

SINKING FUNDS

Pay interest every year on original
Loan amount \Rightarrow remains constant

Net amount of loan = original
Loan amount less sinking fund

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SINKING FUNDS

If Loan = \$1

Same interest on Loan as in Sinking Fund

$$\frac{1}{a\bar{n}|i} = \frac{1}{s\bar{n}|i} + i$$

Diagram showing the components of the equation:

- $\frac{1}{a\bar{n}|i}$ is labeled "Loan amort payment".
- $\frac{1}{s\bar{n}|i}$ is labeled "Sinking Fund Deposit".
- i is labeled "Each Year's Interest".

Arrows point from the labels to their respective terms in the equation.

If Loan = $a\bar{n}|i$

Same interest rate on Loan and S.F.

Annual loan payment would be \$1

Interest on loan = $i(a\bar{n}|i)$

Sinking Fund Deposit = $1 - i(a\bar{n}|i)$

Accum value of S.F.
after n years

$$\begin{aligned} &= (1 - i(a\bar{n}|i))(s\bar{n}|i) \\ &= (1 - (1 - v^n))s\bar{n}|i \\ &= v^n s\bar{n}|i = a\bar{n}|i \end{aligned}$$

SINKING FUNDS

If i' per year on original loan,
and i per year in sinking fund:

$$\begin{aligned}\frac{1}{a\ddot{n}|i'} &= \frac{1}{s\ddot{n}|i} + i' \\ &= \left(\frac{1}{a\ddot{n}|i} - i \right) + i' \\ &= \frac{1}{a\ddot{n}|i} + (i' - i)\end{aligned}$$

Payment is same as before, plus
add'l interest needed for loan amount.

SINKING FUND

<u>n</u>	<u>Int Pd</u>	<u>S. F. Deposit</u>	<u>Interest on S. F.</u>	<u>Amount in S. F.</u>	<u>Net Loan Amount</u>
0	i'	$\frac{1}{s_{n }}$	0	$\frac{1}{s_{n }} = \frac{s_{1 }}{s_{n }}$	$1 - \frac{s_{1 }}{s_{n }}$
1	i'	$\frac{1}{s_{n }}$	$i \frac{s_{1 }}{s_{n }}$	$\frac{s_{2 }}{s_{n }}$	$1 - \frac{s_{2 }}{s_{n }}$
2	i'	$\frac{1}{s_{n }}$	$i \frac{s_{2 }}{s_{n }}$	$\frac{s_{3 }}{s_{n }}$	$1 - \frac{s_{3 }}{s_{n }}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$n-1$	i'	$\frac{1}{s_{n }}$	$i \frac{s_{n-2 }}{s_{n }}$	$\frac{s_{n-1 }}{s_{n }}$	$1 - \frac{s_{n-1 }}{s_{n }}$
n	i'	$\frac{1}{s_{n }}$	$i \frac{s_{n-1 }}{s_{n }}$	$1 = \frac{s_{n }}{s_{n }}$	0
TOTAL ni'		$\frac{n}{s_{n i}}$	$\frac{s_{n } - n}{s_{n }}$	<u> </u>	<u> </u>

KEY CONCEPTS

SECTION IV - LOANS

- 1. Retrospective versus Prospective formula for O/S loan balance**
- 2. Loan amortization schedule**
- 3. Principal vs Interest**
 - a. Single payment (or multiple)**
 - b. Total payments for life of loan
= total principal + total interest**
- 4. Refinance loans based on O/S balance**
 - a. New term**
 - b. New payments**
 - c. New interest rate**
 - d. Any combination**
- 5. Sinking Funds**
 - a. Loan interest at one rate**
 - b. Repay loan principal from S.F.**