GAINS AND LOSSES

Analysis By Source - Active Employees

Assume employee is age x at time 0, and age x+1 at time 1

Assumed Zero for actives

$$
\begin{aligned}
{}_eAL_1 &= (1+i)(NC_0+AL_0) - (BP+int) \\
&= \frac{p_x}{v \cdot p_x}(NC_0+AL_0) \\
&= p_x (NC_0+AL_0)(1+i)/p_x
\end{aligned}
$$

$$
\begin{aligned}
{}_eAL_1 &= p_x^{(T)} \cdot (AL_1) \\
&= (1 - q_x^{(w)} - q_x^{(d)})(AL_1) \\
&= AL_1 - q_x^{(w)} AL_1 - q_x^{(d)} AL_1
\end{aligned}
$$

Typical retrospective definition for AL:

$$AL_0 = \sum_{t=EA}^{X-1} NC_t (1+i)^{x-t} / {}_{x-t}p_t$$

$$AL_1 = \sum_{t=EA}^{X} NC_t (1+i)^{x+1-t} / {}_{x+1-t}p_t$$

GAINS AND LOSSES

Analysis By Source - Active Employees

Logical interpretation of expression:

$$ {}_eAL_1 = AL_1 - q_x^{(w)}AL_1 - q_x^{(d)}AL_1 $$

Case 1: Employee survives

Expected Liability	$AL_1 * p_x^{(T)}$
Actual Liability	AL_1
Experience Loss	$q_x^{(w)}AL_1 + q_x^{(d)}AL_1$
	Termination Loss Mortality Loss

Case 2: Employee dies

Expected Liability	$AL_1 * p_x^{(T)}$
Actual Liability	zero
Experience Gain	$q_x^{(w)}AL_1 - (AL_1 - q_x^{(d)}AL_1)$
	Termination Loss Mortality Gain

GAINS AND LOSSES

Case 3: Employee is vested termination

Expected Liability at time 1

$$_eAL_1 = AL_1 * p_x^{(T)}$$

$$= AL_1 - q_x^{(w)} AL_1 - q_x^{(d)} AL_1$$

Actual Liability at time 1

$$(AB_X)\ (\text{vesting\%})\ \ddot{a}_{RA}^{(12)}\ (v^{RA-X-1}\ {}_{RA-X-1}p_{X+1})$$

Gain = Expected - Actual

Mortality Loss

$$q_x^{(d)} AL_1$$

plus

Termination Gain

$$AL_1 - q_x^{(w)} AL_1$$

$$- (AB_X)(\text{vesting \%})\ \ddot{a}_{RA}^{(12)}(v^{RA-X-1}\ {}_{RA-X-1}p_{X+1})$$

GAINS AND LOSSES

Analysis By Source - Active Employees

Other Types of Questions

1. Salary Scale G/L

   $$G/L = AL_1 \text{ based on expected pay} - AL_1 \text{ based on actual pay}$$

2. Early Retirement G/L

   $$G/L = AL_1 \text{ as active employee} - AL_1 \text{ as retired employee}$$

Item 2 above assumes no retirement decrements

UNIT CREDIT - GAINS AND LOSSES

Actuarially reduced early retirement
➔ No early retirement gain or loss

Actuarially reduced early retirement:

$$ERB_X = AB_X * N^{(12)}_{RA} / N^{(12)}_X$$

Accrued Liability before retirement:

$$AL_{ACT} = AB_X * \ddot{a}^{(12)}_{RA} (v^{RA-X}\ {}_{RA-X}p_X)$$

$$= AB_X * N^{(12)}_{RA} / D_X$$

Accrued Liability after retirement:

$$AL_{RET} = ERB_X * N^{(12)}_X / D_X$$

$$= AB_X * N^{(12)}_{RA} / N^{(12)}_X * (N^{(12)}_X / D_X)$$

$$= AB_X * N^{(12)}_{RA} / D_X$$

G/L = AL_1 as active employee
- AL_1 as retired employee
= Zero

UNIT CREDIT - GAINS AND LOSSES

100% vested termination
➔ No termination gain or loss

Note: assumes no withdrawal decrements,
and same mortality before / after exit

Accrued Liability before withdrawal:

$$AL_{ACT} = AB_X * \ddot{a}_{RA}^{(12)} * (v^{RA-X} \ {}_{RA-X}p_X)$$

Accrued Liability after withdrawal:

$$AL_{DVT} = AB_X * \text{vesting\%} * \ddot{a}_{RA}^{(12)} * (v^{RA-X} \ {}_{RA-X}p_X)$$

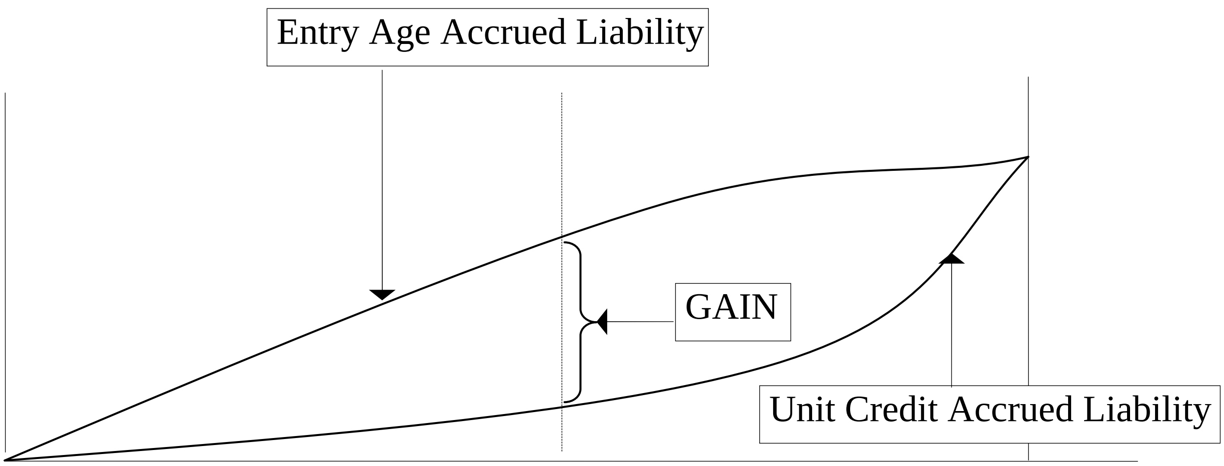
$$= AB_X * \ddot{a}_{RA}^{(12)} * (v^{RA-X} \ {}_{RA-X}p_X)$$

G/L = AL_1 as active employee
- AL_1 as terminated employee
= Zero

If < 100% vested, termination ➔ gain

ENTRY AGE NORMAL
GAINS AND LOSSES

Graphical representation of EAN AL before and after termination:



Entry Age **Current Age** **Retirement Age**

As active, have Entry Age Normal AL.
As termination, have Unit Credit AL.

Actuarially equivalent early retirement benefits ➔ gain upon early retirement.

GAINS AND LOSSES

Analysis by source - Retired Employees

$$_{e}AL_1 = (1+i)(AL_0+NC_0) - (BP+i)$$

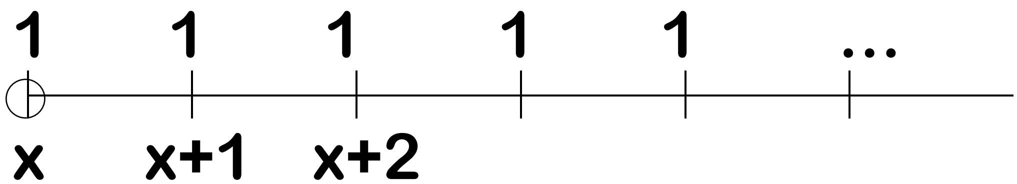
$$= (1+i)(\ddot{a}_x^{(12)}) - (\text{actual } BP+i)$$

Note: NC assumed 0 for retirees

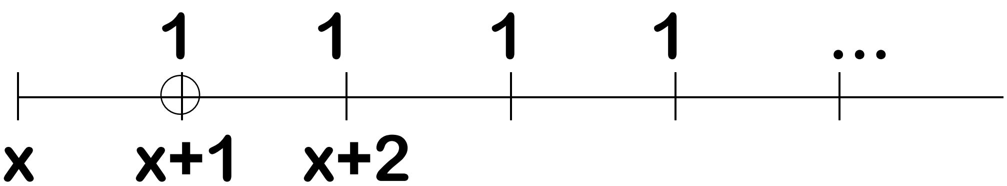
For retirees, both the actual AL_1 and the expected AL_1 depend on status at time 1. A death that occurs at the beginning or end of the year will have different actual benefit payments (monthly), and different values of $_{e}AL_1$.

LIFE CONTINGENCIES REVIEW

$$\ddot{a}_x = 1 + vp_x + v^2 {}_2p_x + \ldots$$



$$\ddot{a}_{x+1} = 1 + vp_{x+1} + v^2 {}_2p_{x+1} + \ldots$$



$$vp_x \ddot{a}_{x+1} = vp_x + v^2 {}_2p_x + \ldots$$
$$= \ddot{a}_x - 1.0$$
$$= a_x$$

Solve for interest rate i:

$$1+i = (p_x \ddot{a}_{x+1}) / (\ddot{a}_x - 1.0)$$

GAINS AND LOSSES

Analysis by source - Retired Employees

SURVIVAL CASE - ANNUAL ANNUITY

$$
\begin{aligned}
{}_eAL_1 &= (1+i)(NC_0 + AL_0) - (\text{actual BP}+i) \\
&= (1+i)(\ddot{a}_x) - (\text{actual BP}+i) \\
&= (1+i)(\ddot{a}_x) - (1+i) \\
&= (1+i)(\ddot{a}_x - 1.0) \\
&= (1+i)\, a_x \\
&= p_x \ddot{a}_{x+1}
\end{aligned}
$$

$$AL_1 = \ddot{a}_{x+1}$$

$$
\begin{aligned}
\text{Loss} &= \ddot{a}_{x+1} - p_x \ddot{a}_{x+1} \\
&= q_x \ddot{a}_{x+1}
\end{aligned}
$$

This is a liability loss

Analysis by source - Retired Employees

SURVIVAL CASE - MONTHLY ANNUITY

$$
\begin{aligned}
{}_eAL_1 &= (1+i)(\ \ddot{a}_x^{(12)}\ ) - (\text{actual BP}+i) \\
&= (1+i)(\ \ddot{a}_x^{(12)}\ ) - [1+\left(\frac{i}{12}\right)\left(\frac{12}{12}+\frac{11}{12}+\ldots+\frac{1}{12}\right)] \\
&= (1+i)(\ \ddot{a}_x^{(12)}\ ) - [1+\left(\frac{13}{24}\right)i] \\
&= (1+i)(\ a_x) - \frac{11}{24} \\
&= \quad p_x\,\ddot{a}_{x+1} - \frac{11}{24} \\
&= \quad p_x\,\ddot{a}_{x+1}^{(12)} - \frac{11}{24}q_x
\end{aligned}
$$

$$AL_1 = \ddot{a}_{x+1}^{(12)}$$

$$\text{Loss} = q_x\,\ddot{a}_{x+1}^{(12)} \quad + \quad \frac{11}{24}q_x$$

liability loss   ben. pmt. loss

GAINS AND LOSSES

Analysis by source - Retired Employees

DEATH CASE - MONTHLY ANNUITY

$_{e}AL_1 \quad = \quad (1+i)(\ \ddot{a}_x^{(12)}\ ) - (\text{actual BP}+i)$

$AL_1 \quad = \quad \text{zero} \quad (\text{straight life annuity})$

$\text{Gain} = \quad {}_{e}AL_1$

Depends on amount of benefits paid, which depends on actual date of death