SoftwarePolish
Rick Groszkiewicz
2964 Nestle Creek Drive
Marietta, GA 30062-4857
Voice/fax (770) 971-8913
email: rickg@softwarepolish.com

SPRING 1995 EA-1B EXAM SOLUTIONS

Copyright © 1996 by
Rick Groszkiewicz FSA EA

Revision History:
05/03/97 Corrected problem 15 - added age 65 annuity in calculation of AL as active employee

THIS PAGE WAS INTENTIONALLY LEFT BLANK

Spring 1995 EA-1B

1 Under the Aggregate method, the PVNC is equal to PV future benefits minus the actuarial value of assets. The normal cost is calculated by amortizing the PVNC over future service as a level dollar amount (or a level percentage of pay, if the benefit is defined based on pay).

With several participants, the easiest way to work the problem is to calculate various items in a column for each participant. Note that Smith and Brown are "identical twins" - you don't need to write down all the intermediate results for both of them, which can save a little time

Item	Smith	Brown	Green	Total
D.O.B.	1/1/60	1/1/60	1/1/50	
1-1-95 age	35	same	45	
D.O.H.	1/1/81	↓	1/1/71	
1-1-95 svc	14	"	24	
Hire age	21	"	21	
Total svc	44	"	44	
Proj. Benefit	12($10)44	"	"	
	= 5280	"	5280	
PVB at 65	5280(8.74)	"	"	
	= 46,147	"	"	
D_x	894,190	"	445,008	
D_{65}/D_x	.1056	"	.2122	
1-1-95 PVB	4,873	"	9,791	19,536
$N_x - N_{65}$	11,496,598	"	4,822,798	
$\ddot{a}_{x:\overline{65-x}\rceil}$	12.8570	"	10.8376	36.5515 avg = 12.1838

PVNC = PVB - AAV = 19,536

$$NC = \frac{PVNC}{\text{avg } \ddot{a}_{x:\overline{65-x}\rceil}} = \frac{19{,}536}{12.1838} = 1{,}603$$

(B)

Spring 1995 EA-1B

2 There is no cost method defined, since this is "simply" a calculation of PV future benefits. With retirement decrements starting at age 62, you have to calculate projected benefits from 62 to 65:

DOB 1/1/33 DOH 1/1/75 $50,000 pay for age 61,
1-1-95 age 62 Svc 20 would use for retirement at 62

Age	(1) Projected Service	(2) Projected Pay	(3) Early Reduction	(1)x.02x(2)x(3) Early Ret Benefit
62	20	50,000	.85	17,000
63	21	52,500 = 50,000 $(1.05)^1$	.90	19,845
64	22	55,125 = 50,000 $(1.05)^2$	.95	23,042
65	23	57,881 = 50,000 $(1.05)^3$	1.00	26,625

The trickier part is calculation of probability of survival each age, and allowing for past retirements:

(1) t	(2) Age $62+t$	(3) v^t	(4) $_tp_{62}^{(T)}$	(5) $q_{62+t}^{(r)}$	(6) $p_{62+t}^{(T)}$	(7) $\ddot{a}_{62+t}^{(12)}$	(8) Early Ret Ben@62+t	(9) (3)x(4)x(5)x(7)x(8) PVB@62+t
0	62	1.0000 = $(1.07)^{-0}$	1.0000	.25	.75	9.18	17,000	39,015
1	63	.9346 = $(1.07)^{-1}$	.7500	.50	.50	8.96	19,845	62,317
2	64	.8734 = $(1.07)^{-2}$	.3750	.75	.25	8.74	23,042	49,472
3	65	.8163 = $(1.07)^{-3}$	.09375	1.00	—	8.51	26,625	17,340
								Σ = 168,143

The key to this problem is that each year's cumulative $_tp_{62}^{(T)}$ is calculated by multiplying $_{t-1}p_{62}^{(T)}$ and $p_{62+t-1}^{(T)}$ together – this allows for the retirements assumed to occur each year.

(C)

Spring 1995 EA-1B

3 This problem uses the Individual Aggregate method, which requires you to calculate the cost for each participant based on the definitions for the Aggregate method. With only one participant, you can think of the cost method as defined to be the Aggregate method:

$PVNC = PVB - AAV \qquad NC = PVNC / \ddot{a}_{x:\overline{NRA-x}|}$

One minor trick to the problem is that PVB includes both retirement and death benefits:

age: 50 51 52 64

$$PVB = \$5000(12)\ddot{a}_{65}^{(12)}\frac{D_{65}}{D_x} + 100{,}000\left(\frac{.005}{1.07} + \frac{.995(.005)}{(1.07)^2} + \frac{(.995)^2.005}{(1.07)^3} + \dots \frac{(.995)^{14}.005}{(1.07)^{15}}\right)$$

1-1-95 Age is 50 based on DOB of 1-1-45

With no other decrements, you can rewrite this based on interest-only functions:

$$PVB \text{ (for Retirement)} = 5000(12)8.74(1.07)^{-15}(.995)^{15} = 176{,}300$$

$$PVB \text{ (for death ben)} = \frac{100{,}000(.005)}{.995}\left[\frac{.995}{1.07} + \left(\frac{.995}{1.07}\right)^2 + \dots + \left(\frac{.995}{1.07}\right)^{15}\right]$$

$$= \frac{100{,}000(.005)}{.995}\left[\frac{1}{1.07538} + \frac{1}{(1.07538)^2} + \dots + \frac{1}{(1.07538)^{15}}\right]$$

$$= 502.51\,(a_{\overline{15}|7.538\%})$$

$$= 4{,}425$$

$PVNC = PVB - AAV = 176{,}300 + 4{,}425 - 71{,}500 = 109{,}225$

$$\ddot{a}_{50:\overline{15}|} = 1 + \frac{.995}{1.07} + \dots + \left(\frac{.995}{1.07}\right)^{14} = \ddot{a}_{\overline{15}|7.538\%} = 9.4703$$

$$NC = \frac{PVNC}{\ddot{a}_{50:\overline{15}|}} = \frac{109{,}225}{9.4703} = 11{,}533$$

(B)

Spring 1995 EA-1B

4 The Unit Credit method is an individual cost method. To work this problem, you must know a "trick": under Unit Credit, for participants who are 100% vested, the valuation results are <u>independent</u> of the withdrawal decrements. You will get the same NC and AL whether you use withdrawal decrements or not.

DOB 1/1/62 1-1-95 Age 33
DOH 1/1/92 Svc 3

The problem asks for the AL due to vested termination benefits. This is calculated as the total AL less the AL due to retirement benefits. The twist to this problem is that the participant will not be 100% vested until age 35. You can calculate the total AL at age 35 ignoring the withdrawal decrements, then discount that back to age 33 with mortality and withdrawal. You can directly calculate the AL for retirement "decrements" at age 33, and the answer is the difference between the two AL figures.

$$AL_{33} = v^2 \, {}_2p_{33}^{(T)} \, (AL_{35})$$

$$p_x^{(T)} = 1 - q_x^{(d)} - q_x^{(w)} = \left(1 - q_x'^{(d)}\right)\left(1 - q_x'^{(w)}\right) = p_x'^{(d)} \cdot p_x'^{(w)}$$

multiple decrement table probabilities — single decrement table rates

$${}_2p_{33}^{(T)} = \left({}_2p_{33}'^{(d)}\right)\left({}_2p_{33}'^{(w)}\right) = \left(\frac{9915}{9952}\right)\left(\frac{7500}{8500}\right) = .8791$$

(4) continued

$$AL_{35} = \sum v^t \, {}_tp_x^{(T)} \, q_{x+t}^{(w)} (AB) \frac{N_{65}^{(12)}}{D_{x+t}} \quad \text{withdrawal}$$
$$+ \; v^{30} \, {}_{30}p_{35}^{(T)} (1.00) \, AB \, \ddot{a}_{65}^{(12)} \quad \text{retirement}$$

Since the participant is 100% vested, you get the same valuation results with no withdrawal decrements, or 100% withdrawal at every age. The reason is that the participant will receive the same benefit at age 65 regardless of when they leave, and only mortality between current age and retirement age will affect the liability.

You can calculate AL_{35} ignoring withdrawal:

$$\text{total } AL_{35} = v^{30} \, {}_{30}p_{35}'^{(d)} (AB) \, \ddot{a}_{65}^{(12)}$$
$$= (1.06)^{-30} \frac{7900}{9915} (3 \times \$20 \times 12) \, 9.439$$
$$= 942.79$$

$$\text{total } AL_{33} = v^2 \, {}_2p_{33}^{(T)} \, AL_{35}$$
$$= (1.06)^{-2} (.8791 \text{ prior page})(942.79)$$
$$= 737.61$$

Retirement portion $AL_{33} = v^{32} \, {}_{32}p_{33}^{(T)} (AB) \, \ddot{a}_{65}^{(12)}$

$$= (1.06)^{-32} \left(\frac{7900}{9952}\right)\left(\frac{2000}{8500}\right)(720)(9.439)$$
$$= .1550 \, (.7938)(.2353)(720)(9.439)$$
$$= 196.70$$

withdrawal portion of $AL = 737.61 - 196.70 = 540.91$ (C)

Spring 1995 EA-1B

5 With side fund problems, you must take the cash surrender value of insurance policies into account as an asset under the given funding method. With this problem you can use the "standard" method of solution:

Step 1 - <u>calculate accrued/projected benefit</u>
This is given as \$400 per month

Step 2 - <u>determine the total amount of insurance</u>
Whole life insurance is 60 times the projected benefit = 60(\$400) = \$24,000

Step 3 - <u>determine the cash surrender value at the assumed retirement age</u>

$$CSV = \frac{550}{1000}(24{,}000) = \$13{,}200$$

Step 4 - <u>convert the CSV to annual benefit</u>
Benefit from insurance = 13,200/10 = 1,320

Step 5 - <u>calculate net projected benefit</u>
Net benefit = 12(400) − 1,320 = 3,480

Step 6 - <u>apply cost method given to the net benefit</u>

Under ILP, the normal cost layer for each change in projected benefit is calculated as

$$\Delta NC = \frac{\Delta PVB}{\ddot{a}_{IA:\overline{RA-IA}|}}$$ where IA is the age when the projected benefit changes

(5) continued

Since the plan has just been established, IA = current age, and you only have one layer of normal cost.

DOB 1/1/56
1-1-95 Age 39

$$ILP\ NC = \frac{3480\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{39}}{\ddot{a}_{39:\overline{26|}}}$$

With only interest, this can be rewritten as follows

$$ILP\ NC = \frac{3480(10)v^{26}}{\ddot{a}_{\overline{26|}.06}} = \frac{34800}{\ddot{s}_{\overline{26|}.06}}$$

$$= 554.97$$

One final trick is that the problem asks for the 12/31 normal cost

$$12/31\ NC = 1.06(554.97)$$

$$= 588.27$$

(E)

Spring 1995 EA-1B

6 This is a standard problem with employee contributions. Under the Aggregate method, PVNC is defined as PVB - AAV. With employee contribution, the PVB should include the liability for refunds of mandatory contributions, and the AAV should include the present value of future mandatory contributions.

The key to working this problem is that voluntary contributions should not be considered anywhere in the funding calculations. In practice, they are placed in a separate account and are accumulated by themselves to provide additional benefits.

PVB = 1,000,000 + 20,000 + 80,000
= 1,100,000

PVEEC = (.5%)(4,800,000)
= 24,000

PVNC = PVB - (AAV + PVEEC)
= 1,100,000 - (30,000 + 150,000 + 24,000)
= 896,000

$$PVE/E = \frac{4,800,000}{600,000} = 8.0$$

$$NC = \frac{896,000}{8.0}$$
= 112,000

(C)

Spring 1995 EA-1B

7 This is an original actuarial equivalence problem! You must write down the equation of value for the original benefit and the modified benefit.

DOB 1-1-36 DOH 1-1-72

1-1-96 Age 60 Svc 24

AB = original accrued benefit at 1-1-96

$= (.02)(FAE_3)(24)$

ERB = original early retirement benefit at 1-1-96

$= .85(AB)$

$$ERB\,\ddot{a}_{60}^{(12)} = 80\%(ERB)\left[\ddot{a}_{60}^{(12)} + .5\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)}\right)\right]$$

$$1 = .8\left[1 + .5\left(\frac{\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)}}{\ddot{a}_{60}^{(12)}}\right)\right]$$

$$\therefore\quad .50 = \frac{\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)}}{\ddot{a}_{60}^{(12)}}$$

MERB = modified early retirement benefit under early out

MAB = modified accrued benefit under early out

$= (.02)(FAE_3)(25)$

$MERB = MAB(1-.03(3))$

$= .02(FAE_3)(25)(.91)$

$$= \frac{25}{24}\left(\frac{.91}{.85}\right) ERB = 1.1152\,ERB$$

Assuming we had \$100/mo of ERB, we now have 111.52/mo of MERB:

$$111.52\,\ddot{a}_{60}^{(12)} = 80.00\left(\ddot{a}_{60}^{(12)} + X\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)}\right)\right)$$

$$111.52 = 80\left(1 + X\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)}\right)/\ddot{a}_{60}^{(12)}\right)$$

$$= 80(1 + .5X)$$

$$111.52/80 = 1 + .5X$$

$$X = .788$$

(B)

Spring 1995 EA-1B

8 Entry Age Normal is an Individual cost method (unless you are told to use the aggregate variation). The normal cost is defined as PVB_{EA} / PVS_{EA}. If the benefit is based on pay, the normal cost should be calculated as a level percentage of pay from entry age. In this problem, the normal cost is calculated as a level dollar amount.

The accrued liability can be calculated as

$AL = PVB - PVNC$ prospective

or $AL = EANC(\ddot{s}_{EA:\overline{CA-EA}|})$ retrospective

	Smith	Brown
DOB	1-1-65	1-1-40
1-1-95 age	30	55
DOH	1-1-90	1-1-80
Entry age = Hire age	25	40
Total service	40	25
Projected benefit	\$200(40) = 8,000	\$200(25) = 5,000
EANC	8(43.04) = 344.32	5(135.85) = 679.25
AL	$EANC\,\ddot{s}_{25:\overline{5}|}$ = $344.32(\ddot{s}_{\overline{5}|.07})$ = 2,118.70	$EANC\,\ddot{s}_{40:\overline{15}|}$ = $679.25(\ddot{s}_{\overline{15}|.07})$ = 18,263.71
old AL	(150/200) 2,118.70 = 1,589.03	(150/200) 18,263.71 = 13,697.78
ΔAL	529.68	4,565.93

$\Sigma = 5{,}095.60$ (C)

Spring 1995 EA-1B

9 This problem uses the Individual Aggregate method, which applies the Aggregate cost method to each plan participant. The assets must be allocated to each participant based on some rule.

$$NC\% = \frac{PVB - \text{Allocated AAV}}{PVE} \Rightarrow NC = \frac{PVB - \text{Alloc AAV}}{(PVE/E)}$$

Since the benefits are pay related, you must calculate the normal cost to be a level % of payroll

DOB 1/1/40 — 1994 pay 90,000 age 54

1-1-95 age 55 — 2004 pay $90,000(1.03)^{10}$ age 64 = 120,952

Projected Ben = .60 (120,952) = 72,571

$$PVB = 72{,}571\ \ddot{a}_{65}^{(12)}\ D_{65}/D_{55}$$
$$= 72{,}571\ (8.74)(1.07)^{-10} \quad \text{no pre-ret decrements}$$
$$= 322{,}433$$

10 terms from age 55-64

$$PVE = 90{,}000(1.03)\left[1 + \frac{1.03}{1.07} + \left(\frac{1.03}{1.07}\right)^2 + \dots + \left(\frac{1.03}{1.07}\right)^9\right]$$
$$= 90{,}000(1.03)\ \ddot{a}_{\overline{10|}j} \quad \text{where } j = (1.07/1.03) - 1 = 3.88\%$$
$$= 785{,}629$$

$$NC = \frac{322{,}433 - 18{,}000}{785{,}629/92{,}700}$$
$$= \frac{322{,}433 - 18{,}000}{\ddot{a}_{\overline{10|}3.88\%}}$$
$$= 35{,}921$$

(B)

Spring 1995 EA-1B

10 The FIL method is an aggregate type of cost method. You normally do not separately calculate gains and losses under aggregate methods, since they become part of the PVNC and affect the normal cost. This problem asks you to calculate the change in the normal cost due to investment gains or losses. If all assumptions are met, the normal cost should be level as a percentage of pay. To work this problem, you should calculate the cost at 1-1-95 assuming every assumption is met, and then produce the final cost reflecting the actual asset value.

	1-1-94 actual	1-1-95 expected
PVB	2,500,000	2,675,000 = 1.07(2,500,000)
VAL	750,000	738,000 = 1.07(750,000+150,000) - 225,000
AAV	250,000	492,500 = 1.07(250,000) + 225,000
PVNC	1,500,000	1,444,500
Pay	4,000,000	4,200,000 = 1.05(4,000,000)
NC	150,000	
NC/Pay	3.75%	
= PVNC/PVE	3.75%	3.75% (Calc using PVE below)
PVE	40,000,000 = 1,500,000/.0375	38,520,000 = 1.07(40,000,000 - 4,000,000)
PVE/E	10.0	9.1714

(next page)

(10) continued

One trick to this problem is that you have a new entrant. You don't know the PVB for the new entrant, and the problem does not require that information. You do have to allow for the impact of the new entrant on the PVE/E ratio, since that directly affects the normal cost.

	1-1-95 "Actual"
PVB	2,675,000
UAL	738,000
AAV	510,000 actual, gain of 17,500
PVNC	1,427,000 lower by 17,500
Pay	4,250,000 = 4,000,000 + 50,000
PVE	39,179,390 = 38,520,000 + 50,000 $\ddot{a}_{\overline{15}|}$ $j=1.90\%$ *
PVE/E	9.2187
ΔNC	1,898 $= \dfrac{\Delta PVNC}{9.2187} = \dfrac{17,500}{9.2187}$ (A)

* PVE for age 50 is (15 terms from age 50 to age 64)

$$50,000\left(1+\frac{1.05}{1.07}+\left(\frac{1.05}{1.07}\right)^2+\cdots+\left(\frac{1.05}{1.07}\right)^{14}\right)$$

$$= 50,000\left(1+(1.019)^{-1}+(1.019)^{-2}+\cdots+(1.019)^{-14}\right)$$

$$= 50,000\ \ddot{a}_{\overline{15}|.019}$$

$$= 659,390$$

Spring 1995 EA-1B

11 The Aggregate method is an aggregate type cost method. In order for the normal cost to remain level from one year to the next, the contribution at the end of the year must equal the normal cost plus interest.

You should calculate projected valuation results at 1-1-95 based on the \$30 plan and verify that the normal cost is a level dollar amount. Then you can calculate the actual normal cost at 1-1-95 based on the \$3 plan and information given.

	1-1-94	\$30 1-1-95 expected	\$35 1-1-95 actual
PVB	900,000	963,000 = 1.07(900,000)	1,123,500 = $\left(\frac{35}{30}\right)$(963,000)
AAV	500,000	573,909 = 1.07(500,000 + 36,364)	573,909
PVNC	400,000	389,091	549,591
avg $\ddot{a}_{x:\overline{RA-x}|}$	11.00	10.70 = 1.07(11.00 − 1.00)	10.70
NC	36,364	36,364 ✓	51,364

(D)

You could determine the average annuity of 10.70 by assuming the normal cost is 36,364 and dividing that into the PVNC of 389,091, but that would not guard against any arithmetic errors.

Note that the percent change in PVB is 16.67%, and the change in normal cost is 41.25%. You can't simply pro-rate values to derive the normal cost.

Spring 1995 EA-1B

12 Entry age normal is an individual type cost method. The normal cost is defined as EANC = PVB_{EA} / PVS_{EA}. For a plan with pay related benefits, you must calculate the normal cost as a level percentage of pay (unless the problem states otherwise).

With no salary scale, the normal cost will be a level dollar amount each year.

"old" $NC = PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA|}}$

$AL = PVB - PVNC$

"old" $PVNC = PVB - AL = 41{,}000 - (10{,}000 + 8{,}000)$

$= 23{,}000$

Δ Benefit $= 7.5\%$ final pay $= 7.5\%$ current pay (no salary scale)

Earn $= 3000/.075 = 40{,}000$

$PVE/E = \dfrac{460{,}000}{40{,}000} = 11.50 = \ddot{a}_{x:\overline{RA-x|}}$

$PVNC = EANC(\ddot{a}_{x:\overline{RA-x|}})$

"old" $EANC = PVNC/\ddot{a}_{x:\overline{RA-x|}} = 23{,}000/11.50$

$= 2{,}000$

"new" $EANC = \left(\dfrac{37.5}{30.0}\right) 2{,}000 = 2{,}500$

(B)

You can use this simple pro-rata approach since the EANC is defined based on PVB_{EA}. Any change in the PVB at entry age produces a proportional change in the EANC.

Spring 1995 EA-1B

13 Under the Aggregate method you normally do not calculate gains and losses separately, since they affect the normal cost directly. This problem asks you to calculate the change in the normal cost due to experience G/L.

You are given the valuation results reflecting the various sources of G/L. You must reconstruct the valuation results on an "expected" basis, and calculate the change in the normal cost.

	After G/L 1-1-95 valn	Before G/L 1-1-95 valn
PVB	149,000	139,518 = 149,000 (1.03/1.10)
AAV	20,000	18,400 = 20,000 - 1,600 gain
PVNC	129,000	121,118
PVE/E	8.0 = 1,200,000/150,000	8.0 unaffected by salary change
NC	16,125	15,140
		$\Delta NC = 985$ (A)

The PVE/E did not change because both the PVE and the Earnings would be multiplied by the (1.03/1.10) ratio, and the effect cancels out. This ratio backs out the actual 10% salary increase, and applies the expected 3% salary increase.

Spring 1995 EA-1B

14 Frozen Initial Liability is an aggregate type cost method. Under FIL, the PVNC is defined as PVB - UAL - AAV. The initial UAL equals the Entry Age Normal AL less any assets. Each year's UAL is defined by writing down the prior year's UAL:

$UAL_1 = (UAL_0 + NC_0)(1+i) - (C+I)$

The key to this problem is that the contribution paid at 12-31-95 equals $1.07\left(NC + \frac{IAL}{\ddot{a}_{\overline{10}|.07}}\right)$

$25{,}000 = 1.07\left(NC + 150{,}000 / \ddot{a}_{\overline{10}|.07}\right)$

$\frac{25{,}000}{1.07} = NC + \frac{150{,}000}{\ddot{a}_{\overline{10}|.07}}$

$NC = \frac{25{,}000}{1.07} - \frac{150{,}000}{\ddot{a}_{\overline{10}|.07}} = 23{,}364 - 19{,}960 = 3{,}405$

$NC = \frac{PVNC}{PVE/E} \Rightarrow PVNC = (3405)\left(\frac{600{,}000}{60{,}000}\right)$

$= 34{,}050$

$PVNC = PVB - UAL - AAV$

$UAL = PVB - PVNC - AAV$

$= 300{,}000 - 34{,}050 - 200{,}000$

$= 65{,}950$

(E)

This is a fairly lengthy problem to get to the final value for the UAL!

Revised 5/13/97

Spring 1995 EA-1B

15 The experience gain/loss upon retirement is calculated as the difference in the accrued liability as an active employee and as a retiree. Under the projected unit credit method, the accrued liability is defined as the present value of the funding accrued benefit. The funding accrued benefit is similar to the actual accrued benefit, but with earnings projected to retirement age:

	Smith	Brown	Green
DOB	1-1-39	1-1-35	1-1-32
1-1-95 Age	56	60	63
Prior year Pay	E_1	E_2	E_3
1-1-95 svc	S_1	S_2	S_3

We are starting with prior year pay because that is what we'll use to calculate the retirement benefit at 1-1-95. We can dispense with the subscripts, since the same values will be used in the accrued benefit and early retirement benefit calculations.

	Smith	Brown	Green
F.A.B.	$.02(S)(E)(1.03)^9$	$.02(S)(E)(1.03)^5$	$.02(S)(E)(1.03)^2$
active A.L.	$[\,.02(S)(E)(1.03/1.09)^9$	$.02(S)(E)(1.03/1.09)^5$	$.02(S)(E)(1.03/1.09)^2\,]\ddot{a}_{65}^{(12)}$
	$=(.02SE)(5.0403)$	$=(.02SE)(6.3214)$	$=(.02SE)(7.4918)$
Accrued Ben	$.02SE$	$.02SE$	$.02SE$
Early Ret Ben	$(.02SE)(1-\frac{5}{15}-\frac{4}{30})$	$(.02SE)(1-\frac{5}{15})$	$.02SE(1-\frac{2}{15})$
retired PVB	$(.02SE)(\frac{16}{30})(9.84)$	$(.02SE)(\frac{10}{15})(9.25)$	$(.02SE)(\frac{13}{15})(8.71)$
	$=(.02SE)(5.2480)$	$=(.02SE)(6.1667)$	$=(.02SE)(7.5487)$
Gain if AL>PVB	NO	YES	NO

(B)

Spring 1995 EA-1B

16 Aggregate is an aggregate type cost method. PVNC is defined as PVB − AAV.

In this problem, you should calculate the change in the asset due to the investment experience different than assumed. Then you can easily calculate the effect on NC.

	1-1-95 Actual	1-1-95 Expected	Δ
PVB	1,200,000	1,200,000	
AAV	$1.04(AAV_{94})$	$1.07(AAV_{94})$	$.03(AAV_{94})$
PVNC	$1{,}200{,}000 - 1.04(AAV_{94})$		$.03(AAV_{94})$ = LOSS
PVE/E	$12.6667 = \frac{9{,}500{,}000}{750{,}000}$	12.6667	

$$NC \quad \frac{1{,}200{,}000 - 1.04(AAV_{94})}{12.6667}$$

$$\Delta NC = 100 = \frac{.03(AAV_{94})}{12.6667}$$

$$\therefore AAV_{94} = 1266.67/.03$$

$$= 42{,}222$$

$$\text{1-1-95 AAV} = 1.04(42{,}222)$$

$$= 43{,}911$$

(E)

Spring 1995 EA-1B

17 This is basically a benefit calculation problem. You must calculate the benefit based on service and earnings at both ages 65 and 66, and calculate the actuarial increase from age 65 to age 66.

Partic. DOB 1-1-29
1-1-95 age 66
DOH 1-1-64
1-1-95 svc 31

Spouse DOB 1-1-29
Spouse age 66

	Age 65	Age 66
Pay at 65	—	32,000
Pay at 64	45,000	45,000
Pay at 63	42,000	42,000
Pay at 62	30,000	—
3 years' pay	117,000	119,000
FAE3	39,000	39,667

Service	30	31
Accrued Ben	.02(30)39,000 = 23,400	.02(31)(39,667) = 24,593

Act'l Increase $(N_{65}^{(12)}/N_{66}^{(12)})23{,}400$

$= \left(\frac{824{,}779}{734{,}109}\right)23{,}400$

$= 26{,}290$ this benefit is greater

$\ddot{a}_{66}^{(12)} = (734{,}109/86{,}246) = 8.5118$

100% J+S benefit $= 26{,}290\left(\ddot{a}_{66}^{(12)} / \left[\ddot{a}_{66}^{(12)} + 1.00(\ddot{a}_{66}^{(12)} - \ddot{a}_{66:66}^{(12)})\right]\right)$

$= 26{,}290 / \left[1 + \frac{8.5118-6.3}{8.5118}\right]$

$= 20{,}868$

(B)

Spring 1995 EA-1B

18 Entry age normal is an individual cost method. You are told to calculate the normal cost as a level dollar amount, but this is what you would assume anyway, based on the information in the problem.

The EANC is defined as $PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$.
The EAN AL can be calculated two ways:

$AL = EANC(\ddot{s}_{EA:\overline{CA-EA}|})$ retrospective

$AL = PVB - EANC(\ddot{a}_{CA:\overline{RA-CA}|})$ prospective

DOB	1-1-50	DOH	1-1-80
1-1-95 Age	45	SVC	15
Entry Age	30		

$$EANC = \frac{12(2500)\ddot{a}_{65}^{(12)} D_{65}/D_{30}}{(N_{30}-N_{65})/D_{30}} = \frac{30{,}000\,\ddot{a}_{65}^{(12)}(D_{65})}{N_{30}-N_{65}}$$

$$EAN\ AL = EANC\,\ddot{s}_{30:\overline{15}|}$$

$$= EANC(N_{30}-N_{45})/D_{45}$$

$$= 30{,}000\,\ddot{a}_{65}^{(12)}\left(\frac{D_{65}}{D_{45}}\right)\left(\frac{N_{30}-N_{45}}{N_{30}-N_{65}}\right)$$

$$= 30{,}000(8.74)\frac{94{,}414}{445{,}008}\left(\frac{17{,}887{,}840-5{,}690{,}850}{17{,}887{,}840-868{,}052}\right)$$

$$= 30{,}000(8.74)(.2122)(.7166)$$

$$= 39{,}866$$

(D)

Spring 1995 EA-1B

19 Frozen Initial Liability is an Aggregate cost method. The PVNC is defined as PVB - AAV - UAL. The initial UAL is based on the Entry Age Normal AL less any assets. Each subsequent year's UAL is written down as $(1+i)(UAL_0 + NC_0) - (C+I)$.

The trick to this problem is that you do NOT use actual investment return. The write down of the UAL is always based on the valuation rate of interest. Another trick is that you must add the UAL layer at 1-1-94 due to the plan amendment.

Date	UAL_0	NC_0	$(1+i)(NC_0+UAL_0)$	C_0	Date	(C_0+I)
1-1-92	10,000,000	3,000,000	13,780,000	4,500,000	1-1-92	4,770,000
1-1-93	9,010,000	3,200,000	12,942,600	5,000,000	9 months 4-1-93	5,225,000
before 1-1-94	7,717,600					
after 1-1-94	12,717,600	3,500,000	17,190,656	5,500,000	9 months 4-1-94	5,747,500
1-1-95	11,443,156					

(D)

Spring 1995 EA-1B

20 Entry Age Normal is an individual cost method. The normal cost is defined as PVB_{EA}/PVS_{EA}. Since the benefit is not based on pay, the EANC will be calculated as a level dollar amount.

The definition of the gain or loss due to retirement is the active AL less the retired AL (which is always the retired PVB). There are two tricks to this problem. First is the assumed retirement age of 62. The second is that the active AL must be calculated assuming that 80% of Smith is married and 20% is single.

DOB	1-1-35	Spouse DOB	1-1-35	
1-1-95 age	60	age	60	
DOH	1-1-85	Past service	10	- use for retired PVB
Entry age	50	Total service	12	- use for active AL

$$EANC = PVB_{EA} / \ddot{a}_{EA:\overline{RA-EA}|}$$

$$= \frac{(\text{Proj Ben})(\text{PV factor at } 62)\ D_{62}/D_{50}}{\ddot{a}_{50:\overline{12}|}}$$

$$= \frac{(\text{Proj Ben})(\text{PV factor at } 62)}{\ddot{s}_{\overline{12}|.07}} \quad \text{no pre-ret decrements}$$

$$AL = EANC\,(\ddot{s}_{50:\overline{10}|})$$

$$= (\text{Proj Ben})(\text{PV factor at } 62)\ \frac{\ddot{s}_{\overline{10}|.07}}{\ddot{s}_{\overline{12}|.07}}$$

$$\text{Proj Ben} = 12(\$120)[1-12(.5\%)3] \quad \text{reduction at ARA 62}$$

$$= 1440(.82)$$

$$= 1180.80$$

Spring 1995 EA-1B

(20) continued

Active PV Factor @ 62

$20\% (\ddot{a}_{62}^{(12)})$ — single

$+ 80\% (\ddot{a}_{62}^{(12)} + 1.0(\ddot{a}_{62}^{(12)} - \ddot{a}_{62:62}^{(12)})$ — married

$= 1.8(\ddot{a}_{62}^{(12)}) - .8(\ddot{a}_{62:62}^{(12)})$

$= 1.8(9.0) - .8(7.5)$

$= 10.20$

Active EAN AL $= 1180.80(10.20)(\ddot{s}_{\overline{10}|.07} / \ddot{s}_{\overline{12}|.07})$

$= 1180.80(10.20)(14.7836/19.1406)$

$= 9{,}303$

Retired PVB at 60: (Early ret ben) x (PV factor at 60)

Early ret ben $= 10(\$120)[1 - 12(.5\%) 5]$

$= 1200(.70)$

$= 840$

PV factor at 60 $= \ddot{a}_{60}^{(12)} + 1.0(\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)})$

$= 10.0 + 10.0 - 8.0$

$= 12.0$

Retired PVB at 60 $= 840(12)$

$= 10{,}080$

Loss on retirement $= 10{,}080 - 9{,}303$

$= 777$

(B)