ANCILLARY BENEFITS

Generally calculate PV as follows:

$$\sum_{t=0}^{NRA-X-1} v^{t+1}\ {}_tp_x^{(T)}\ q_{x+t}^{(i)}\ Ben_{x+t}\ PVfact_{x+t+1}$$

Based on these assumptions:

1. Exits occur at end of year
2. i is decrement type - could be mortality, termination, disability
3. Ben_{x+t} is benefit amount payable due to decrement type i
4. $PVfact_{x+t+1}$ is appropriate present value factor
   a) deferred versus immediate annuity
   b) may be mortality & interest only
   c) could use “completely different” post-exit assumptions

UNIT CREDIT - EFFECT OF WITHDRAWAL DECREMENTS

Employee 100% vested ➔ NC, AL both independent of withdrawal decrement

$$\begin{aligned} AL &= \text{PV of accrued benefit (unit credit)} \\ &= \text{PV-Retirement} + \text{PV-Withdrawal} \\ &= AB_X * \ N_{RA}^{(12)} \ / \ D_x^{(T)} \ + \\ &\quad AB_X * \sum_{t=0}^{RA-X-1} (D_{x+t}^{(T)} / D_x^{(T)}) v q_{x+t}^{(w)} (N_{RA}^{(12)} / D_{x+t+1}) \\ &= AB_X * \ v^{RA-X} \ {}_{RA-X}p_X^{(T)} \ \ddot{a}_{RA}^{(12)} \ + \\ &\quad AB_X * \ \sum_{t=0}^{RA-X-1} v^{t+1} \ {}_tp_X^{(T)} \ q_{x+t}^{(w)} \ [{}_{RA-x-t|}\ddot{a}_{x+t+1}^{(12)}] \end{aligned}$$

See next page for simplified example. Formula for AL simplifies to one with no withdrawal decrements at all.

UNIT CREDIT - EFFECT OF WITHDRAWAL DECREMENTS

$$AL = AB_X * \ v^{RA-X} \ {}_{RA-X}p_X^{(T)} \ \ddot{a}_{RA}^{(12)} \ +$$

$$AB_X * \sum_{t=0}^{RA-X-1} v^{t+1} \ {}_tp_X^{(T)} \ q_{x+t}^{(w)} \ [{}_{RA-x-t|}\ddot{a}_{x+t+1}^{(12)}]$$

Simplified example - assume single withdrawal decrement at age Z

$$AB_X \, [\, v^{RA-X} \ {}_{Z-X}p_X^{(T)} \ [1 - q_z^{(w)} - q_z^{(d)}]_{RA-1-z}p_{z+1}^{(d)} \ddot{a}_{RA}^{(12)}$$

$$+ \quad v^{Z-X} \ {}_{Z-X}p_X^{(T)} \ [v \, q_z^{(w)}] \ {}_{RA-1-z}p_{z+1}^{(d)} v^{RA-Z-1} \ \ddot{a}_{RA}^{(12)}]$$

$$= AB_X \, [v^{RA-X} \ {}_{Z-X}p_X^{(T)} \ [1 - q_z^{(d)}]_{RA-1-z}p_{z+1}^{(d)} \ddot{a}_{RA}^{(12)}$$

$$- \quad v^{RA-X} \ {}_{Z-X}p_X^{(T)} \ [\, q_z^{(w)}] \ {}_{RA-1-z}p_{z+1}^{(d)} \ \ddot{a}_{RA}^{(12)}$$

$$+ \ v^{RA-X} \ {}_{Z-X}p_X^{(T)} \ [\, q_z^{(w)}] \ {}_{RA-1-z}p_{z+1}^{(d)} \ \ddot{a}_{RA}^{(12)}]$$

$$= AB_X * v^{RA-X} \ {}_{Z-X}p_X^{(T)} \ [\, p_z^{(d)}] \ {}_{RA-1-z}p_{z+1}^{(d)} \ \ddot{a}_{RA}^{(12)}$$

$$= AB_X * v^{RA-X} \ {}_{RA-X}p_X^{(d)} \ \ddot{a}_{RA}^{(12)}$$

ANCILLARY BENEFITS

Complex example - ERISA pre-retirement "Joint and Survivor" death benefit

BENEFIT

Upon employee death, after attaining early retirement age, plan must provide death benefit. Plan can charge employee for cost of coverage.

TASK

Create a set of true actuarial equivalent factors that vary based on age at election of coverage and age of retirement. Assume reduction factor is applied to benefit for employees who survive to retirement age.

ANCILLARY BENEFITS

Assume annual life annuity
RA = age of retirement
OA = age option is elected, but not less than earliest retirement age
P = percent per year reduction factor

PV of benefits - no death benefit election:

$$ERB_{RA}(\ddot{a}_{RA} v^{RA-OA} {}_{RA-OA}p_{OA}^{(T)})$$

PV of benefits with election:

$$ERB_{RA}(\ddot{a}_{RA} v^{RA-OA} {}_{RA-OA}p_{OA}^{(T)})\ [1 - P(RA-OA)]$$
$$+ \sum_{t=0}^{RA-OA} v^{t+1}\ {}_tp_{OA}^{(T)}\ q_{OA+t}^{(d)} DB_{OA+t}\ \ddot{a}_{BEN+t+1}$$

Solve for P by equating the present values. P will vary based on each combination of OA and RA, and definition of death benefit (may be unreduced accrued, early retirement benefit, reduced for J&S, etc.)

ANCILLARY BENEFITS

Some inspection problems show answers where D_x and N_x are based on interest and mortality only. This requires modifying the standard formula:

$$\sum_{t=0}^{NRA-X-1} v^{t+1} \, {}_tp_x^{(T)} \, q_{x+t}^{(i)} \, Ben_{x+t} \, PVfact_{x+t+1}$$

(1) Change limits of summation

$$\sum_{t=x}^{NRA-1} v^{t+1-x} (l_t/l_x) \, q_t^{(i)} \, Ben_t \, PVfact_{t+1}$$

(2) PV factor is life annuity at NRA

$$\sum_{t=x}^{NRA-1} v^{t+1-x} (l_t/l_x) \, q_t^{(i)} \, Ben_t \, (N_{NRA}^{(12)} / D_{t+1})$$

(3) Rearrange terms of summation

$$(N_{NRA}^{(12)} / l_x) \sum_{t=x}^{NRA-1} q_t^{(i)} \, Ben_t \, v^{t+1-x} (l_t / D_{t+1})$$