

SOCIETY OF ACTUARIES
AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2011 EA-1 EXAMINATION

CONDITIONS GENERALLY APPLICABLE TO ALL EA-1 EXAMINATION QUESTIONS

If applicable, the following conditions should be considered a part of the data for each question, unless otherwise stated or implied:

- (1) The normal retirement age is 65.
- (2) Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.
- (3) There are no pre-retirement death or disability benefits.
- (4) Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.
- (5) Interest rates that are compounded more frequently than annually are expressed as nominal rates.
- (6) Where multiple lives are involved, future lifetimes are assumed to be independent of each other.
- (7) The term “gross single premium” is equivalent to “contract single premium;” the term “net single premium” is equivalent to “single benefit premium;” the term “gross annual premium” is equivalent to “annual contract premium;” the term “net annual premium” is equivalent to “annual benefit premium.”
- (8) There are no policy loans in effect.
- (9) For a bond, the face amount and the redemption value are the same.
- (10) Interest rate equals yield rate.
- (11) The term “duration” means “Macaulay duration”.

2011

Data for Question 1 (3 points)

Given the following spot rates from a yield curve:

	Annual <u>yield</u>
1 year	2.00%
2 years	4.00%
3 years	5.00%

A three-year bond with annual coupons of X is purchased for \$5,000 and is redeemable for \$5,000.

Question 1

In what range is X ?

- (A) Less than \$243
- (B) \$243 but less than \$245
- (C) \$245 but less than \$247
- (D) \$247 but less than \$249
- (E) \$249 or more

2011

Data for Question 2 (2 points)

Selected probabilities:

x	p_x
55-59	0.99
60-64	0.95
65	0.90

Interest rate: 7% per year, compounded annually

$$a_{65} = 8.194$$

Question 2

In what range is ${}_{10|}\ddot{a}_{55}$?

- (A) Less than 3.1
- (B) 3.1 but less than 3.2
- (C) 3.2 but less than 3.3
- (D) 3.3 but less than 3.4
- (E) 3.4 or more

2011

Data for Question 3 (3 points)

Smith deposits \$1,300 into a fund. Smith makes no further deposits or any withdrawals during the 17-year period beginning with the date of deposit.

Interest is credited on Smith's account as follows:

Discount rate during the first 6 years 7.0% per year, compounded quarterly.

Interest rate during the next 5 years 8.0% per year, compounded semiannually.

Force of interest during the next 6 years 6.0% per year.

X = the value of the fund at the end of the 17-year period.

Question 3

In what range is X ?

- (A) Less than \$4,200
- (B) \$4,200 but less than \$4,220
- (C) \$4,220 but less than \$4,240
- (D) \$4,240 but less than \$4,260
- (E) \$4,260 or more

2011

Data for Question 4 (3 points)

Data relative to a 4-year temporary life annuity immediate with annual payments:

Age at purchase	65.
Present value at purchase	\$100,000.

Interest rate: 7% per year, compounded annually.

Selected values from a 3-year select mortality table:

x	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}
62	10,000,000	9,844,074	9,662,671	9,460,642
63	9,831,050	9,662,801	9,465,583	9,247,191
64	9,642,924	9,463,045	9,251,733	9,017,426
65	9,435,643	9,245,121	9,020,841	8,771,863
66	9,206,484	9,007,739	8,773,003	8,511,918
67	8,963,417	8,757,765	8,512,335	8,238,915
68	8,703,733	8,492,221	8,237,205	7,952,671
69	8,414,800	7,922,545	7,648,142	7,340,327

Question 4

In what range is the annual payment?

- (A) Less than \$31,200
- (B) \$31,200 but less than \$31,300
- (C) \$31,300 but less than \$31,400
- (D) \$31,400 but less than \$31,500
- (E) \$31,500 or more

2011

Data for Question 5 (3 points)

Data about an annuity:

Form of annuity	Life annuity-due.
Amount of annuity	\$1,000 per month.
To whom payable	Smith, age 65.

In lieu of this annuity, Smith and Jones (age 63) elect an actuarially equivalent joint and survivor annuity-due providing:

- I. P per month while both are alive.
- II. If Smith dies before Jones, $0.50P$ for Jones' remaining lifetime after Smith's death.
- III. If Jones dies before Smith, \$1,000 per month for Smith's remaining lifetime after Jones's death.

Given:

$$\ddot{a}_{63}^{(12)} = 12.1$$

$$\ddot{a}_{65}^{(12)} = 11.5$$

$$\ddot{a}_{63:65}^{(12)} = 9.0$$

Question 5

In what range is P ?

- (A) Less than \$780
- (B) \$780 but less than \$810
- (C) \$810 but less than \$840
- (D) \$840 but less than \$870
- (E) \$870 or more

2011

Data for Question 6 (4 points)

Terms of an annuity:

Term	12 years.
Payment period	Annual, with payments at the end of each year.
Payments	First payment equals \$10,000. Subsequent payments will be indexed to the excess of the percentage increase in the Consumer Price Index (CPI) over 3%.
Interest rate	8% per year, compounded annually.

X = the present value of the annuity if the annual rate of increase in the CPI is 6%.

Y = the present value of the annuity if the annual rate of increase in the CPI is 4%.

Question 6

In what range is $|X - Y|$?

- (A) Less than \$7,745
- (B) \$7,745 but less than \$7,925
- (C) \$7,925 but less than \$8,105
- (D) \$8,105 but less than \$8,285
- (E) \$8,285 or more

Data for Question 7 (2 points)

Selected values:

$$\ddot{a}_{\overline{2}|}^{(12)} = 1.892447$$

$$a_{\overline{2}|}^{(12)} = 1.883280$$

i = interest rate, compounded annually

Question 7

In what range is i ?

- (A) Less than 5.5%
- (B) 5.5% but less than 5.9%
- (C) 5.9% but less than 6.3%
- (D) 6.3% but less than 6.7%
- (E) 6.7% or more

Data for Question 8 (3 points)

Terms of a loan:

Amount of loan	\$1,000,000.
Term	30 years.
Payments	Level monthly, payable at the end of each month.
Interest rate	
For first 15 years	7.0% per year, compounded annually.
For last 15 years	11.0% per year, compounded annually.

X = the amount of interest paid in the 204th repayment.

Question 8

In what range is X ?

- (A) Less than \$5,120
- (B) \$5,120 but less than \$5,240
- (C) \$5,240 but less than \$5,360
- (D) \$5,360 but less than \$5,480
- (E) \$5,480 or more

2011

Data for Question 9 (3 points)

Terms of a bond:

Par value	\$1,000.
Redemption value	\$1,100.
Term of bond	10 years.
Coupon rate	R per year, payable semiannually.

Issue price:

P , if yield to maturity is 4% per year, compounded annually

$(P - \$95.50)$, if yield to maturity is 5% per year, compounded annually

Question 9

In what range is R ?

- (A) Less than 7.2%
- (B) 7.2% but less than 7.7%
- (C) 7.7% but less than 8.2%
- (D) 8.2% but less than 8.7%
- (E) 8.7% or more

2011

Data for Question 10 (3 points)

A portfolio consists of the following two bonds:

	<u>Bond 1</u>	<u>Bond 2</u>
Face amount	\$1,000	\$1,000
Term	10 years	13 years
Coupon amount	\$90	None
Coupon frequency	Annually	N/A
Modified duration	6.42 years	
Yield rate	9% per year, compounded annually	9% per year, compounded annually

X = the modified duration of the portfolio.

Question 10

In what range is X ?

- (A) Less than 7.9 years
- (B) 7.9 years but less than 8.4 years
- (C) 8.4 years but less than 8.9 years
- (D) 8.9 years but less than 9.4 years
- (E) 9.4 years or more

2011

Data for Question 11 (4 points)

On 1/1/2011 Smith (age 30) establishes a fund to provide for Smith's retirement at age 65. The following information is given:

Contributions	5% of monthly salary at the end of each month.
2011 annual salary	\$40,000.
Assumed salary increases	4% per year assumed to occur on the first day of each calendar year.
Assumed annual rate of investment return	5%.
Mortality before retirement	None.

X = the accumulated value of the fund at retirement.

Question 11

In what range is X ?

- (A) Less than \$315,000
- (B) \$315,000 but less than \$319,000
- (C) \$319,000 but less than \$323,000
- (D) \$323,000 but less than \$327,000
- (E) \$327,000 or more

2011

Data for Question 12 (2 points)

Data from a select and ultimate mortality table:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}
71	0.057	0.070	0.082
72	0.060	0.072	0.085
73	0.062	0.075	0.088
74	0.063	0.077	0.090

Question 12

In what range is ${}_4q_{[71]+1}$?

- (A) Less than 0.270
- (B) 0.270 but less than 0.280
- (C) 0.280 but less than 0.290
- (D) 0.290 but less than 0.300
- (E) 0.300 or more

Data for Question 13 (4 points)

A mortality table has a 3-year select period.

You are given the following:

$$\ell_6 = 90,000$$

$$d_x = 5000, x \geq 3$$

$$q_{[0]} = 0.1667$$

$${}_5p_{[1]} = 0.8000$$

$${}_3p_{[0]+1} = 0.9{}_3p_{[1]}$$

Question 13

In what range is $\ell_{[0]}$?

- (A) Less than 120,000
- (B) 120,000 but less than 132,000
- (C) 132,000 but less than 144,000
- (D) 144,000 but less than 156,000
- (E) 156,000 or more

2011

Data for Question 14 (2 points)

Smith's will provides for \$1,000,000 to be paid to Smith's favorite charity upon Smith's death. On each December 31 following Smith's death, the charity is required to pay Smith's surviving spouse \$70,000 for the spouse's remaining lifetime.

Smith dies on January 1. His spouse is age 65 when he dies.

Selected commutation functions:

x	D_x	N_x
65	9,957	106,068
66	9,360	

X = the present value of the charity's interest in the fund as of Smith's date of death.

Question 14

In what range is X ?

- (A) Less than \$250,000
- (B) \$250,000 but less than \$275,000
- (C) \$275,000 but less than \$300,000
- (D) \$300,000 but less than \$325,000
- (E) \$325,000 or more

2011

Data for Question 15 (4 points)

There are 9,800 members in a stationary population. Each year, there are four times as many deaths at age 25 and over as there are deaths at ages under 25. The average age at death for the former group is 66. The average age at death for the latter group is 16.

X = the number of members who die younger than age 25 each year.

Question 15

In what range is X ?

- (A) Less than 25
- (B) 25 but less than 28
- (C) 28 but less than 31
- (D) 31 but less than 34
- (E) 34 or more

2011

Data for Question 16 (3 points)

A pension plan provides for a death benefit after 5 years of service and for a retirement benefit for anyone who terminates after attainment of age 55 with 5 years of service.

Under a two decrement table used for the pension valuation the absolute rate of death is 2% and the absolute rate of termination is 4% for all ages.

An employee was hired at age 58 and is currently age 60.

X = the probability that the employee receives either a death or retirement benefit from the plan.

Question 16

In what range is X ?

- (A) Less than 0.828
- (B) 0.828 but less than 0.830
- (C) 0.830 but less than 0.832
- (D) 0.832 but less than 0.834
- (E) 0.834 or more

2011

Data for Question 17 (3 points)

The following annuities are actuarially equivalent:

- I. A life annuity of \$1,000 per month payable at the beginning of each month starting at age 55.
- II. A life annuity that provides for the payment of X per month payable at the beginning of each month from age 55 to age 62 and $(X - \$800)$ per month thereafter.

Selected actuarial values:

$$\ddot{a}_{55}^{(12)} = 11.33$$

$$\ddot{a}_{55:\overline{7}|}^{(12)} = 5.50$$

Question 17

In what range is X ?

- (A) Less than \$1,150
- (B) \$1,150 but less than \$1,250
- (C) \$1,250 but less than \$1,350
- (D) \$1,350 but less than \$1,450
- (E) \$1,450 or more

2011

Data for Question 18 (3 points)

33.0% of those who die between ages 30 and 70 die before age 50.

The probability that a person aged 30 dies before age 50 is 0.20.

X = the probability that a person aged 50 will survive to age 70.

Question 18

In what range is X ?

- (A) Less than 0.42
- (B) 0.42 but less than 0.47
- (C) 0.47 but less than 0.52
- (D) 0.52 but less than 0.57
- (E) 0.57 or more

2011

Data for Question 19 (5 points)

$$q_{[40]+s}^{(w)} = q_{[40]}^{(w)} - 0.02s; \quad s = 0, 1$$

$$q_x^{(w)} = 0.10 - 0.003(x - 40)$$

All rates of withdrawal are less than 0.50.

Number of participants as of 1/1/2011:

<u>Years of service</u>	<u>Age 40</u>	<u>Age 41</u>
0	200	0
1	0	150
2+	900	0

120 of these individuals are expected to terminate employment in 2012.

$$X = q_{[40]}^{(w)}$$

Question 19

In what range is X ?

- (A) Less than 0.175
- (B) 0.175 but less than 0.185
- (C) 0.185 but less than 0.195
- (D) 0.195 but less than 0.205
- (E) 0.205 or more

2011

Data for Question 20 (4 points)

Smith (age 65) is eligible to retire with a monthly pension benefit payable under one of the following forms of payment:

- I. A straight life annuity of \$15,000.
- II. An annuity of \$13,500 payable to Smith, with \$6,750 continuing to Smith's surviving spouse after Smith's death.
- III. An annuity of P payable to Smith, with $0.75P$ continuing to Smith's surviving spouse after Smith's death.

All annuity payments are made at the beginning of each month.

Question 20

In what range is P ?

- (A) Less than \$12,200
- (B) \$12,200 but less than \$12,500
- (C) \$12,500 but less than \$12,800
- (D) \$12,800 but less than \$13,100
- (E) \$13,100 or more

2011

Data for Question 21 (3 points)

Terms of an annuity:

Date of first payment	1/1/2012.
Frequency of payments	Monthly.
Amount of each payment	
First 5 years	\$500 per month.
Next 5 years	\$650 per month.
Final payment	\$10,000 on 1/1/2022.
Interest rate	7% per year, compounded annually.

X = the present value of the annuity as of 1/1/2011.

Question 21

In what range is X ?

- (A) Less than \$50,500
- (B) \$50,500 but less than \$51,500
- (C) \$51,500 but less than \$52,500
- (D) \$52,500 but less than \$53,500
- (E) \$53,500 or more

2011

Data for Question 22 (3 points)

Smith (age 65) purchases an annuity that pays \$1,000 at the end of each year. Payment ceases at the earlier of Smith's death or 25 years from purchase date.

Assumptions used for the annuity:

Interest rate

5%, compounded annually.

Mortality

$p_x = 0.95$, for $x < 75$;

Mortality rates at ages 75 and greater are double those under age 75.

Y = the present value of this annuity.

Question 22

In what range is Y ?

- (A) Less than \$6,500
- (B) \$6,500 but less than \$7,500
- (C) \$7,500 but less than \$8,500
- (D) \$8,500 but less than \$9,500
- (E) \$9,500 or more

2011

Data for Question 23 (4 points)

As of 1/1/2011, a portfolio of assets consists of the following:

- I. An annuity certain with 20 annual payments of \$2,000 beginning on 12/31/2021.
- II. A \$10,000 zero coupon bond maturing on 12/31/2015.

Interest rate: 8% per year, compounded annually.

X = the modified duration of the portfolio as of 1/1/2011.

Question 23

In what range is X ?

- (A) Less than 10.7 years
- (B) 10.7 years but less than 11.2 years
- (C) 11.2 years but less than 11.7 years
- (D) 11.7 years but less than 12.2 years
- (E) 12.2 years or more

2011

Data for Question 24 (4 points)

Assume a uniform distribution of decrement over each interval $[x, x + 1]$.

$${}_{0.5}q_{40.4} = 0.025$$

$${}_{0.9}p_{41} = 0.955$$

$$\mu_{42.2} = 0.05$$

$$\ell_{43} = 100,000$$

Question 24

In what range is ℓ_{40} ?

- (A) Less than 116,000
- (B) 116,000 but less than 116,500
- (C) 116,500 but less than 117,000
- (D) 117,000 but less than 117,500
- (E) 117,500 or more

2011

Data for Question 25 (4 points)

On 1/1/2011, Smith borrows a sum of money repayable in annual payments at the end of each year for 20 years. Each annual payment contains principal and interest.

The principal in the payment at the end of year t equals $\$100t$ (for $t = 1, 2, 3, \dots, 20$).

The payment at the end of the year also includes the interest accrued during the year.

Interest rate: 6.0%, compounded annually.

X = the present value as of 1/1/2011 of interest payments made during the 20 years of the loan.

Question 25

In what range is X ?

- (A) Less than \$9,000
- (B) \$9,000 but less than \$10,000
- (C) \$10,000 but less than \$11,000
- (D) \$11,000 but less than \$12,000
- (E) \$12,000 or more

2011

Data for Question 26 (2 points)

Terms of a bond:

Face amount \$1,000.

Coupon rate 7.0% per year, payable semiannually on each 6/30 and 12/31.

Amortized value of bond as of 6/30/2011 (before payment of coupon): \$939.33

Amortized value of bond as of 12/31/2011 (before payment of coupon): \$943.78

X = the annual effective yield rate for the bond.

Question 26

In what range is X ?

- (A) Less than 8.00%
- (B) 8.00% but less than 8.25%
- (C) 8.25% but less than 8.50%
- (D) 8.50% but less than 8.75%
- (E) 8.75% or more

2011

Data for Question 27 (4 points)

Market value of a pension fund at various dates in a calendar year:

<u>Date</u>	<u>Market Value</u>	<u>Contributions</u>	<u>Benefit Payments</u>
1/1	\$ 50,000		
3/31	\$ 60,000		
4/1			$\$P$
6/30	\$ 45,000		
7/1		\$ 17,000	$\$P$
9/30	\$ 40,000		
10/1		\$ 55,000	$\$P$
12/31	\$ 65,000		

Dollar-weighted rate of return for the year: 7.0%.

X = time-weighted rate of return for the year.

Question 27

In what range is X ?

- (A) Less than 6.0%
- (B) 6.0% but less than 8.0%
- (C) 8.0% but less than 10.0%
- (D) 10.0% but less than 12.0%
- (E) 12.0% or more

Data for Question 28 (4 points)

In a triple decrement table you are given the following:

$$\ell_x^{(1)} = 3(100 - x), 0 \leq x \leq 100$$

$$\ell_x^{(2)} = 4(110 - x), 0 \leq x \leq 110$$

$$\ell_x^{(3)} = 5(120 - x), 0 \leq x \leq 120$$

Question 28

In what range is $\mu_{50}^{(2)}$?

- (A) Less than 0.0050
- (B) 0.0050 but less than 0.0090
- (C) 0.0090 but less than 0.0130
- (D) 0.0130 but less than 0.0170
- (E) 0.0170 or more

Data for Question 29 (4 points)

Smith (age 55) is entitled to an annual payment of X at the beginning of each year guaranteed for 10 years and continuing as long as Smith is alive. Instead, Smith elects an actuarially equivalent annuity that pays the following as long as Smith is alive:

- I. \$10,000 at the beginning of each year for the first 5 years,
- II. \$7,500 at the beginning of each year for the next 5 years, and
- III. \$5,000 at the beginning of each year thereafter.

Interest rate: 7%, compounded annually.

Selected actuarial factors:

$$\ddot{a}_{55} = 11.2751$$

$$\ddot{a}_{60} = 10.2758$$

$$\ddot{a}_{65} = 9.1301$$

$$\ddot{a}_{55:\overline{5}|} = 4.3122$$

$$\ddot{a}_{60:\overline{5}|} = 4.2707$$

Question 29

In what range is X ?

- (A) Less than \$7,375
- (B) \$7,375 but less than \$7,425
- (C) \$7,425 but less than \$7,475
- (D) \$7,475 but less than \$7,525
- (E) \$7,525 or more

2011

Data for Question 30 (3 points)

Smith (age 50) takes out a 3-year term insurance policy:

Sum insured \$100,000.

Payable	At the end of year of Smith's death.
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Selected probabilities:

$${}_2p_{50} = 0.9955$$

$${}_kq_{50} = 1.08^k \cdot q_{50}$$

Interest: 6.0% per year, compounded annually.

X = the net single premium for this policy.

Question 30

In what range is X ?

- (A) Less than \$630
- (B) \$630 but less than \$640
- (C) \$640 but less than \$650
- (D) \$650 but less than \$660
- (E) \$660 or more

2011

Data for Question 31 (2 points)

The following assumed rates of retirement are used in the actuarial valuation of a defined benefit pension plan:

<u>Age</u>	<u>Rate of retirement</u>
62	40%
63	25%
64	25%
65	100%

Retirement is assumed to occur at integral ages only.

No other decrements apply from ages 62 through 65.

X = the weighted average assumed retirement age for the pension plan.

Question 31

In what range is X ?

- (A) Less than 63.25
- (B) 63.25 but less than 63.45
- (C) 63.45 but less than 63.65
- (D) 63.65 but less than 63.85
- (E) 63.85 or more

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EA-1 Spring 2011
Answer Key

Question	Points	Solution
1	3	C
2	2	E
3	3	B
4	3	B
5	3	D
6	4	B
7	2	C
8	3	A
9	3	B
10	3	A
11	4	C
12	2	C
13	4	D
14	2	D
15	4	E
16	3	D
17	3	D
18	3	C
19	5	E
20	4	D
21	3	B
22	3	C
23	4	C
24	4	B
25	4	D
26	2	E
27	4	D
28	4	B
29	4	A
30	3	A
31	2	B

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