot{a}_{60:\overline{5|}})/\ddot{a}_{65}$$

$$\begin{aligned} D_{70}/D_{60} &= (D_{70}/D_{65})(D_{65}/D_{60}) \\ &= \left(\frac{\ddot{a}_{65} - \ddot{a}_{65:\overline{5|}}}{\ddot{a}_{70}}\right)\left(\frac{\ddot{a}_{60} - \ddot{a}_{60:\overline{5|}}}{\ddot{a}_{65}}\right) \\ &= \left(\frac{10.8207 - 4.2985}{9.7262}\right)\left(\frac{11.7952 - 4.3393}{10.8207}\right) \\ &= (.6706)(.6890) \\ &= .4621 \end{aligned}$$

C

27. The key to working this problem is knowing the formula for converting a nominal rate of interest to an effective rate:

$$\left(1+\frac{i^{(m)}}{m}\right) = 1+i$$

$$\left(1+\frac{.06}{12}\right)^{12} = 1.06168 \qquad \left(1+\frac{.07}{4}\right)^{4} = 1.0719 \qquad \left(1+\frac{.08}{2}\right)^{2} = 1.0816$$

$$(1+x)^5 = (1.06168)(1.06168)(1.0719)(1.0816)(1.0816)$$

$$= 1.4134$$

$$X = 7.16\%$$

D

28 This is a typical question on identities and interest rates. The key definitions for nominal interest and discount rates are:

$$\left[1+\frac{i^{(m)}}{m}\right]^m = 1+i = \frac{1}{1-d} = \left[1-\frac{d^{(m)}}{m}\right]^{-m}$$

$$v^{11} = \tfrac{1}{2}$$
$$v = \left(\tfrac{1}{2}\right)^{\frac{1}{11}}$$
$$= .9389$$

$$1+i = 1.06504 = \left(1+\frac{i^{(4)}}{4}\right)^4$$
$$1+\frac{i^{(4)}}{4} = (1.06504)^{\frac{1}{4}}$$
$$i^{(4)} = \left[(1.06504)^{\frac{1}{4}}-1\right]4$$
$$= .06351$$

$$1.060504 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$$
$$1-\frac{d^{(4)}}{4} = (1.06504)^{-\frac{1}{4}}$$
$$d^{(4)} = \left[1-(1.06504)^{-\frac{1}{4}}\right]4$$
$$= .06252$$

$$X = 100\left|i^{(4)}-d^{(4)}\right|$$
$$= 100(.000993)$$
$$= .099$$

B

2009

29 This is a typical exam question on refinancing a loan. There are several steps in the solution:

1. Determine the original loan payment (every other year)
2. Determine the outstanding loan balance after the 10^{th} payment
3. Determine the revised loan payment based on payments every four years
4. The ~~total~~ interest paid is equal to the difference between the total loan payments, and the original loan amount of 100,000

Step 1

payment			A		A	...	A		...	A		A
year	0	1	2	3	4	...	10	11	- -	38	39	40
pmt #	0		1		2		5			19		20

To calculate the payment, I prefer to convert the interest rate compounding period to match the payment period

$$A\left(a_{\overline{20}|j}\right) = 100{,}000 \quad \text{where} \quad 1+j = (1.06)^2 = 1.1236$$

$$A = \frac{100{,}000}{a_{\overline{20}|12.36\%}}$$

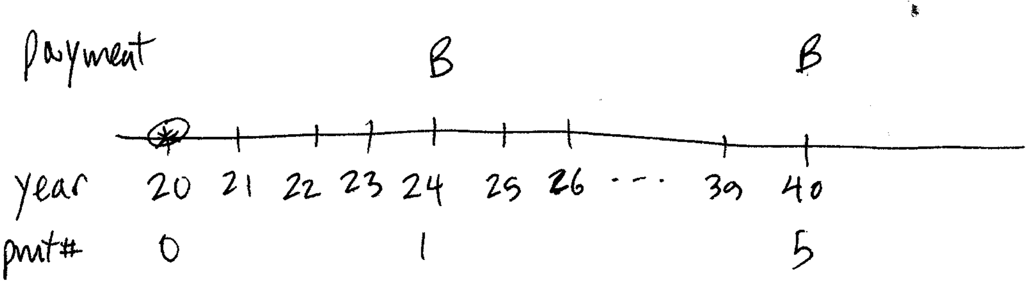
$$= 13{,}691$$

(29) STEP 2

The outstanding loan balance after the 10^{th} payment is $A(a_{\overline{10}|12.36\%}) = 13{,}691(5.5679)$
$= 76{,}231$

STEP 3

Now calculate the new payment based on one payment every four years:



Now I need to convert the interest rate based on four years for the payment period:

$$B(a_{\overline{5}|k}) = 76{,}231 \quad \text{where } 1+k = (1.06)^4 = 1.2625$$
$$B = 76{,}231 / a_{\overline{5}|26.25\%}$$
$$= 29{,}074$$

STEP 4

There are 10 payments of 13,691 plus five payments of 29,074, for a total of 282,283. After subtracting the principal payments of 100,000 (the original loan balance), the interest paid is 182,283. C

30 This is a typical EA-1 question on probability calculations. You need to write down formulas for each of the items given in the problem.

$$X = 1 - {}_5p_{40} = {}_5q_{40}$$

$$.40 = ({}_{15}p_{55})({}_{15}p_{40})$$

$$.44 = 1 - {}_{25}p_{45}$$

The key to working the problem is looking at the ages in the second item carefully. Or, you could simply express everything in terms of l_x values

$$X = 1 - {}_5p_{40} = (l_{40} - l_{45})/l_{40} = 1 - l_{45}/l_{40}$$
$$.40 = ({}_{15}p_{55})({}_{15}p_{40}) = (l_{70}/l_{55})(l_{55}/l_{40})$$
$$= l_{70}/l_{40}$$
$$.44 = 1 - {}_{25}p_{45} = (l_{45} - l_{70})/l_{45} = 1 - l_{70}/l_{45}$$

If you take the ratio of the two given items, you almost have what you want. Simply rearrange the terms of the last item to give the value of $l_{70}/l_{45} = .56$, then do the ratio:

$$\frac{l_{70}/l_{40}}{l_{70}/l_{45}} = \frac{l_{45}}{l_{40}} = \frac{.40}{.56} \Rightarrow 1 - \frac{l_{45}}{l_{40}} = X = .2857$$

B

31 This is one of the rare theoretical/calculus based questions on the EA-1 exam. You are given values for the probability of survival from age zero for males and females.

One trick to the problem is that you need to be careful how you write the integral for the complete expectation of life.

$$\int_{80}^{100} {}_tp_{0:0}\,dt \quad \text{this is WRONG!}$$

The expression above ignores the fact that the two lives survived from age zero to age 80. What the problem is asking for must be written differently:

$$_{20}\mathring{e}_{80:80} = \int_0^{20} {}_tp_{80:80}\,dt$$

The trick to the problem is that you are not given a formula for the value of $_tp_{80}$, but you are given $_tp_0$ instead.

$$_tp_0 = l_t/l_0 = (l_{80}/l_0)(l_t/l_{80}) \text{ for ages } t \geq 80$$
$$= ({}_{80}p_0)({}_{t-80}p_{80})$$

$$\therefore {}_{t-80}p_{80} = \frac{{}_tp_0}{{}_{80}p_0}$$

(31) continued

Now you can write formulas for the probability of survival from age 80, separately for males and females

${}_{t-80}p_{80}$ for males $= \dfrac{1-.01t}{1-.01(80)}$

${}_{t-80}p_{80}$ for females $= \dfrac{(1-.01t)^2}{[1-.01(80)]^2}$

${}_{t-80}p_{80:80}$ for the male/female joint life status is the product of the two formulas above.

$$ {}_{20}\overset{\circ}{e}_{80:80} = \int_{80}^{100} {}_{t-80}p_{80:80}\, dt $$

$$ = \int_{80}^{100} \frac{(1-.01t)^3}{(1-.01(80))^3}\, dt $$

$$ = \frac{1}{(.2)^3} \int_{80}^{100} (1-.01t)^3\, dt $$

$$ = \frac{1}{(.2)^3} \left[\frac{(1-.01t)^4}{4(-.01)} \Bigg|_{80}^{100} \right] $$

The above result is based on using the chain rule, and knowing a bit of calculus.

(31) continued

$$ {}_{20}\mathring{e}_{80:80} = \frac{1}{(.2)^3}\left[\frac{[1-.01(100)]^4}{-.04} - \frac{[1-.01(80)]^4}{-.04}\right] $$

$$ = \frac{1}{.008}\left[0 + \frac{(.2)^4}{.04}\right] $$

$$ = \frac{1}{.008}\left(\frac{.0016}{.04}\right) $$

$$ = \frac{.20}{.04} $$

$$ = 5.0 $$

(B)

Unfortunately, there does not seem to be any simpler method of solution for this problem. You could try using the formula for ${}_{t-80}p_{80:80}$, and approximating the integral using a summation:

$$ {}_{t-80}p_{80:80} = (1-.01t)^3/(.2)^3 $$

$$ {}_{20}\mathring{e}_{80:80} \doteq .5 + \frac{1}{(.2)^3}\left[(1-.01(81))^3 + (1-.01(82))^3 + \dots (1-.01(99))^3 + (1-.01(100))^3\right] $$

I don't see any nifty evaluation technique for that summation!

32 This is an unusual problem on multiple decrement tables. One key concept is the difference between a rate and a probability. If the problem asked for the probability of suspension, that would be based on the multiple decrement table. Since it asks for the rate of suspension, that is based on the single decrement table.

Single decrement rates are shown with a "prime", but probabilities are not. Here are the standard formulas that relate probabilities to rates:

$$p_x^{(T)} = 1 - q_x^{(1)} - q_x^{(2)} - q_x^{(3)} - q_x^{(4)} \quad \text{multiple decrement table}$$
$$= (1 - q_x'^{(1)})(1 - q_x'^{(2)})(1 - q_x'^{(3)})(1 - q_x'^{(4)}) \quad \text{single decrement rates}$$
$$= p_x'^{(1)} \cdot p_x'^{(2)} \cdot p_x'^{(3)} \cdot p_x'^{(4)}$$

In this problem, you are told that retirements occur at the beginning of the year, and non-renewals occur at the end of each year. That means that during each year, you have only two decrements that are "competing" to knock out a participant: death and suspension. You can use the standard formulas for a two decrement table to calculate the single decrement table rate of suspension.

(next page)

(32) With a two decrement table, you have these formulas:

$$p_x^{(T)} = 1 - q_x^{(1)} - q_x^{(2)}$$
$$= [1 - q_x'^{(1)}][1 - q_x'^{(2)}]$$

$$q_x'^{(1)} \doteq \frac{q_x^{(1)}}{1 - \frac{1}{2} q_x^{(2)}} \qquad q_x'^{(2)} \doteq \frac{q_x^{(2)}}{1 - \frac{1}{2} q_x^{(1)}}$$

These last two formulas illustrate the concept that the rate of decrement will exceed the probability in the multiple decrement table. These formulas are based on the assumption of uniform distribution of decrements in the multiple decrement table.

Let $q_x^{(d)}$ be the probability of death, and $q_x^{(s)}$ be the probability of suspension. Based on the data given, you can calculate both probabilities:

lives
1000
−10 retirements
990
−8 deaths
−12 suspensions
970
−42 non renewals
(don't care about final value)

(next page)

(32) During the year, the number who survive the initial retirement decrement drops from 990 to 970. The average number exposed to the death and suspension decrement is 980. I will use this value to calculate the probability of death and suspension in the multiple decrement table:

$$q_x^{(d)} \doteq \frac{8}{980} = .008163 \qquad q_x^{(s)} \doteq \frac{12}{980} = .012245$$

$$q_x'^{(s)} \doteq \frac{q_x^{(s)}}{1-\frac{1}{2}q_x^{(d)}}$$

$$= \frac{.012245}{1-.5(.008163)}$$

$$= .012295$$

(B)

Note that the multiple decrement probability of suspension also falls within answer range B. So you could get the correct answer range by incorrectly calculating only the probability of suspension.

33 The key point of this question is whether you understand the terminology of "one year term cost". This is the present value of benefits for the expected exits in the 12 months following the valuation date.

The benefit is 50% of salary, and it is paid at mid-year. When you set up the calculation, allow for either simple or compound interest. You should get in the same answer range for either approach.

$$
\begin{aligned}
X &= .15(50\%)(5{,}000{,}000)(1.08)^{-.5} \\
&\quad + .10(50\%)(9{,}000{,}000)(1.08)^{-.5} \\
&\quad + .05(50\%)(6{,}000{,}000)(1.08)^{-.5} \\
&= 50\%(1.08)^{-.5}[.15(5)+.10(9)+.05(6)](1{,}000{,}000) \\
&= .5(.96225)(1.95)(1{,}000{,}000) \\
&= 938{,}194
\end{aligned}
$$

B

If you used simple interest, then you get a different value for X:

$$
\begin{aligned}
X &= 50\%(1.04)^{-1}(1.95)(1{,}000{,}000) \\
&= 937{,}500
\end{aligned}
$$

2009

34. This is a typical question on rate of return calculations. You are given fund values and cash flows for each quarter. You don't use the interim fund balances in this problem (but you do use them to calculate a time-weighted rate of return).

First, write down the cash flows on a time line diagram:

cash flow	500,000	-200,000	-350,000	800,000	
Date	1-1	4-1	7-1	10-1	12-31
market value	10,000,000				11,400,000

Now you can write down a formula that accumulates the beginning market value and all the cash flows. For simplicity, I will use simple interest:

$$10{,}500{,}000(1+i) - 200{,}000\left(1+\tfrac{3}{4}i\right) - 350{,}000\left(1+\tfrac{2}{4}i\right) + 800{,}000\left(1+\tfrac{i}{4}\right) = 11{,}400{,}000$$

$$10{,}500{,}000\,i - 200{,}000\left(\tfrac{3}{4}i\right) - 350{,}000\left(\tfrac{2}{4}i\right) + 800{,}000\left(\tfrac{1}{4}i\right)$$

$$= 11{,}400{,}000 - 10{,}500{,}000 + 200{,}000 + 350{,}000 - 800{,}000 = 650{,}000$$

$$i = 650{,}000 \Big/ \left[\tfrac{4}{4}(10{,}500{,}000) - \tfrac{3}{4}(200{,}000) - \tfrac{2}{4}(350{,}000) + \tfrac{1}{4}(800{,}000)\right]$$

$$= 650{,}000/10{,}375{,}000 = 6.265\%$$

D

The final formula for i is an exposure formula. The numerator is the total investment return, and the denominator shows the weights for each of the cash flows.

35. There have been a few questions in recent years on realized and unrealized gains and losses.
A realized gain is the balancing item from one year's book value to the next year. This realized gain is the result of selling an investment, and it increases both the book value and the market value of assets.

An unrealized gain is the excess of the market value over the book value. A reconciliation of the book value of assets between years includes the realized gain (or loss). A reconciliation of market value of assets includes both the realized gain (or loss) and <u>the change</u> in the unrealized gain (or loss).

The problem gives you the starting book value and the realized gain. You can directly determine Y:

Book value + contrib + income − (benefits + expenses) + RG

$Y = 5{,}000{,}000 + 600{,}000 + 315{,}000 - (250{,}000 + 60{,}000) + 465{,}000$
$= 6{,}070{,}000$

Now you can write down the reconciliation of market value:

$5{,}335{,}000 = X + 600{,}000 + 315{,}000 - (250{,}000 + 60{,}000) + 465{,}000$ RG
$- 535{,}000$ ΔUG

$X = 5{,}335{,}000 - 535{,}000 = 4{,}800{,}000$

$X - Y = -1{,}270{,}000$

A