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SPRING 1989 EA-1B EXAM SOLUTIONS

Revision History:
02/12/00 Changed problem 16 - expanded to show quicker alternative solution

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Problem 1

This problem has salary scales, which really tests your ability to work carefully. Under the Entry Age Normal method, the salary scales complicate the calculations even more. You must calculate the normal cost percentage, which is defined as the PVB at entry age divided by the PV of salary at entry age. This year's normal cost equals this percentage multiplied by this year's salary.

The steps in working this problem are:

(i) calculate the final average pay
(ii) calculate the projected benefit
(iii) calculate the PVB at entry age
(iv) calculate the PV of pay at entry age
(v) calculate the normal cost as a percentage of pay.

Calculate the final average salary by projecting pay from current age to age 64, then multiplying by $\ddot{a}_{\overline{3}|}/3$:

1/1/89 valuation

Age = 30, Age 30 pay = 10,000
Entry age = 25

$$\text{Age 64 pay} = 10{,}000(1.04)^{34} = 37{,}943$$
$$\text{FAE3} = \text{Age 64 pay} * \left(\ddot{a}_{\overline{3}|.04} / 3 \right)$$
$$= 36{,}503$$

projected benefit = .5(36,503) = 18,251

$$PVB_{ea} = 18{,}251 * \ddot{a}^{(12)}_{65} * \left(D_{65} / D_{25} \right)$$
$$= 18{,}251(9.345)(1{,}738) \div (22{,}499)$$
$$= 13{,}175$$

To calculate the PVE, write down the expression with salary scales and commutation functions.

$$\text{Entry age pay} = 10{,}000(1.04)^{-5} = 8{,}219$$

$$PVE_{ea} = 8{,}219 \left({}^{S}N_{ea} - {}^{S}N_{65} \right) / {}^{S}D_{ea}$$
$$= 8{,}219 \left(1{,}920{,}504 - 296{,}192 \right) \div 59{,}979$$
$$= 222{,}589$$

$$\text{EANC\%} = 13{,}175 / 222{,}589 = 5.92\%$$
$$\text{NC} = 10{,}000 * .0592$$
$$= 592$$

answer is B

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Problem 2

Attained Age Normal is an aggregate cost method. The Unit Credit method was used to establish the initial Accrued Liability. When there is a change in plan benefits, you use the Unit Credit method to calculate the change in the Accrued Liability. The change in the unfunded liability is equal to the change in the Unit Credit Accrued Liability.

In making the calculations for this problem, Smith will have no effect. The reason is that Smith has zero years of service since Smith's hire date is 01/01/89. Smith's Unit Credit Accrued Liability is zero under both the old plan and the new plan, so Smith has been ignored:

	Brown		Green		
current age	40		50		
hire age	30		30		
past service	10		20		
$10 / mo accrued	1,200		2,400		
$15 / mo accrued	1,800		3,600		
change in accrued	600		1,200		
D_x at current age	9,205		4,968		
$D_{65}\ \ddot{a}^{(12)}_{65} / D_x$	1.7644		3.2692		
P.V. at 1/1/89	1,059	+	3,923	=	4,982

The present value of the change in the accrued benefit is the change in the Unit Credit Accrued Liability. This is also the change in the unfunded liability under the Attained Age Normal method.

answer is A

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Problem 3

This is a straightforward life contingencies problem. The formula for the present value of an annual K% joint and survivor benefit is

$$\ddot{a}_x + (K/100)(\ddot{a}_y - \ddot{a}_{x:y}).$$

In this problem, the benefits are payable monthly on a 50% joint and survivor basis. The participant is age 60 and will not retire for five years. The present value must allow for the survival of the participant for five years. If the spouse does not survive the five years, the present value will be based on a single life annuity for the participant. If the spouse survives the five years, the present value will be based on a 50% joint and survivor annuity for the spouse and participant.

$$[\, v^5\, {}_5p_x \,]\,[\, {}_5p_y \,]\,[\, \ddot{a}^{(12)}_{x+5} + \tfrac{1}{2}\,(\, \ddot{a}^{(12)}_{y+5} - \ddot{a}^{(12)}_{x+5:y+5} \,)\,]$$

$$+\ [\, v^5\, {}_5p_x \,]\,[\, 1 - {}_5p_y \,]\ \ddot{a}^{(12)}_{x+5}$$

For a participant age 60 (x) with a spouse age 65 (y) this becomes

$$[\, v^5\, {}_5p_{60} \,]\,[\, {}_5p_{65} \,]\,[\, \ddot{a}^{(12)}_{65} + \tfrac{1}{2}\,(\, \ddot{a}^{(12)}_{70} - \ddot{a}^{(12)}_{65:70} \,)\,]$$

$$+\ [\, v^5\, {}_5p_{60} \,]\,[\, 1 - {}_5p_{65} \,]\ \ddot{a}^{(12)}_{65}$$

The only item not given is ${}_5p_{60}$ but it can be derived easily. Since ${}_{10}p_{60} = {}_5p_{60} \times {}_5p_{65}$ you have ${}_5p_{60} = {}_{10}p_{60} \div {}_5p_{65}$.

Substituting values in the formula above, the present value is

$$\begin{aligned} PVB &= 600\,[(1.06)^{-5}\,(.80/.87)]\,[\,.87\,]\ [\,112 + .5\,(\,97 - 79\,)] \\ &\quad + 600\,[(1.06)^{-5}\,(.80/.87)]\,[\,1-.87\,]\ [\,112\,] \\ &= 43{,}401 + 6003 = 49{,}404 \end{aligned}$$

answer is A

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Problem 4

This problem can be solved without worrying about the special aspects of the Projected Unit Credit cost method. The key is that under any variation of the Unit Credit method, the Accrued Liability is calculated as the present value of the accrued benefit. A change in the mortality rate will have a direct effect on the present value factor used.

For a sole participant at age 45, we have the following:

$$AL = D_{65}\ \ddot{a}_{65}\ (FAB) \div D_{45} = v^{20}\ {}_{20}p_{45}\ \ddot{a}_{65}\ (FAB)$$

$$= p_{45}\ [\ v^{20}\ {}_{19}p_{46}\ \ddot{a}_{65}\ (FAB)\ ]$$

The change in mortality rate at age 45 will affect the Accrued Liability by the ratio of p_{45} before and after the change. Since the new value is .9966 and the old value was .9660, the new Accrued Liability will be larger.

$$\text{New AL} = 300{,}000\ (\ .9966\ /\ .9660\ )$$
$$= 309{,}503$$

answer is C

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Problem 5 - Page 1

Under the Aggregate method, the PVNC is calculated on an aggregate basis as PVB - AAV. The normal cost is calculated using the average temporary annuity to normal retirement age.

The problem states that there is an actuarial gain due to the deaths of three retired participants. This is confusing, because you normally don't think of experience gains and losses in conjunction with aggregate cost methods.

The best way to think of this is that the normal cost will remain level as a percentage of pay if all assumptions are met, and if the normal cost is paid at the date it is calculated. This "experience gain" tells you that the present value of benefits is 30,000 lower than expected.

1-1-89 valuation

	ASSETS		LIABILITIES
AAV	500,000	PVB - actives	900,000
PVNC	500,000	PVB - retired	100,000
	1,000,000		1,000,000

NC = PVNC / (PVE/E)
= 31,250 = 3.125% of 1,000,000

PVNC / PVE = .03125

PVE = 500,000 ÷ .03125
= 16,000,000

You must set up the expected balance sheet at 01/01/90. The expected earnings will be based on the salary scale for one year. The expected PVE is calculated by considering the PVE as an annuity that contains a series of increasing earnings pieces:

Since the PVE can be expressed as
EARN * (1 + $[(1+s)/(1+i)]^1$ + $[(1+s)/(1+i)]^2$ + ...),

the expected PVE_1 is $(1+i)*(PVE_0-EARN_0)$.

Problem 5 - Page 2

1-1-90 valuation

expected PVE = 1.06 [16,000,000 - 1,000,000] = 15,900,000
expected PAY = 1.05 (1,000,000) = 1,050,000

expected PVE/E = 15,900,000 ÷ 1,050,000 = 15.1429
expected NC = .03125 * (1,050,000) = 32,813
actual NC = 32,813 - 30,000 ÷ 15.1429 = 30,832

answer is B

There is a way to check the result for this problem by constructing the total balance sheet for 1990:

1-1-90 valuation (expected)

exp PVB 1,060,000 - (BP+I) = 1.06 (900,000 + 100,000) - (BP+I)
exp AAV 563,125 - (BP+I) = 1.06 (500,000 + 31,250) - (BP+I)

exp PVNC 496,875 = 1.06 (500,000 - 31,250)
= exp PVB - exp AAV

exp NC = PVNC / (PVE/E)
= 496,875 ÷ 15.1429 = 32,812 using expected PVE/E calculated above

actual PVNC = 466,875 actual NC = 30,832

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Problem 6 - Page 1

This gain and loss problem can't be worked using the standard formula for non-investment G/L. The reason is that the problem does not ask for the total gain or loss during the year. We should compare the accrued liability before and after the participant retires to calculate the gain or loss due to early retirement.

At 12-31-88, the participant is age 59 with 21 years of service. Under the Entry Age Normal method, the accrued liability is defined based on the normal cost. The normal cost is defined as the present value of future benefits at entry age, divided by the present value of future salary or lives at entry age. The problem gives us N_x and D_x values that we must use in the calculations.

The key to this problem is that the assumed retirement age is not age 65. The projected benefit and all funding must be based on the assumed retirement age of 60. With no deaths or terminations prior to retirement, we discount liabilities prior to age 60 at interest only. We can use annuities certain for the temporary annuity to retirement age.

1/1/89 valuation

Age 59, entry age 38, past service 21

$$\text{EANC} = \frac{(\text{proj ben})\ \ddot{a}^{(12)}_{60}\ (D_{60}/D_{38})}{(N_{38}-N_{60})/D_{38}}$$

$$= \frac{(\text{proj ben})(N^{(12)}_{60} \div D_{60}) * v^{22}}{\ddot{a}_{\overline{22}|.06}}$$

$$\text{Proj ben} = 22(\$15)(12)[\ 1 - .02(5)\ ]$$
$$= 3{,}564$$

$$\text{EANC} = \frac{3{,}564(2{,}751/260)(1.06)^{-22}}{12.7641}$$

$$= 819.85$$

Problem 6 - Page 2

EAN AL = retrospective accumulation of normal costs
$= EANC * (N_{ea} - N_{ca}) / D_{ca}$
$= 1.06 * s_{\overline{21}|.06} * 819.85$
$= 34,755$

As a retired employee, the accrued liability is simply the present value of the early retirement benefit:

retired AL $= 21(\$15)(12)[1-.02(6)](N^{(12)}_{59}/D_{59})$
$= 3,780(.88)(3,019/275)$
$= 36,518$

The effect of this early retirement is a loss of

$34,755 - 36,518 = 1,763$

answer is A

You can check the prospective definition of the accrued liability to be sure of your answer:

active AL $= 3,564(N^{(12)}_{60}/D_{60}) * v - 820$
$= 3,564(2,751/260) \div 1.06 - 820$
$= 34,755$

If you use the value of D59 to discount the present values from age 60, your answers will be slightly different. The reason is that the ratio of the D's from 59 to 60 should be 1.06 if there are no pre-retirement deaths or terminations. Instead the result is 275/260 = 1.0577, which means there is probably mortality in the D's.

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Problem 7 - Page 1

This problem has salary scales, which really tests your ability to work carefully. Under the Entry Age Normal method, the salary scales complicate the calculations even more. We must calculate the normal cost percentage, which is defined as the PVB at entry age divided by the PV of salary at entry age.

With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

The steps in working this problem are:

(i) calculate the final average pay
(ii) calculate the projected benefit
(iii) calculate the PVB at entry age
(iv) calculate the PV of pay at entry age
(v) calculate the normal cost as a percentage of pay.

This year's normal cost equals this percentage multiplied by this year's earnings. The accrued liability can be calculated both prospectively and retrospectively as an arithmetic check.

1/1/89 valuation

Age = 55 Age 54 pay = 30,000
PS = 20 Age 55 pay = 31,500 = 30,000 * 1.05
TS = 30

Age 64 pay = $30,000(1.05)^{10} = 48,867$

projected benefit = 30 * .01 * 48,867 = 14,660

$PVB_{ea} = 14,660 * \ddot{a}^{(12)}_{65} * (1.07)^{-30}$
$= 14,660(10.0)(.1314)$
$= 19,258$

To calculate the PVE at entry age, you must calculate the pay at entry age. Then write down the expression for PVE with interest rates and salary scales. Then the expression can be evaluated as an annuity certain:

Entry age = 35 Age 35 pay = $30,000 \div (1.05)^{-19}$
$= 11,872$

PVE $= 11,872 [1 + (1.05/1.07)^{1} + \ldots + (1.05/1.07)^{29}]$ (30 terms)
$= 11,872 (\ddot{a}_{\overline{30}|1.9\%})$
$= 11,872 (23.1248) = 274,537$

Problem 7 - Page 2

EANC% = 19,258 / 274,537 = 7.01%
NC = 31,500 * .0701 = 2,210

$PVB_{Ca} = 14,660 * \ddot{a}^{(12)}_{65} * (1.07)^{-10}$
= 74,525
PVNC = 2,210 ($\ddot{a}_{\overline{10}|1.9\%}$)
= 20,328
AL = 74,525 - 20,328 = 54,197

answer is B

You can check your answer by also calculating the accrued liability under the retrospective definition. This would equal the normal costs from entry age 35 through age 54 accumulated to age 55:

$AL = .0701*11,872*[(1.07)^{20} + (1.05)(1.07)^{19} + ... + (1.05)^{19}(1.07)]$
$= .0701*11,872*(1.07)^{20}[1 + (1.05/1.07)^{1} + ... + (1.05/1.07)^{19}]$
$= 832.81 * (1.07)^{20} * \ddot{a}_{\overline{20}|1.9\%}$

= 54,197

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Problem 8 - Page 1

Attained Age Normal is an aggregate cost method. The Unit Credit method was used to establish the initial Accrued Liability. When there is a change in plan benefits, you use the Unit Credit method to calculate the change in the Accrued Liability. The change in the unfunded liability is equal to the change in the Unit Credit Accrued Liability.

The actual unfunded liability is defined equal to the expected unfunded liability. You must complete the valuation at 01/01/88 to calculate the normal cost in order to derive the unfunded at 01/01/89.

1-1-88 valuation

	ASSETS		LIABILITIES
AAV	250,000	PVB - actives	840,000
UAL	300,000	PVB - retired	250,000
PVNC	540,000		
	1,090,000		1,090,000

Since the plan benefit definition is not based on earnings, the normal cost should be calculated as a level dollar amount:

$$NC = PVNC / (PVL/L)$$
$$= 540,000 / (100 \div 10) = 54,000$$

One aspect of this problem that may lead you astray is the large difference in valuation results between 01/01/88 and 01/01/89. There has been a large number of retirements from the active population. You still must calculate the expected unfunded liability based on the standard formula:

1-1-89 valuation

$$e^{UAL_1} = (1+i)(NC_0+UAL_0) - (contrib+int)$$
$$= (1.06)(54,000+300,000) - 88,000$$
$$= 287,240$$

Problem 8 - Page 2

The tricky part of the problem is calculating the change in the unfunded. For the actives, use the Unit Credit accrued liability. For the inactives, you must use the present value of benefits.

	Actives UC AL		Inactives PVB			
$50 benefit	320,000		570,000			
$54 benefit	345,600		615,600			(use simple pro-rata)
$4 increase	25,600	+	45,600	=	71,200	

New UAL = 287,240 + 71,200 = 358,440

	ASSETS		LIABILITIES	
AAV	345,000	PVB - actives	594,000	= (54/50)(550,000)
UAL	358,440	PVB - retired	615,600	= (54/50)(570,000)
PVNC	506,160			
	1,209,600		1,209,600	

NC = PVNC / (PVL/L)
= 506,160 / (96 ÷ 8) = 42,180

answer is D

Problem 9

Under the Individual Level Premium cost method, benefit changes are funded prospectively from the date of change. When the projected benefit under the plan changes, either due to plan amendments or salary changes, a new layer of normal cost is created.

In this problem, we should calculate the initial normal cost based on amortization from the effective date of the plan. The 1989 normal cost will be that amount decreased by the new layer of normal cost due to the decrease in salary.

With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

01/01/88 valuation

Age 43

$$\text{ILP NC} = .40(200{,}000)(v^{22})\ \ddot{a}_{65}^{(12)} \ / \ \ddot{a}_{\overline{22}|.06}$$
$$= 80{,}000(.2775)(9.35)/12.7641$$
$$= 16{,}262$$

The calculation of the projected benefit in the 01/01/89 valuation is tricky. If the benefit formula was 40% of final compensation, you would use .4(170,000). Instead, you must calculate the highest 3 year average compensation:

01/01/89 valuation

Age 44

$$\text{Revised projected benefit} = .40(200{,}000+170{,}000+170{,}000)/3$$
$$= .40(180{,}000)$$

$$\text{chg ILP NC} = .40(-20{,}000)(v^{21})\ \ddot{a}_{65}^{(12)} \ / \ \ddot{a}_{\overline{21}|.06}$$
$$= -8{,}000(.2942)(9.35)/12.4699$$
$$= -1{,}764$$

$$\text{1/1 ILP NC} = 16{,}262 - 1{,}764 = 14{,}498$$

answer is B

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Problem 10

This problem can be worked using the formula for non-investment G/L. Since you are asked for the gain due to Green's death, you should not waste time doing any calculations for Smith and Brown.

non-inv G/L = $_eAL_1 - AL_1$

Since Green has died at 01/01/89, the actual accrued liability is zero. The total gain is equal to Green's expected accrued liability:

$_eAL_1 = (1+i)(NC_0 + AL_0)$ - (actual ben pmts + interest)

At 01-01-88, Green is age 60 with 10 years of service. Under the Entry Age Normal method, the accrued liability is defined based on the normal cost. The normal cost is defined as the present value of future benefits at entry age, divided by the present value of future salary or lives at entry age. The problem gives us N_x and D_x values that must be used in the calculations.

<u>1/1/88 valuation</u>

Age 60, entry age 50, past service 10

$$EANC = \frac{(\text{proj ben})\ \ddot{a}^{(12)}_{65}\ (D_{65}/D_{50})}{(N_{50}-N_{65})/D_{50}}$$

Proj ben = 15($20)(12) = 3,600

$$EANC = \frac{3,600(9.35)(178)}{(6,723-1,745)} = 1,204$$

EAN AL = retrospective accumulation of normal costs
= EANC * ($N_{50} - N_{60}$) / D_{60}
= 1,204(6,723-2,870)/260
= 17,836

mortality gain = expected AL = 1.06(NC+AL) = 1.06(1,204+17,836) = 20,183

answer is C

You should check the prospective calculation of the accrued liability:

AL = PVB - PVNC = 3,600(9.35)(178/260) - 1,204(2,870-1,745)/260 = 17,835

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Problem 11

Under the Unit Credit method, the accrued liability is defined as the present value of the accrued benefit. In this problem, the computation of the accrued liability is complicated by having retirement decrements. Instead of having a single retirement age, the present value factor is a summation

$$\Sigma\ v^t\ {}_tp_x^T\ q_{x+t}^{(r)}\ \text{ERF}\ \ddot{a}_{x+t}^{(12)}$$

where ERF is the early retirement reduction factor. In this problem early retirement benefits are reduced 6 2/3% per year.

1/1/89 valuation

Age 62, service 10 yrs

Under the old assumptions, the accrued liability is the present value of the age 62 accrued benefit payable at age 65:

$$AL = 10(\$25)(12)(9.35)(1.06)^{-3}$$
$$= 23{,}551$$

Under the new assumptions, 40% of the participant is assumed to retire immediately, with the remainder retiring at age 65:

$$AL = 3{,}000\ [\ v^0\ (1.0)\ (.40)\ 10.10\ (1\text{-}3/15)\quad \text{(age 62)}$$
$$+\ v^3\ (.60)\ (1.0)\ \ 9.35\ ]\quad \text{(age 65)}$$
$$= 3{,}000\ [\ 3.232 + 4.710\ ]$$
$$= 23{,}827$$

The difference in the accrued liability under the two sets of assumptions is 23,827 - 23,551, or 276.

answer is D

You can check on the reasonableness of the final result by comparing the weighted PV factors. The age 62 PV factor is 10.10(.80) = 8.08. The age 65 PV factor is 9.35/[(1.06)^3] = 7.85. The ratio of the two factors is 8.08/7.35 = 1.0292. Based on the original AL calculation, if 100% of the employees retired at age 62, you would expect the change in AL to be 2.92% of 23,551. With 40% retiring at 62, you expect .0292(.4)(23,551) which equals 275. This is consistent with the previous results.

Problem 12 - Page 1

This problem has salary scales, which really tests your ability to work carefully. Under the Entry Age Normal method, the salary scales complicate the calculations even more. You must calculate the normal cost percentage, which is defined as the PVB at entry age divided by the PV of salary at entry age. This year's normal cost equals this percentage multiplied by this year's salary.

The steps in working this problem are:

(i) calculate the final year's pay
(ii) calculate the projected benefit
(iii) calculate the PVB at entry age
(iv) calculate the PV of pay at entry age
(v) calculate the normal cost as a percentage of pay.

The accrued liability can be calculated on a retrospective or prospective basis using the normal cost. The tricky part of this problem is that you are not given the current pay, nor are you given the value of $\ddot{a}_{65}$.

1/1/89 valuation

Age = 50, Entry age = 33, Past service = 17, Total service = 32

Let pay at age 50 be represented by PAY50, and let pay at age 33 be PAY33.

Age 64 pay = $PAY50(1.03)^{14}$
Proj Ben = $.25*PAY50*(1.03)^{14}$

$$PVB_{50} = .25*PAY50*(1.03)^{14}* \ddot{a}_{65}^{(12)} * (1.06)^{-15} = 110,000$$

$$PVB_{ea} = PVB_{50} * D_{50} / D_{33} = 110,000 * 1.06^{-17} = 40,850$$

Problem 12 - Page 2

To calculate the PVE, write down the expression with salary scales and commutation functions:

$$\begin{aligned}
PVE &= PAY33\ [\ 1 + (1.03/1.06)^{1} + \ \ldots + (1.03/1.06)^{31}]\quad (\ 32\ \text{terms}\ ) \\
&= PAY33\ (\ \ddot{a}_{\overline{32}|2.9\%}) \\
&= PAY50 * (1.03)^{-17} * \ddot{a}_{\overline{32}|2.9\%} \\
&= PAY50 * 12.8471 \\
EANC\% &= PVB_{ea} \ / \ PVE_{ea} \\
&= 40{,}850 \ / \ (\ PAY50 * 12.8471\ ) \\
&= 3{,}180 \ / \ PAY50 \\
\\
NC &= PAY50 * (\ 3{,}180 \ / \ PAY50\ ) \\
&= 3{,}180 \\
\\
PVNC &= 3{,}180\ (\ \ddot{a}_{\overline{15}|2.9\%}) = 39{,}313 \\
AL &= 110{,}000 - 39{,}313 = 70{,}687
\end{aligned}$$

answer is B

You can check your answer by also calculating the accrued liability under the retrospective definition. This would equal the normal costs from entry age 33 through age 49 accumulated to age 50:

$$\begin{aligned}
AL &= (3{,}180/PAY50)*(PAY33)*[(1.06)^{17}+(1.03)(1.06)^{16}+\ldots+\ (1.03)^{16}(1.06)] \\
&= [3{,}180*(1.03)^{-17}]\ *\ [(1.06)^{17}+(1.03)(1.06)^{16}+\ldots+\ (1.03)^{16}(1.06)] \\
&= 3{,}180 * (1.06/1.03)^{17}[\ 1 + (1.03/1.06)^{1} + \ \ldots\ + (1.03/1.06)^{16}\ ] \\
&= 5{,}181\ * \ddot{a}_{\overline{17}|2.9\%} \\
&= 70{,}687
\end{aligned}$$

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Problem 13 - Page 1

Under the Individual Level Premium cost method, benefit changes are funded prospectively from the date of change. When the projected benefit under the plan changes, either due to plan amendments or salary changes, a new layer of normal cost is created.

In this problem, we should calculate the initial normal cost based on amortization from the effective date of the plan. The accrued liability is defined retrospectively based on the accumulation of prior normal costs. The change in assumptions will affect the amount of the normal cost, but it does not create a new layer of normal cost.

With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

<u>01/01/88 valuation</u>

Age 35

$$\text{ILP NC} = .50(24{,}000)(1.08)^{-30}\, \ddot{a}^{(12)}_{65} \,/\, \ddot{a}_{\overline{30}|.08}$$
$$= 12{,}000(.09938)(8.1958)/12.1584$$
$$= 803.87$$

<u>01/01/89 valuation</u>

Age 36

Accrued Liability (old assumptions) = 1.08 (803.87) = 868

Under the new assumptions, the accrued liability is still defined as the retrospective accumulation of the 1988 normal cost. It is necessary to recalculate the 1988 normal cost under the new assumptions:

$$\text{ILP NC} = .50(24{,}000)(1.06)^{-30}\, \ddot{a}^{(12)}_{65} \,/\, \ddot{a}_{\overline{30}|.06}$$
$$= 12{,}000(.17411)(9.3452)/14.5907$$
$$= 1{,}338.19$$

Accrued Liability (new assumptions) = 1.06 (1,338.19) = 1,418

Change in accrued liability due to assumption change is 1,418 - 868 = 550

answer is A

The real trick to this question is that the change in benefit at 1/1/89 does not have to be taken into account. The reason is that the change in the PVB is funded through the normal cost, so the change in the PVNC is the same amount. The net effect on the accrued liability is zero. This will be demonstrated after the answer is checked.

Problem 13 - Page 2

To check the answer, calculate the accrued liability on a prospective basis under both the old and the new assumptions:

Accrued Liability (old assumptions-no benefit change)

$$\text{PVB} = .50(24{,}000)(1.08)^{-29}\,\ddot{a}^{(12)}_{65}$$
$$= 12{,}000(.10733)(8.1958)$$
$$= 10{,}556$$
$$\text{PVNC} = 803.87 * \ddot{a}_{\overline{29}|.08}$$
$$= 9{,}687$$
$$\text{AL} = 869$$

Accrued Liability (new assumptions-no benefit change)

$$\text{PVB} = .50(24{,}000)(1.06)^{-29}\,\ddot{a}^{(12)}_{65}$$
$$= 12{,}000(.18456)(9.3452)$$
$$= 20{,}697$$
$$\text{PVNC} = 1{,}338 * \ddot{a}_{\overline{29}|.06}$$
$$= 19{,}278$$
$$\text{AL} = 1{,}418$$

Assumption change affects AL by 1,418 - 869 = 549 (close enough)

You can demonstrate that the accrued liability is not affected by the change in the projected benefit under the ILP method:

Accrued Liability (new assumptions-with benefit change)

$$\text{add'l NC} = .50(4{,}800)(1.06)^{-29}\,\ddot{a}^{(12)}_{65} \,/\, \ddot{a}_{\overline{29}|.06}$$
$$= 2{,}400(.18456)(9.3452)/14.4062$$
$$= 287.33$$

$$\text{PVB} = .50(28{,}800)(1.06)^{-29}\,\ddot{a}^{(12)}_{65}$$
$$= 14{,}400(.18456)(9.3452)$$
$$= 24{,}836$$
$$\text{PVNC} = (287.33+1338.19) * \ddot{a}_{\overline{29}|.06}$$
$$= 23{,}418$$
$$\text{AL} = 1{,}419$$

This is almost identical to the AL without the benefit change.

Problem 14

Under the Individual Aggregate method, the normal cost is calculated on an individual basis. The assets are allocated to each participant, and the PVNC is calculated directly as PVB - allocated AAV. With no deaths or terminations prior to retirement, we discount liabilities prior to age 65 at interest only. We can use annuities certain for the temporary annuity to retirement age.

With a side fund, it is necessary to calculate the benefit provided by the insurance at retirement age. The computation of the normal cost is based on benefits in excess of those provided by the insurance contract.

This problem is unusual in that you are given the allocated assets at 01/01/90. The problem mostly tests your ability to handle the side fund.

	Smith		Brown	
monthly P.B.	500		200	
insurance	50,000		20,000	
CSV at age 65	50(200)		20(250)	
	= 10,000		= 5,000	
total P.V. 1/1/90	41,900		14,072	
current age	60		57	
discount factor	v^5		v^8	
total P.V. at 65	56,072		22,429	
net P.V. at 65	46,072		17,429	(less CSV)
net P.V. 1/1/90	34,427		10,935	
alloc. assets	7,500		-0-	
PVNC = PVB - AAV	26,927		10,935	
PVL/L	$\ddot{a}_{\overline{5}\rceil .06}$		$\ddot{a}_{\overline{8}\rceil .06}$	
	= 4.4651		6.5824	
Normal cost	6,031		1,661	
Insurance premium	3,000		1,000	
Total cost	9,031	+	2,661	= 11,692

answer is C

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Problem 15

Under the Aggregate method, the PV of future normal costs is defined as equal to PVB less the AAV. This problem requires calculation of the PVB and normal cost based on retirement at age 64 and at age 65.

With no pre-retirement decrements, you can use interest only to discount benefits to current age. The PVL factor can be calculated using an annuity certain.

1/1/89 valuation

Age 55, past service 25

	NRA 65	NRA 64
svc at age NRA	35	34
Proj ben @ NRA	8,400	8,160
$\ddot{a}^{(12)}_{NRA}$	8.14	8.35
Total P.V. @ NRA	68,376	68,136
discount factor	v^{10}	v^{9}
PVB at 1/1/90	31,671	34,085
AAV	10,000	10,000
PVNC = PVB - AAV	21,671	24,085
PVL/L	$\ddot{a}_{\overline{10}\rceil .08}$	$\ddot{a}_{\overline{9}\rceil .08}$
=	7.2469	6.7466
Normal cost	2,990	3,570

The difference in the normal cost under the two sets of assumptions is 3,570 - 2,990 or 580.

answer is A

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Problem 16 - Page 1

Attained Age Normal is an aggregate cost method. The Unit Credit method is used to establish the initial Accrued Liability. When there is a change in plan benefits, you use the Unit Credit method to calculate the change in the Accrued Liability. The change in the unfunded liability is equal to the change in the Unit Credit Accrued Liability.

The actual unfunded liability is defined equal to the expected unfunded liability. You must complete the valuation at 01/01/88 to calculate the normal cost in order to derive the unfunded at 01/01/89. This problem requires you to do valuations at 01/01/88 under both Unit Credit (for the initial UAL), and under Attained Age Normal (for the normal cost).

1/1/88 valuation - Unit credit

current age	50
hire age	27
past service	23
$25 / mo accrued	6,900
D_x at current age	322
$D_{65}\ \ddot{a}^{(12)}_{65} / D_x$	2.6656
PV AB at 1/1/88	18,393

The present value of the accrued benefit is the Unit Credit Accrued Liability. This is also the initial unfunded liability under the Attained Age Normal method.

1/1/88 valuation - Attained age normal

total service	38
projected benefit	11,400
PV Future benefit	30,388

ASSETS		LIABILITIES	
AAV	-0-	PVB - actives	30,388
UAL	18,393		
PVNC	11,995		
	30,388		30,388

Problem 16 - Page 2

Since the plan benefit definition is not based on earnings, the normal cost should be calculated as a level dollar amount:

$(N_{50} - N_{65}) / D_{50} = 9.3106$

NC = PVNC / (PVL/L)
= 11,995 / 9.3106 = 1,288.32

At this point, you are finished with the problem. If all of the assumptions are met, then the normal cost would be level on a dollar per participant basis. We would expect p_{50} survivors next year:

$p_{50} = .99950 = (1+i)(D_{51} / D_{50}) = 1.08 * 298 / 322$

1/1/89 $_ePVNC = .9995 (1288.32) (N_{51} - N_{65}) / D_{51}$
PVL/L = $(N_{51} - N_{65}) / D_{50} = 8.9799$
1/1/89 $_ePVNC = 11,563$
1/1/89 $_eAAN\ NC = 1,287.68 = 1,288.32 * .9995$

1/1/89 PVNC = 11,563 / .9995 = 11,569
1/1/89 AAN NC = 1,288.32

answer is D

Here is an attempt to reach the same result, but going the long way!

<u>1/1/89 valuation - Attained age normal</u>

$_eUAL_1 = (1+i)(NC_o+UAL_o) - (contrib+int)$
= (1.08)(1,288+18,393) - 3,000
= 18,256

total service	38
projected benefit	11,400
D_x at current age	298
$D_{65}\ \ddot{a}^{(12)}_{65} / D_x$	2.8803
PV Future benefit	32,835

Problem 16 - Page 3

	ASSETS		LIABILITIES
AAV	3,000	PVB - actives	32,835
UAL	18,256		
PVNC	11,580		
	32,835		32,835

Since the plan benefit definition is not based on earnings, the normal cost should be calculated as a level dollar amount:

$PVL = (N_{51} - N_{65}) / D_{50}$

$= 8.9799$

$NC = PVNC / (PVL/L)$

$= 11,580 / 8.9799 = 1,289$

answer is D

The answer is very close to the prior result, but not exactly the same. The results would be identical if the data given in the problem had six digits of precision. Since the values at age 65 only have two or three significant digits, the answer can have no more than two or three significant digits. Expressed as accurate to only three significant digits, the answers are both equal to 1290.

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Problem 17 - Page 1

Under the Individual Level Premium cost method, benefit changes are funded prospectively from the date of change. When the projected benefit under the plan changes, either due to plan amendments or salary changes, a new layer of normal cost is created.

In this problem, we should calculate the initial normal cost based on amortization from the effective date of the plan. The accrued liability is defined retrospectively based on the accumulation of prior normal costs.

<u>01/01/84 valuation</u>

Age 45, 5 years past service, 25 years total service

Projected benefit = 12(15)(25) = 4,500

D_x at current age = 694

$D_{65}\ \ddot{a}^{(12)}_{65} / D_{45}$ = 2.5648

PVB at 1/1/84 = 11,542

ILP NC = PVB ÷ [($N_{45} - N_{65}$) / D_{45}]

= 11,542 ÷ [(9,789 - 1,741) / 694]
= 995

<u>01/01/89 valuation</u>

Age 50, 10 years past service

AL retrospective = 995 * [($N_{45} - N_{50}$) / D_{50}]
= 995 * [(9,789 - 6,712) / 508]
= 6,028

answer is C

Problem 17 - Page 2

To check the answer, calculate the accrued liability on a prospective basis:

D_x at current age = 508

D_{65} $\ddot{a}^{(12)}_{65}$ / D_{50} = 3.5039

PVB at 1/1/89 = 15,768

PVNC at 1/1/89 = $NC * [(N_{50} - N_{65}) / D_{50}]$

= 995 * [(6,712 - 1,741) / 508]

= 9,739

AL prospective = 15,768 - 9,739 = 6,029

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Problem 18

This problem does not require calculation of a normal cost or accrued liability, so the cost method is not given. In this problem, the computation of the present value of benefits is complicated by having retirement decrements. Instead of having a single retirement age, the present value is a summation

$$\Sigma\ v^t\ {}_tp_x^T\ \ q_{x+t}^{(r)}\ \text{ERB}\ \ \ddot{a}_{x+t}^{(12)}$$

where ERB is the early retirement benefit at age x+t. In this problem early retirement benefits are reduced 6% per year prior to age 65.

1/1/89 valuation

Age 63, service 16 yrs

Under these assumptions, 50% of the participant is assumed to retire immediately, with 20% retiring at age 64 and the remainder retiring at age 65. You need to set up the probability of surviving from age 63 to age 65 before the present value of benefits can be calculated:

t	x+t	l_{x+t}	$q_{x+t}^{(r)}$	${}_tp_x^T$
0	63	1,000	.50	1.0
1	64	500	.20	.50
2	65	400	1.0	.40

age		v^t	${}_tp_x^T$	$q_{x+t}^{(r)}$	$\ddot{a}_{x+t}^{(12)}$	ERB
63	PVB = [	v^0	(1.0)	(.50)	9.85	15*12*16*(1-.06(2))
64		+ v^1	(.50)	(.20)	9.60	15*12*17*(1-.06(1))
65		+ v^2	(.40)	(1.0)	9.35	15*12*18*(1-.06(0))]

= 12,482 + 2,605 + 10,785

= 25,872

answer is D

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Problem 19

This problem does not require calculation of a normal cost or accrued liability, so the cost method is not given. In this problem, the present value of death benefits is done using "multiple" decrements. The present value is a summation:

$$\Sigma\ v^t * {}_tp_x^T * q_{x+t}^{(d)} * .90 * .50 * ERB\ \ \ddot{a}_{x+t}^{(12)}$$

where ERB is the early retirement benefit at age x+t. In this problem early retirement benefits are reduced 5% per year prior to age 65. The death benefit to the spouse is 50% of the early retirement benefit. Only 90% of the participants are assumed to be married at death. You are given factors that should be assumed on a unisex basis. Since the spouses are the same age as the participants, no age adjustment is necessary in the annuity factor.

1/1/89 valuation

Age 63, service 30 yrs

Under these assumptions, 2% of the participant is assumed to die at the beginning of each year. At age 65, the participant retires at the beginning of the year. You need to set up the probability of surviving from age 63 to age 65 before the present value of benefits can be calculated:

t	x+t	l_{x+t}	$q_{x+t}^{(d)}$	${}_tp_x^T$
0	63	1,000	.02	1.0
1	64	980	.02	.98
2	65	960	0.0	.9604

age		v^t	${}_tp_x^T$	$q_{x+t}^{(r)}$	$\ddot{a}_{x+t}^{(12)}$	ERB	
63	PVB = .90 * .50 * [	v^0	(1.0)	(.02)	9.85	20*12*30*(1-.05(2))	
64	+	v^1	(.98)	(.02)	9.60	20*12*31*(1-.05(1))]	

$$= .45 * (1,277 + 1,255)$$
$$= 1,139$$

answer is B

Note that there is no present value of death benefits for the year the participant attains age 65. Compare the calculation of retirement benefits at age 65 in the previous problem.

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Problem 20 - Page 1

Under the Aggregate method, the PVNC is calculated on an aggregate basis as PVB - AAV. The normal cost is calculated using the average temporary annuity to normal retirement age.

You are given valuation results based on a life annuity normal form. You must adjust the present value of benefits to reflect the change in normal form of benefit payment. The new normal form should only be available to retirements after 1988, so it does not affect the retiree liability of 400,000.

The actives PVB must be derived based on the information given on the life annuity normal form.

1-1-89 valuation - life annuity

NC = PVNC / (PVE/E)

PVNC = (PVE / E) * NC
= (4,200,000 / 350,000) * 85,000
= 1,020,000

	ASSETS		LIABILITIES
AAV	600,000	PVB - actives	X
PVNC	1,020,000	PVB - retired	400,000
	1,620,000		X + 400,000

X = 1,620,000 - 400,000 = 1,220,000

(see next page)

Problem 20 - Page 2

The actives PVB is based on a life annuity factor of 9.90. If all the actives were assumed to be married at retirement, the new PVB for actives would be based on the 50% Joint and Survivor factor:

$$\left[\ddot{a}^{(12)}_{65} + \tfrac{1}{2} \left(\ddot{a}^{(12)}_{65} - \ddot{a}^{(12)}_{65:65} \right) \right]$$

$$= 9.90 + .50 \ (9.90 - 7.82)$$
$$= 10.94$$

Since only 85% of the participants are assumed to be married at retirement, the present value of benefits at age 65 should be based on a weighted factor:

$$85\% * 10.94 + 15\% * 9.90 = 10.784$$

Adjusted PVB for actives = 1,220,000 (10.784 ÷ 9.90)
= 1,328,937

1-1-89 valuation - J & S form

PVNC = (PVE / E) * NC
= (4,200,000 / 350,000) * 85,000
= 1,020,000

	ASSETS		LIABILITIES
AAV	600,000	PVB - actives	1,328,937
PVNC	1,128,937	PVB - retired	400,000
	1,728,937		1,728,937

NC = PVNC / (PVE/E)
= 1,728,937 / 12
= 94,078

answer is B

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