SOCIETY OF ACTUARIES

AMERICAN SOCIETY OF PENSION ACTUARIES

JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2013 EA-1 EXAMINATION

CONDITIONS GENERALLY APPLICABLE TO ALL EA-1 EXAMINATION QUESTIONS

If applicable, the following conditions should be considered a part of the data for each question, unless otherwise stated or implied:

(1) The normal retirement age is 65.

(2) Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.

(3) There are no pre-retirement death or disability benefits.

(4) Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.

(5) Interest rates that are compounded more frequently than annually are expressed as nominal rates.

(6) Where multiple lives are involved, future lifetimes are assumed to be independent of each other.

(7) The term "gross single premium" is equivalent to "contract single premium;" the term "net single premium" is equivalent to "single benefit premium;" the term "gross annual premium" is equivalent to "annual contract premium;" the term "net annual premium" is equivalent to "annual benefit premium."

(8) There are no policy loans in effect.

(9) For a bond, the face amount and the redemption value are the same.

(10) Interest rate equals yield rate.

(11) The term "duration" means "Macaulay duration".

2013

Data for Question 1 (2 points)

The present value of a 15-year monthly annuity-immediate is $20,600.

Payments are as follows:

Years	Monthly annuity payment
$1-7$	X
$8-15$	$X + \$300$

Interest rate: 8.0% per year, compounded annually.

Question 1

In what range is X?

(A) Less than $74.50

(B) $74.50 but less than $75.50

(C) $75.50 but less than $76.50

(D) $76.50 but less than $77.50

(E) $77.50 or more

Data for Question 2 (3 points)

Consider the following two last survivor annuities:

	Annuity A	Annuity B
Payment Timing	Beginning of Year	End of Year
Payment while both x and y are alive	$\boldsymbol{R}$	$\boldsymbol{R}$
Payment after the first death of x and y	$2\boldsymbol{R}$	$\frac{1}{2}\boldsymbol{R}$
Net single premium	\$116	$\boldsymbol{X}$

Selected actuarial values:

$$\ddot{a}_x = 10.00$$
$$\ddot{a}_y = 15.00$$
$$\ddot{a}_{xy} = 7.00$$

Question 2

In what range is $\boldsymbol{X}$?

(A) Less than \$32

(B) \$32 but less than \$36

(C) \$36 but less than \$40

(D) \$40 but less than \$44

(E) \$44 or more

Data for Question 3 (3 points)

Smith (age 65) purchases an annuity with the following provisions:

Frequency of annuity payments	Monthly, with the first payment made on the purchase date
Amount of each annuity payment	\$50
Death benefit	\$10,000, payable at end of year of death if death occurs during first 10 years, \$0 otherwise

Interest rate: 7.0% per year, compounded annually.

Selected commutation functions:

x	D_x	N_x
65	965	8,872
75	346	2,379

$\boldsymbol{X}$ = the net single premium.

Question 3

In what range is $\boldsymbol{X}$?

(A) Less than \$6,500

(B) \$6,500 but less than \$7,500

(C) \$7,500 but less than \$8,500

(D) \$8,500 but less than \$9,500

(E) \$9,500 or more

2013

Data for Question 4 (3 points)

Fund balance as of 1/1/2013	$12,000.
Deposits to the fund:	60 deposits of $100 on the last day of each month beginning 1/31/2013
Withdrawals from the fund:	20 withdrawals of $1,000 on the first day of each quarter beginning 1/1/2020
Interest rate:	8.0% per year, compounded monthly.

X = the fund balance as of 12/31/2024.

Question 4

In what range is ***X***?

(A) Less than $13,500

(B) $13,500 but less than $15,000

(C) $15,000 but less than $16,500

(D) $16,500 but less than $18,000

(E) $18,000 or more

Data for Question 5 (2 points)

A $1,000 mortgage is repaid over 20 years by level annual payments at the end of each year computed at 4.0% per year compounded annually.

X = the modified duration of the mortgage.

Question 5

In what range is ***X***?

(A) Less than 9.0

(B) 9.0 but less than 9.3

(C) 9.3 but less than 9.6

(D) 9.6 but less than 9.9

(E) 9.9 or more

Data for Question 6 (3 points)

An insurance company uses a 3-year select period in its calculations.

Given the following survival model for ultimate mortality:

$$l_x = (31 - x), \quad 0 \le x \le 31$$

For the 3-year select period:

$$q_{[x]+t} = \left(\frac{t+1}{t+2}\right) q_{x+t}, \; t = 0, 1, 2$$

Smith was insured one year ago, at age 20.

X = the probability that Smith will die between age 23 and age 24.

Question 6

In what range is ***X***?

(A) Less than 0.0925

(B) 0.0925 but less than 0.1000

(C) 0.1000 but less than 0.1075

(D) 0.1075 but less than 0.1150

(E) 0.1150 or more

Data for Question 7 (3 points)

Given the following:

I. The probability that two independent lives, a 20-year old and a 40-year old, both survive 20 years is 0.733333

II. Out of 800 lives at age 20, 96 are expected to die by age 30

Question 7

In what range is ${}_{30}q_{30}$?

(A) Less than 0.150

(B) 0.150 but less than 0.155

(C) 0.155 but less than 0.160

(D) 0.160 but less than 0.165

(E) 0.165 or more

Data for Question 8 (4 points)

Given the following selected mortality values:

$${}_{n}p_{xx} = 0.25$$

$$p_{x+n} = 0.50$$

$$X = {}_{n}q_{x} + {}_{n}q_{xx} - {}_{n|}q_{xxx}$$

Question 8

In what range is X?

(A) Less than 1.14

(B) 1.14 but less than 1.16

(C) 1.16 but less than 1.18

(D) 1.18 but less than 1.20

(E) 1.20 or more

Data for Question 9 (3 points)

Selected annuity values:

$$\ddot{a}_{\overline{n+2}|} = 14.030$$
$$\ddot{s}_{\overline{n}|} = 52.344$$

Question 9

In what range is the effective annual interest rate?

(A) Less than 5.00%

(B) 5.00% but less than 5.25%

(C) 5.25% but less than 5.50%

(D) 5.50% but less than 5.75%

(E) 5.75% or more

Data for Question 10 (4 points)

Consider the following annuities:

Annuity 1	Annuity 2
10-year decreasing annuity with annual payments beginning at the end of the first year. Payments begin at \$100 and decline by \$10 each year.	15-year increasing annuity with annual payments beginning at the end of the first year. The first payment equals \$30 and each subsequent payment is 5.0% greater than the one preceding it.

Interest rate: 6.5% per year, compounded annually.

$\boldsymbol{X}$ = the present value of Annuity 1.

$\boldsymbol{Y}$ = the present value of Annuity 2.

Question 10

In what range is $|\boldsymbol{X} - \boldsymbol{Y}|$?

(A) Less than \$22

(B) \$22 but less than \$29

(C) \$29 but less than \$36

(D) \$36 but less than \$43

(E) \$43 or more

Data for Question 11 (3 points)

Smith (age 65) purchases an annuity-immediate of $100,000 per annum on 1/1/2013 payable annually for two years.

Mortality rates for 2012 are as follows:

x	q_x
65	0.0156
66	0.0176

To compute the single premium for this annuity, the seller uses projected mortality and assumes mortality rates will decrease at a rate of 1.5% per year.

Interest rate: 5.0% per year, compounded annually.

Question 11

In what range is the single premium for this annuity?

(A) Less than $180,000

(B) $180,000 but less than $181,550

(C) $181,550 but less than $183,100

(D) $183,100 but less than $184,650

(E) $184,650 or more

Data for Question 12 (4 points)

Consider a 2-decrement table, reflecting mortality and withdrawal:

$$q'^{(mortality)}_x = 0.03$$
$$\mu^{(withdrawal)}_x = 0.20$$

Mortality is uniformly distributed on the single table.

Withdrawal has a constant force of decrement on the single table.

Question 12

In what range is $q^{(mortality)}_x$?

(A) Less than 0.0270

(B) 0.0270 but less than 0.0275

(C) 0.0275 but less than 0.0280

(D) 0.0280 but less than 0.0285

(E) 0.0285 or more

Data for Question 13 (3 points)

A pension plan has a shortfall of $1,000,000.

The shortfall is to be amortized in level annual installments over 7 years using the yield curve below, with the first payment due immediately.

Duration	Spot rate
1	6.19%
2	7.32%
3	7.83%
4	8.03%
5	8.18%
6	8.33%
7	8.50%

X = the annual amortization payment.

Question 13

In what range is ***X***?

(A) Less than $175,000

(B) $175,000 but less than $180,000

(C) $180,000 but less than $185,000

(D) $185,000 but less than $190,000

(E) $190,000 or more

Data for Question 14 (3 points)

An organization has 125 members all exact age 62 as of 01/01.

From this group, the following members left during the calendar year, for the reasons shown:

Deaths, all on 06/01:	3
Disablements, all on 07/01:	5
Terminations, all on 08/01:	10
Retirements, all on 11/01:	4

In addition, 15 new members, each exact age 62 on 01/01, joined the group on 10/01.

None of the 15 new members died, terminated, became disabled, or retired during the year.

q_{62} = the organization's rate of mortality at age 62.

Question 14

In what range is q_{62}?

(A) Less than 0.0255

(B) 0.0255 but less than 0.0260

(C) 0.0260 but less than 0.0265

(D) 0.0265 but less than 0.0270

(E) 0.0270 or more

Data for Question 15 (3 points)

Smith (age 62) has the option to choose one of the following actuarially equivalent benefits:

I. A life annuity-due of \$30,000 per year, with payments beginning immediately

II. A \$50,000 lump sum payable immediately; plus
a 5-year term certain annuity-due that pays ***X*** per year starting at age 65, if Smith survives to age 65

Interest rate: 7.0% per year, compounded annually.

Selected actuarial values:

$$\ddot{a}_{62} = 12.68$$

$$_{n}p_{x} = 0.99^{n}; \quad x \le 65$$

Question 15

In what range is ***X***?

(A) Less than \$80,000

(B) \$80,000 but less than \$85,000

(C) \$85,000 but less than \$90,000

(D) \$90,000 but less than \$95,000

(E) \$95,000 or more

Data for Question 16 (3 points)

Given the following actuarial values:

$$q_x = 0.03$$
$$q_{x+1} = 0.04$$

Deaths are uniformly distributed throughout each year.

$$\boldsymbol{X} = {}_{0.25|1.50}q_x$$

Question 16

In what range is $\boldsymbol{X}$?

(A) Less than 0.050

(B) 0.050 but less than 0.051

(C) 0.051 but less than 0.052

(D) 0.052 but less than 0.053

(E) 0.053 or more

Data for Question 17 (3 points)

Smith borrows $1,000 from Jones.

The terms of the loan follow:

Repayment period	25 years
Payments	Level annual payments
Interest	4.0% per year, compounded annually

When Jones receives each repayment from Smith, Jones immediately reinvests the entire repayment at a nominal interest rate of 5.0% per year, compounded semiannually.

X = the equivalent level annual yield Jones earns over the 25 years.

Question 17

In what range is ***X***?

(A) Less than 4.59%

(B) 4.59% but less than 4.61%

(C) 4.61% but less than 4.63%

(D) 4.63% but less than 4.65%

(E) 4.65% or more

Data for Question 18 (3 points)

Smith retires at age 60 and can elect one of the following actuarially equivalent annuity options:

Option 1	A life annuity of $1,000 per month, payable at the beginning of each month, commencing at age 60, with the first 60 payments guaranteed
Option 2	A deferred life annuity of ***X*** per month, payable at the beginning of each month, commencing at age 65

Selected actuarial value:

$${}_{5|}\ddot{a}_{60}^{(12)} = 8.88$$

Interest: 5.0% per year, compounded annually.

Question 18

In what range is ***X***?

(A) Less than $1,485

(B) $1,485 but less than $1,495

(C) $1,495 but less than $1,505

(D) $1,505 but less than $1,515

(E) $1,515 or more

Data for Question 19 (4 points)

Terms of a bond:

Face amount	$1,000
Term	20 years
Coupons	4.0% per year, payable quarterly
Redemption	At par
Yield to maturity	4.5% per year, compounded semiannually

X = the duration of the bond.

Question 19

In what range is ***X***?

(A) Less than 12.75

(B) 12.75 but less than 13.00

(C) 13.00 but less than 13.25

(D) 13.25 but less than 13.50

(E) 13.50 or more

Data for Question 20 (3 points)

Smith pays $950 for an investment that has the following payouts:

At the end of year 3.....$500
At the end of year 4.....$700

The purchase price of this investment is based on a 2-year spot rate of 5.0% and a 4-year spot rate of 7.0%.

X = the three-year deferred, one year spot rate.

Question 20

In what range is ***X***?

(A) Less than 7.0%

(B) 7.0% but less than 7.6%

(C) 7.6% but less than 8.2%

(D) 8.2% but less than 8.8%

(E) 8.8% or more

Data for Question 21 (3 points)

A company provides for a lump sum on termination of employment.

The lump sum is equal to 6 months of salary for employees under age 45 who terminate during the year.

All terminations occur at the beginning of the year.

The lump sum is paid in the middle of the year.

Data for all employees under age 45 at 1/1/2013:

x	$q_x^{(termination)}$	Total annual salary
30	0.15	$5,000,000
35	0.10	$9,000,000
40	0.05	$6,000,000

There are no causes of decrement other than termination.

Interest rate: 8.0% per year, compounded annually.

X = the one-year term cost of the 2013 "termination benefit" at 1/1/2013.

Question 21

In what range is ***X***?

(A) Less than $925,000

(B) $925,000 but less than $945,000

(C) $945,000 but less than $965,000

(D) $965,000 but less than $985,000

(E) $985,000 or more

Data for Question 22 (2 points)

Given the following net single premiums:

$$A_{76} = 0.80$$
$$_1E_{76} = 0.90$$

Interest rate: 3.0% per year, compounded annually.

Question 22

In what range is A_{77}?

(A) Less than 0.806

(B) 0.806 but less than 0.809

(C) 0.809 but less than 0.812

(D) 0.812 but less than 0.815

(E) 0.815 or more

Data for Question 23 (4 points)

Given the following survival model:

$$l_x = (5-x)^2, \quad 0 \le x < 5$$

Question 23

In what range is ${}_{1|}q_{\overline{2:3}}$?

(A) Less than 0.35

(B) 0.35 but less than 0.40

(C) 0.40 but less than 0.45

(D) 0.45 but less than 0.50

(E) 0.50 or more

Data for Question 24 (4 points)

Given the following mortality rates for lives as of 01/01/2013:

x	q_x
50	0.0148
51	0.0159
52	0.0170
53	0.0183

Rates of mortality are expected to decrease at each age by 2.0% per year, compounded annually.

Interest rate: 4.0% per year, compounded annually.

$\boldsymbol{X}$ = the present value of a 3-year annuity due of \$10,000 per year, payable annually on a life age 50 on 01/01/2013.

$\boldsymbol{Y}$ = the present value of a 3-year annuity due of \$10,000 per year, payable annually on a life age 50 on 01/01/2023.

Question 24

In what range is $|\boldsymbol{X} - \boldsymbol{Y}|$?

(A) Less than \$76

(B) \$76 but less than \$80

(C) \$80 but less than \$84

(D) \$84 but less than \$88

(E) \$88 or more

Data for Question 25 (4 points)

Smith borrows $\$\boldsymbol{X}$ at a rate of 12.5%, and makes level payments at the end of each year for n years.

The interest portion of the final payment is \$153.86.

The total principal repaid after the $(n-1)$ payment is \$6,009.12.

$\boldsymbol{Y}$ = the principal repaid in the first payment.

Question 25

In what range is $\boldsymbol{Y}$?

(A) Less than \$470

(B) \$470 but less than \$478

(C) \$478 but less than \$486

(D) \$486 but less than \$494

(E) \$494 or more

Data for Question 26 (2 points)

Consider the following bond:

Face value	$1,000
Bond price:	$980
Coupon rate:	10.0% annual, payable at end of year
Risk-free yield rate	7.0%, compounded annually
Term	1 year

X = the implicit probability of default.

Question 26

In what range is ***X***?

(A) Less than 0.0450

(B) 0.0450 but less than 0.0455

(C) 0.0455 but less than 0.0460

(D) 0.0460 but less than 0.0465

(E) 0.0465 or more

Data for Question 27 (2 points)

Selected actuarial values:

$$l_x = 1000$$
$$q'^{(1)}_x = 0.05$$
$$q'^{(2)}_x = 0.03$$
$$q'^{(3)}_x = 0.30$$

Decrements (1) and (3) are uniformly distributed throughout the year.

Decrement (2) occurs at the end of the year.

Question 27

In what range is $d_x^{(2)}$?

(A) Less than 19.5

(B) 19.5 but less than 20.0

(C) 20.0 but less than 20.5

(D) 20.5 but less than 21.0

(E) 21.0 or more

Data for Question 28 (3 points)

Smith has the option to choose one of the following actuarially equivalent annuities:

I. A life annuity of $1,250 per month payable to Smith.

II. An annuity that provides for the payment of $1,000 per month to Smith, with $500 per month continuing to Smith's surviving spouse commencing upon Smith's death.

III. An annuity that provides for the payment of X per month to Smith, with $0.75X$ continuing to Smith's surviving spouse commencing upon Smith's death.

Question 28

In what range is X?

(A) Less than $875

(B) $875 but less than $885

(C) $885 but less than $895

(D) $895 but less than $905

(E) $905 or more

Data for Question 29 (3 points)

Terms of a perpetuity:

$1.00 at the end of year 2;
$2.00 at the end of year 4;
$3.00 at the end of year 6;
…
$$k$ at the end of year $2k$;
…

No payments are made in any other year.

Interest rate: 6.0% per year, compounded annually.

X = the present value of this perpetuity.

Question 29

In what range is ***X***?

(A) Less than $73.00

(B) $73.00 but less than $76.00

(C) $76.00 but less than $79.00

(D) $79.00 but less than $82.00

(E) $82.00 or more

Data for Question 30 (3 points)

Values from a mortality table:

x	q_x
$0 \le x \le 35$	$\frac{0.01}{1.01}$
$35 < x \le 75$	$\frac{0.02}{1.02}$
$x > 75$	1.00

Question 30

In what range is e_0?

(A) Less than 48.85

(B) 48.85 but less than 49.35

(C) 49.35 but less than 49.85

(D) 49.85 but less than 50.35

(E) 50.35 or more

Data for Question 31 (3 points)

Smith (age 65) has a beneficiary, Jones (age 62).

Smith has the option to choose one of the following actuarially equivalent annuities, with the first payment being made immediately:

I. $10,000 per year, payable to Smith at the beginning of each year.

II. $\boldsymbol{X}$ per year, payable at the beginning of each year while both Smith and Jones are alive, decreasing to $0.75\boldsymbol{X}$ upon the first death. All payments cease upon the second death.

Selected actuarial values:

x	$\ddot{a}_x$	$\ddot{a}_{x:x-3}$
62	10.6974	9.4604
63	10.4827	9.2108
64	10.2642	8.9590
65	10.0426	8.7060

Question 31

In what range is $\boldsymbol{X}$?

(A) Less than $8,700

(B) $8,700 but less than $8,800

(C) $8,800 but less than $8,900

(D) $8,900 but less than $9,000

(E) $9,000 or more

Data for Question 32 (5 points)

Consider the following fund:

Date	Market value	Contribution	Withdrawals
01/01	$100,000	$0	$0
03/31	-	0	20,000
04/01	90,000	0	0
07/01	95,000	0	0
09/30	-	75,000	0
10/01	185,000	0	0
12/31	180,000	0	0

Rates of return on fund:

Time weighted ..... ***t***
Dollar weighted..... ***d***
Dollar weighted assuming cash flows at mid-year ..... ***m***

Question 32

Which of the following relationships, if any, is true?

(A) $t > m > d$

(B) $t > d > m$

(C) $m > t > d$

(D) $d > t > m$

(E) The correct answer is not given by (A), (B), (C), or (D) above

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Answer Key

Question	Points	Solution
1	**C**	
2	E	
3	B	
4	E	
5	A	
6	C	
7	E	
8	B	
9	E	
10	E	
11	C	
12	B	
13	B	
14	A	
15	E	
16	C	
17	B	
18	C	
19	E	
20	E	
21	B	
22	C	
23	D	
24	B	
25	C	
26	E	
27	B	
28	E	
29	B	
30	B	
31	D	
32	B	

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