

SOCIETY OF ACTUARIES
AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2009 EA-1 EXAMINATION

**Conditions Generally Applicable to
All EA-1 Examination Questions**

If applicable, the following conditions should be considered a part of the data for each question, unless otherwise stated or implied:

- (1) The normal retirement age is 65.
- (2) Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.
- (3) There are no pre-retirement death or disability benefits.
- (4) Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.
- (5) Interest rates that are compounded more frequently than annually are expressed as nominal rates.
- (6) Where multiple lives are involved, future lifetimes are assumed to be independent of each other.
- (7) The term “gross single premium” is equivalent to “contract single premium;” the term “net single premium” is equivalent to “single benefit premium;” the term “gross annual premium” is equivalent to “annual contract premium;” the term “net annual premium” is equivalent to “annual benefit premium.”
- (8) There are no policy loans in effect.
- (9) For a bond, the face amount and the redemption value are the same.
- (10) Interest rate equals yield rate.
- (11) The term “duration” means “Macaulay duration”.

2009

Data for Question 1 (2 points)

Terms of a 5-year annuity-certain:

Payment amount	\$100
Payment frequency	Monthly, with payments at the beginning of each month
Interest rate	5% per year, compounded quarterly

Question 1

In what range is the present value of the annuity?

- (A) Less than \$5,190
- (B) \$5,190 but less than \$5,260
- (C) \$5,260 but less than \$5,330
- (D) \$5,330 but less than \$5,400
- (E) \$5,400 or more

2009

Data for Question 2 (3 points)

$$d_{45} = 502$$

$$\ell_{47} = 89,472$$

$${}_1q_{45} = 0.006141$$

Question 2

In what range is d_{46} ?

- (A) Less than 512
- (B) 512 but less than 532
- (C) 532 but less than 552
- (D) 552 but less than 572
- (E) 572 or more

2009

Data for Question 3 (3 points)

Terms of a 10-year annuity-certain:

Payments	\$100 per year payable annually with the first payment on January 1, 2012
Interest	The interest rates used to value the annuity as of January 1, 2009 are given by the following annual spot rates:

<u>Years</u>	<u>Spot rate</u>
2009-2013	5%
2014-2018	6%
2019-2023	7%

X = the present value of the annuity as of January 1, 2009.

Question 3

In what range is X ?

- (A) Less than \$650
- (B) \$650 but less than \$685
- (C) \$685 but less than \$720
- (D) \$720 but less than \$755
- (E) \$755 or more

2009

Data for Question 4 (3 points)

Selected values from a basic mortality table:

x	q_x
84	0.0762
85	0.0852
86	0.0953
87	0.1075
88	0.1204
89	0.1341
90	0.1493

The basic mortality table is adjusted using a 1-year set forward for male lives and using a 2-year set back for female lives.

$x =$ male life age 85

$y =$ female life age 86

Question 4

In what range is ${}_3p_{xy}^-$?

- (A) Less than 0.9075
- (B) 0.9075 but less than 0.9150
- (C) 0.9150 but less than 0.9225
- (D) 0.9225 but less than 0.9300
- (E) 0.9300 or more

2009

Data for Question 5 (4 points)

$$q_{20} = 0.10$$

$$q_x = 0.05 \text{ for all } x > 20$$

$i = 4.0\%$, compounded annually

Question 5

In what range is \ddot{a}_{20} ?

- (A) Less than 9.7
- (B) 9.7 but less than 10.2
- (C) 10.2 but less than 10.7
- (D) 10.7 but less than 11.2
- (E) 11.2 or more

2009

Data for Question 6 (3 points)

$$\mu(x) = \frac{1}{100-x}, \quad 0 \leq x \leq 100$$

$$i = 0.0\%$$

Question 6

In what range is \ddot{a}_{40} ?

- (A) Less than 29.9
- (B) 29.9 but less than 30.4
- (C) 30.4 but less than 30.9
- (D) 30.9 but less than 31.4
- (E) 31.4 or more

2009

Data for Question 7 (3 points)

Employee Smith earns an annual salary in 2009 of \$50,000.

Assumed salary increases: 3% each year, beginning 1/1/2010.

At the end of each year, beginning 12/31/2009, 5% of Smith's annual salary is deposited into a fund earning an annual rate of interest of 6%.

X = the amount in Smith's fund on 1/1/2030.

Question 7

In what range is X ?

- (A) Less than \$122,000
- (B) \$122,000 but less than \$126,000
- (C) \$126,000 but less than \$130,000
- (D) \$130,000 but less than \$134,000
- (E) \$134,000 or more

2009

Data for Question 8 (2 points)

$$\ddot{a}_{\overline{n}|} = 6.091836$$

$$\ddot{a}_{\overline{n+1}|} = 6.381005$$

Question 8

In what range is $\ddot{s}_{\overline{n}|}$?

- (A) Less than 20.10
- (B) 20.10 but less than 20.50
- (C) 20.50 but less than 20.90
- (D) 20.90 but less than 21.30
- (E) 21.30 or more

2009

Data for Question 9 (3 points)

Terms of a five-year bond issued on 1/1/2009:

Face amount	\$1,000
Redemption amount	\$1,000
Coupon rate	5.0% per year, payable annually
Yield rate	5.0% per year, compounded annually

X = the duration of this bond in years.

Question 9

In what range is X ?

- (A) Less than 3.75
- (B) 3.75 but less than 4.00
- (C) 4.00 but less than 4.25
- (D) 4.25 but less than 4.50
- (E) 4.50 or more

2009

Data for Question 10 (4 points)

A service table has two sources of decrement.

$$q_x^{(1)} = 4q_x^{(2)}$$

$$q_x^{(T)} = 0.24$$

Question 10

In what range is $q_x^{(1)}$?

- (A) Less than 0.195
- (B) 0.195 but less than 0.198
- (C) 0.198 but less than 0.201
- (D) 0.201 but less than 0.204
- (E) 0.204 or more

2009

Data for Question 11 (4 points)

Selected net annual premiums for whole life insurance policy:

$$P_{50} = 0.03550$$

$$P_{51} = 0.03712$$

$$I = 4.0\%$$

N = Out of the 100,000 alive at age 50, the number expected to die after age 51.

Question 11

In what range is N ?

- (A) Less than 93,800
- (B) 93,800 but less than 95,300
- (C) 95,300 but less than 96,800
- (D) 96,800 but less than 98,300
- (E) 98,300 or more

2009

Data for Question 12 (3 points)

The following is an extract from a table with a 3-year select period:

\underline{x}	$\underline{q_{[x]}}$	$\underline{q_{[x]+1}}$	$\underline{q_{[x]+2}}$	$\underline{q_{x+3}}$	$\underline{x+3}$
50	0.074	0.094	0.114	0.126	53
51	0.076	0.096	0.116	0.128	54
52	0.078	0.098	0.118	0.130	55
53	0.080	0.100	0.120	0.132	56
54	0.083	0.103	0.123	0.135	57

$$\ell_{55} = 13,200$$

Question 12

In what range is $d_{[53]+1}$?

- (A) Less than 1,460
- (B) 1,460 but less than 1,480
- (C) 1,480 but less than 1,500
- (D) 1,500 but less than 1,520
- (E) 1,520 or more

2009

Data for Question 13 (3 points)

You have the following information about a group of participants in a pension plan:

Number of active participants at exact age 50	1,000
Number of deaths between exact ages 50 and 51	10
Number of decrements other than death between exact ages 50 and 51	57

All decrements other than death occur one-third of the way during the period between consecutive ages.

X = the rate of mortality at age 50 in the associated single decrement mortality table.

Question 13

In what range is X ?

- (A) Less than 0.01025
- (B) 0.01025 but less than 0.01035
- (C) 0.01035 but less than 0.01045
- (D) 0.01045 but less than 0.01055
- (E) 0.01055 or more

2009

Data for Question 14 (3 points)

Interest = 6% per year, compounded annually.

$X =$ Present value of a perpetuity that pays \$1 at the end of the 2nd year, \$2 at the end of the 4th year, \$3 at the end of the 6th year, continuing to pay \$ k at the end of the $2k^{\text{th}}$ year.

Question 14

In what range is X ?

- (A) Less than \$73.00
- (B) \$73.00 but less than \$76.00
- (C) \$76.00 but less than \$79.00
- (D) \$79.00 but less than \$82.00
- (E) \$82.00 or more

2009

Data for Question 15 (5 points)

Terms of a serial bond:

Face amount of bond	\$20,000
Terms of redemption	20 equal annual installments at par payable at the end of the 11 th through the 30 th years
Coupons	8%, payable semi-annually
Yield rate	9%, compounded semi-annually

X = the purchase price of the serial bond.

Question 15

In what range is X ?

- (A) Less than \$18,100
- (B) \$18,100 but less than \$18,150
- (C) \$18,150 but less than \$18,200
- (D) \$18,200 but less than \$18,250
- (E) \$18,250 or more

2009

Data for Question 16 (2 points)

An annuity provides level annual payments of \$1,000 at the end of each year for four years.

Term structure of interest rates as of 1/1/2009:

<u>Length of investment</u>	<u>Spot rate</u>
1 year	4.00%
2 years	5.00%
3 years	5.75%
4 years	6.25%

X = the present value of the payments as of 1/1/2009.

Question 16

In what range is X ?

- (A) Less than \$3,460
- (B) \$3,460 but less than \$3,510
- (C) \$3,510 but less than \$3,560
- (D) \$3,560 but less than \$3,610
- (E) \$3,610 or more

2009

Data for Question 17 (2 points)

The following assumed rates of retirement are used in the actuarial valuation of a defined benefit pension plan:

<u>Age</u>	<u>Retirement rate at exact age</u>
62	40%
63	25%
64	25%
65	100%

No other decrements apply from ages 62 through 65.

All active participants are currently under age 62.

X = the weighted average assumed retirement age for the pension plan.

Question 17

In what range is X ?

- (A) Less than 63.25
- (B) 63.25 but less than 63.45
- (C) 63.45 but less than 63.65
- (D) 63.65 but less than 63.85
- (E) 63.85 or more

2009

Data for Question 18 (3 points)

Smith (age 45) purchases a single premium annuity with the following characteristics:

Single premium	\$100,000
Monthly payment	Z at the beginning of each month
Payment period	For Smith's lifetime, with payments guaranteed for the first 120 months
Interest rate	3% per year, compounded annually

Selected commutation functions:

\underline{x}	$\underline{D_x}$	$\underline{N_x}$
45	2,392,905	
...
55	1,639,330	24,032,177

Question 18

In what range is Z ?

- (A) Less than \$440.00
- (B) \$440.00 but less than \$445.00
- (C) \$445.00 but less than \$450.00
- (D) \$450.00 but less than \$455.00
- (E) \$455.00 or more

2009

Data for Question 19 (2 points)

Values for a double decrement service table:

$$q_x^{(1)} = 0.015, \quad 50 \leq x \leq 70$$

$$q_x^{(2)} = 0.050, \quad 50 \leq x \leq 70$$

No other decrements exist.

$X =$ the probability that a 53-year old will still be employed by age 62.

Question 19

In what range is X ?

- (A) Less than 0.5465
- (B) 0.5465 but less than 0.5475
- (C) 0.5475 but less than 0.5485
- (D) 0.5485 but less than 0.5495
- (E) 0.5495 or more

2009

Data for Question 20 (2 points)

Terms of a 9-year annuity due:

Payments	\$100 per year payable annually	
Interest	Applicable annual spot rates:	
	<u>Years</u>	<u>Applicable spot rate</u>
	1 - 5	5.00%
	6 - 9	6.00%

X = the present value of the annuity.

Question 20

In what range is X ?

- (A) Less than \$693
- (B) \$693 but less than \$713
- (C) \$713 but less than \$733
- (D) \$733 but less than \$753
- (E) \$753 or more

2009

Data for Question 21 (4 points)

Terms of a loan:

Date of loan	1/1/2009
Amount of loan	\$100,000
Frequency of payments	Annual
First payment	12/31/2009
Term of loan	5 years

X = the sum of payments under a level annual payment schedule, with interest rate of 6.0% per year compounded annually.

Y = the sum of payments under the sinking fund method, where the lender receives interest annually at 6.0% per year, and a sinking fund accumulates annually at 5.0% per year.

Question 21

In what range is $|X - Y|$?

- (A) Less than \$500
- (B) \$500 but less than \$1,000
- (C) \$1,000 but less than \$1,500
- (D) \$1,500 but less than \$2,000
- (E) \$2,000 or more

2009

Data for Question 22 (3 points)

Terms of an immediate annuity payable for life to a life age x on 1/1/2009:

Payment \$100,000 per year payable annually

Interest 5% per year, compounded annually

Mortality rates before improvement:

$$q_x = 0.051$$

$$q_{x+1} = 0.057$$

$$q_{x+2} = 0.063$$

Mortality rates are projected to improve by 1% per year, compounded annually, beginning 1/1/2010.

$Z =$ the present value as of 1/1/2009 of the third payment.

Question 22

In what range is Z ?

- (A) Less than \$72,570
- (B) \$72,570 but less than \$72,600
- (C) \$72,600 but less than \$72,630
- (D) \$72,630 but less than \$72,660
- (E) \$72,660 or more

2009

Data for Question 23 (3 points)

Smith (age 60) and Jones (age 61) are joint annuitants entitled to a joint and 100% survivor annuity paying \$1,000 at the beginning of each year commencing 1/1/2009.

Selected commutation functions:

\underline{x}	$\underline{D_x}$
60	285
61	267
...	...
64	219
65	205

$i = 6\%$, compounded annually.

$X =$ the present value of 5th payment as of 1/1/2009.

Question 23

In what range is X ?

- (A) Less than \$720
- (B) \$720 but less than \$750
- (C) \$750 but less than \$780
- (D) \$780 but less than \$810
- (E) \$810 or more

2009

Data for Question 24 (3 points)

The following actuarially equivalent annuities are available to Smith:

- Annuity 1: Monthly payments of \$100 for life
- Annuity 2: Monthly payments of \$94 for life, with monthly payments of \$47 continuing for the life of Smith's surviving spouse
- Annuity 3: Monthly payments of X for life, with monthly payments of 75% of X continuing for the life of Smith's surviving spouse

Question 24

In what range is X ?

- (A) Less than \$90.52
- (B) \$90.52 but less than \$91.52
- (C) \$91.52 but less than \$92.52
- (D) \$92.52 but less than \$93.52
- (E) \$93.52 or more

2009

Data for Question 25 (2 points)

Given the following values from a single decrement table:

x	q_x
46	0.07000
47	0.06500
48	0.06000
49	0.05000

A 2-year select mortality table based on this single decrement table has the following characteristics:

$$q_{[x]} = 1.5q_x$$

$$q_{[x]+1} = 1.3q_{x+1}$$

Question 25

In what range is ${}_1|q_{[46]+1}$?

- (A) Less than 0.05400
- (B) 0.05400 but less than 0.05450
- (C) 0.05450 but less than 0.05500
- (D) 0.05500 but less than 0.05550
- (E) 0.05550 or more

Data for Question 26 (3 points)

$$\ddot{a}_{60:\overline{5}|} = 4.3393$$

$$\ddot{a}_{65:\overline{5}|} = 4.2985$$

$$\ddot{a}_{60} = 11.7952$$

$$\ddot{a}_{65} = 10.8207$$

$$\ddot{a}_{70} = 9.7262$$

Question 26

In what range is ${}_{10}E_{60}$?

- (A) Less than 0.4000
- (B) 0.4000 but less than 0.4500
- (C) 0.4500 but less than 0.5000
- (D) 0.5000 but less than 0.5500
- (E) 0.5500 or more

2009

Data for Question 27 (2 points)

On 1/1/2009, Smith purchases a 5-year Certificate of Deposit that yields:

6% interest per year, compounded monthly for years 1 and 2

7% interest per year, compounded quarterly for year 3

8% interest per year, compounded semiannually for years 4 and 5

X = the equivalent annual rate of interest compounded annually.

Question 27

In what range is X ?

- (A) Less than 6.700%
- (B) 6.700% but less than 6.900%
- (C) 6.900% but less than 7.100%
- (D) 7.100% but less than 7.300%
- (E) 7.300% or more

2009

Data for Question 28 (2 points)

$$v^{11} = 1/2$$

$$X = 100|i^{(4)} - d^{(4)}|$$

Question 28

In what range is X ?

- (A) Less than 0.07
- (B) 0.07 but less than 0.14
- (C) 0.14 but less than 0.21
- (D) 0.21 but less than 0.28
- (E) 0.28 or more

2009

Data for Question 29 (3 points)

Terms of a loan:

Initial amount of loan	\$100,000
Term of loan	40 years
Level payments	Every other year with first payment in two years
Interest rate	6% per year compounded annually

Immediately after the 10th payment, the loan is renegotiated to make level payments every 4 years, with the first such payment four years after the renegotiation. The original term of the loan remains unchanged.

$I =$ the total interest paid over the entire term of the loan.

Question 29

In what range is I ?

- (A) Less than \$150,000
- (B) \$150,000 but less than \$170,000
- (C) \$170,000 but less than \$190,000
- (D) \$190,000 but less than \$210,000
- (E) \$210,000 or more

2009

Data for Question 30 (3 points)

Smith is 55, Jones is 45, and Brown is 40.

The underlying mortality is the same for all three individuals.

There is a 40% probability that both Smith and Brown will be alive in 15 years.

There is a 44% probability that Jones will die before age 70.

X = the probability that Brown will die before age 45.

Question 30

In what range is X ?

- (A) Less than 0.250
- (B) 0.250 but less than 0.300
- (C) 0.300 but less than 0.350
- (D) 0.350 but less than 0.400
- (E) 0.400 or more

2009

Data for Question 31 (4 points)

$${}_tP_0^{(\text{Male})} = 1 - 0.01t, \quad t \leq 100$$

$${}_tP_0^{(\text{Female})} = (1 - 0.01t)^2, \quad t \leq 100$$

X = the complete joint expectation of life for a male and a female, both age 80.

Question 31

In what range is X ?

- (A) Less than 4.5
- (B) 4.5 but less than 7.0
- (C) 7.0 but less than 9.5
- (D) 9.5 but less than 12.0
- (E) 12.0 or more

2009

Data for Question 32 (2 points)

Data for members of a professional association:

Members as of 1/1/2008	1,000
Retirements during 2008	10
Deaths during 2008	8
Suspensions during 2008	12
Non-renewals during 2008	42

Retirements occur at the beginning of each calendar year.

Deaths and suspensions occur uniformly throughout each calendar year.

Non-renewals occur at the end of each calendar year.

There are no other exits from the population.

X = the rate of suspension for 2008, given the data presented.

Question 32

In what range is X ?

- (A) Less than 0.012200
- (B) 0.012200 but less than 0.012400
- (C) 0.012400 but less than 0.012600
- (D) 0.012600 but less than 0.012800
- (E) 0.012800 or more

2009

Data for Question 33 (2 points)

A company provides for a lump sum severance benefit equal to 6 months salary for employees under age 45 who terminate during the next year. Assume that the benefits are paid mid-year.

Data for all employees:

x	$q_x^{(\text{Termination})}$	Total salary
30	0.15	\$5,000,000
35	0.10	\$9,000,000
40	0.05	\$6,000,000

$i = 8\%$, compounded annually.

$X =$ the one-year term cost of the severance benefit as of 1/1/2009.

Question 33

In what range is X ?

- (A) Less than \$925,000
- (B) \$925,000 but less than \$945,000
- (C) \$945,000 but less than \$965,000
- (D) \$965,000 but less than \$985,000
- (E) \$985,000 or more

2009

Data for Question 34 (3 points)

Pension fund information for 2009:

<u>Date</u>	<u>Fund Balance</u>	<u>Contribution</u>	<u>Distribution</u>
12/31/2008	\$10,000,000	-	-
01/01/2009	-	\$500,000	-
03/31/2009	\$10,250,000	-	-
04/01/2009	-	-	\$200,000
06/30/2009	\$10,500,000	-	-
07/01/2009	-	-	\$350,000
09/30/2009	\$10,400,000	-	-
10/01/2009	-	\$800,000	-
12/31/2009	\$11,400,000	-	-

X = the dollar-weighted rate of return in 2009.

Question 34

In what range is X ?

- (A) Less than 6.00%
- (B) 6.00% but less than 6.10%
- (C) 6.10% but less than 6.20%
- (D) 6.20% but less than 6.30%
- (E) 6.30% or more

2009

Data for Question 35 (2 points)

A pension trust statement reported the following information:

	<u>12/31/2008</u>	<u>12/31/2009</u>
Book value of assets	\$5,000,000	Y
Market value of assets	X	\$5,335,000

Activity during 2009:

Contributions	\$600,000
Investment income	\$315,000
Benefit payments	\$250,000
Plan expenses	\$60,000
Realized gains (losses)	\$465,000
Change in unrealized gains (losses)	(\$535,000)

Question 35

In what range is $(X - Y)$?

- (A) Less than (\$900,000)
- (B) (\$900,000) but less than (\$300,000)
- (C) (\$300,000) but less than \$300,000
- (D) \$300,000 but less than \$900,000
- (E) \$900,000 or more

****END OF EXAMINATION****

2009

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EA-1 Spring 2009
Answer Key

Question	Solution
1	C
2	D
3	A
4	E
5	D
6	C
7	C
8	D
9	E
10	C
11	E
12	A
13	C
14	B
15	C
16	B
17	B
18	D
19	E
20	C
21	D
22	B
23	D
24	B
25	C
26	C
27	D
28	B
29	C
30	B
31	B
32	B
33	B
34	D
35	A

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