SECTION 430(h)

Commutation Functions

Present Values

§430(h)(3)
MORTALITY TABLES

A. Secretary will prescribe tables
B. Secretary will update tables every 10 years
C. Substitute mortality table based on actual plan experience
D. Separate mortality tables for disabled (both pre-1995 and post-1994)

1.430(h)(3)-1
MORTALITY TABLES

<u>Proposed regulation April 2007</u>
Defines mortality basis for PPA valuations

Two available options are given - generational mortality tables or static mortality tables.

<u>IRS Notice 2013-49</u>
Static mortality tables for 2014 and 2015

<u>Final regulation 1.430(d)-1(f)(2)</u>
Plan with fewer than 100 participants not in pay status may assume no pre-retirement mortality - only if that assumption would be a reasonable assumption

1.430(h)(3)-1
MORTALITY TABLES

Generational mortality tables

- **Projection scale AA**
- **Base annuitant mortality (2000)**
- **Base non-annuitant mortality (2000)**

<u>Small plans - less than 500 participants</u>
If using static mortality tables, have option to use "combined" table (no separate mortality for annuitants and non-annuitants)

1.430(h)(3)-2
MORTALITY TABLES

Plan-specific mortality tables

- **Must obtain approval in writing**
- **Separate mortality tables for each gender**
- **Need "credible mortality experience" for each gender – at least 1,000 deaths within a 4 year period**
- **If insufficient deaths, can use standard table for one gender and plan-specific table for other gender**
- **May have separate tables for annuitants versus non-annuitants**
- **Only need "credible mortality experience" for either annuitants or non-annuitants**

1.430(h)(3)-2
MORTALITY TABLES

Plan-specific mortality tables

- All plans within controlled group must use plan-specific mortality tables

- If insufficient deaths, can use standard table for one plan and plan-specific tables for other plans

- Must demonstrate lack of "credible mortality experience" for plan to continue use of standard table

§430(h)(4)
OPTIONAL FORMS

Present value calculations must reflect

- **Probability that benefits will be paid under optional forms of benefit**
- **Any difference in present value due to use of different actuarial assumptions**

EXAMPLES:

- **Availability of early retirement benefits**
- **PV of early retirement benefit based on reduction factors > actuarial equivalent**

Not really new concept – covered by ASOPs

§430(h)(5)
CHANGES IN ACTUARIAL ASSUMPTIONS

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a) Approval required for any change in assumptions used for Funding Target

b) Applies if single employer plan, and
   i) Aggregate UVB in controlled group plans ñ $50 million
      a) Determined at end of prior year
      b) PBGC basis §4006
      c) Must ignore plans with no UVB
   ii) AND assumption change reduces *Funding Shortfall* by
      a) More than $50 million, <u>or</u>
      b) More than both $5 million and 5% of Funding Target before the change

NOTE:
Funding shortfall is defined as
Funding target – (AAV–CB–PB)

§430(h)
PRESENT VALUE CALCULATION EXAMPLES

Segment interest rates are <u>not</u> the same as Select and ultimate rates

First example shows calculations using select and ultimate interest rates. Next example uses three segment rates.

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SELECT AND ULTIMATE INTEREST RATES

4.0% for 5 years, then 4.5% for 15 years, and 5.0% thereafter. Multiple rates are used to value benefit payments occurring more than 5 years after valuation date.

Participant age x, receives <u>annual</u> benefits

In general, $PV = \sum_{t=0}^{\omega} v^t ({}_tp_x)(\text{Benefit Payment}_{x+t})$

$$\sum_{t=0}^{4} (1.04)^{-t}({}_tp_x)(\text{Benefit Payment}_{x+t}) +$$

$$\sum_{t=5}^{19} (1.04)^{-5}(1.045)^{-(t-5)}({}_tp_x)(\text{Benefit Payment}_{x+t}) +$$

$$\sum_{t=20}^{\omega} (1.04)^{-5}(1.045)^{-15}(1.05)^{-(t-20)}({}_tp_x)(\text{Benefit Payment}_{x+t})$$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE SEGMENT INTEREST RATES

4.0% for 5 years, then 4.5% for 15 years, and 5.0% thereafter.

Rates for any segment are ONLY used to value benefit payments occurring during that segment.

Participant age x, receives <u>annual</u> benefits

$$\sum_{t=0}^{4} (1.040)^{-t}({}_tp_x)(\text{Benefit Payment}_{x+t}) +$$

$$\sum_{t=5}^{19} (1.045)^{-t}({}_tp_x)(\text{Benefit Payment}_{x+t}) +$$

$$\sum_{t=20}^{\omega-x} (1.050)^{-t}({}_tp_x)(\text{Benefit Payment}_{x+t})$$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

4.0% for 5 years, then 4.5% for 15 years, and 5.0% thereafter.

Rates for any segment are ONLY used to value benefit payments occurring during that segment.

Participant age x, receives annual benefits

Rewrite prior results using annual annuities instead of summations

$\ddot{a}_{x:\overline{5|}}$ at 4.0%

$+ (1.045)^{-5}({}_5p_x)\ \ddot{a}_{x+5:\overline{15|}}$ at 4.5%

$+ (1.050)^{-20}({}_{20}p_x)\ \ddot{a}_{x+20}$ at 5.0%

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

4.0% for 5 years, then 4.5% for 15 years, and 5.0% thereafter.

Rates for any segment are ONLY used to value benefit payments occurring during that segment.

Participant age x, receives annual benefits

Alternate expression

$$\ddot{a}_{x:\overline{5}|} \text{ at } 4.0\%$$
$$+\ \ddot{a}_{x:\overline{20}|} \text{ at } 4.5\% - \ddot{a}_{x:\overline{5}|} \text{ at } 4.5\%$$
$$+\ \ddot{a}_{x} \text{ at } 5.0\% - \ddot{a}_{x:\overline{20}|} \text{ at } 5.0\%$$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE

Participant age x, receives <u>monthly</u> benefits

$\ddot{a}^{(12)}_{x:\overline{5}|}$ at 4.0%

$+\ \ddot{a}^{(12)}_{x:\overline{20}|}$ at 4.5% $-\ \ddot{a}^{(12)}_{x:\overline{5}|}$ at 4.5%

$+\ \ddot{a}^{(12)}_{x}$ at 5.0% $-\ \ddot{a}^{(12)}_{x:\overline{20}|}$ at 5.0%

§430(h)
PRESENT VALUE CALCULATION EXAMPLE

Life contingencies review

$$\ddot{a}_x^{(12)} = \ddot{a}_x - \frac{11}{24}$$

$$\begin{aligned}
\ddot{a}_{x:\overline{n|}}^{(12)} &= \ddot{a}_x^{(12)} - v^n {}_np_x \ddot{a}_{x+n}^{(12)} \\
&= \{\ddot{a}_x - \frac{11}{24}\} - v^n {}_np_x\{\ddot{a}_{x+n} - \frac{11}{24}\} \\
&= \{\ddot{a}_x - v^n {}_np_x \ddot{a}_{x+n}\} - \frac{11}{24}\{1 - v^n {}_np_x\} \\
&= \ddot{a}_{x:\overline{n|}} - \frac{11}{24}\{1 - v^n {}_np_x\}
\end{aligned}$$

PRESENT VALUE CALCULATION EXAMPLE

Equivalent expression using annual annuities with monthly adjustment

$$\ddot{a}_{x:\overline{5}|} \text{ at } 4.0\% + \ddot{a}_{x:\overline{20}|} \text{ at } 4.5\% - \ddot{a}_{x:\overline{5}|} \text{ at } 4.5\%$$

$$+ \ddot{a}_{x} \text{ at } 5.0\% - \ddot{a}_{x:\overline{20}|} \text{ at } 5.0\%$$

$$-\tfrac{11}{24}[1 - (1.040)^{-5}{}_{5}p_x + (1.045)^{-5}{}_{5}p_x]$$

$$-\tfrac{11}{24}[\quad - (1.045)^{-20}{}_{20}p_x + (1.050)^{-20}{}_{20}p_x]$$

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

With three segment interest rates, problems may give you commutation functions

Easy to value streams of payments:

$D_x = v^x(l_x)$

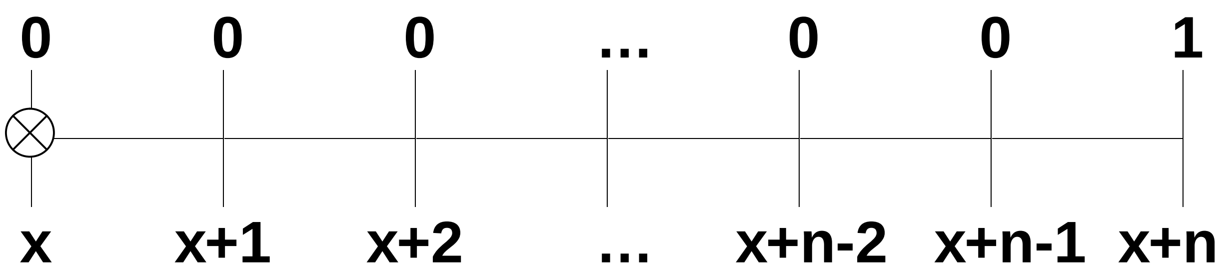
Interpret D_x as single payment upon survival to age x

$N_x = \sum_{t=0}^{\mu - x} D_{x+t}$

Interpret N_x as stream of annual payments for life, starting at age x

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

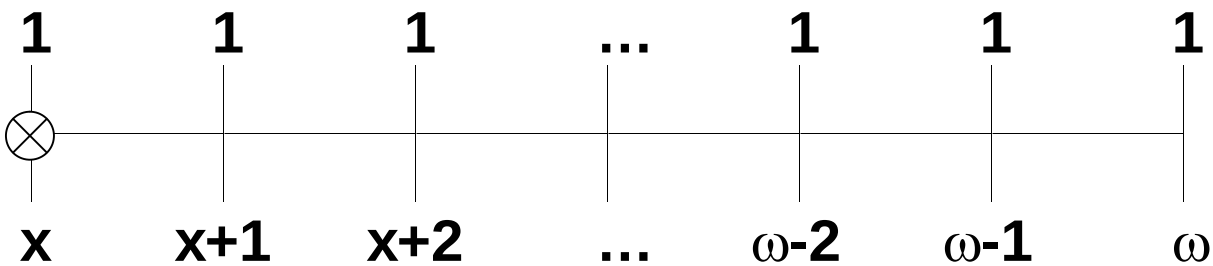
Pure Endowment:
Value of \$1 payable n years from now



$$
\begin{aligned}
PV &= {}_nE_x \\
&= \frac{D_{x+n}}{D_x} \\
&= \frac{v^{x+n}(l_{x+n})}{v^x(l_x)} \\
&= \frac{v^n(l_{x+n})}{l_x} \\
&= v^n({}_np_x)
\end{aligned}
$$

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

Life annuity due:
Value of $1 per annum until death



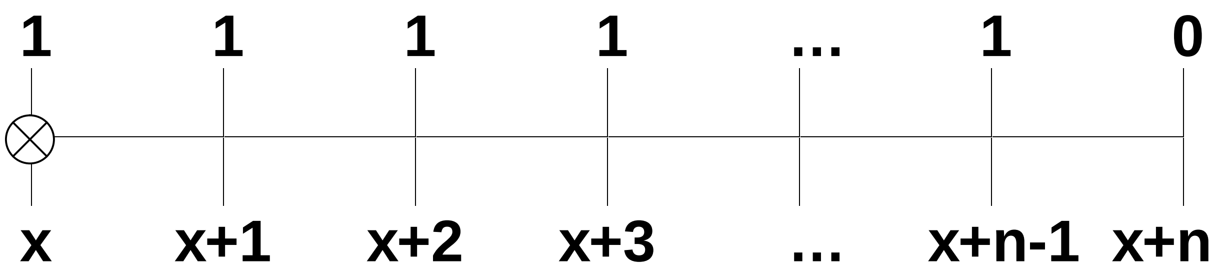
$$PV = \ddot{a}_x$$

$$= \sum_{t=0}^{\mu - x} v^t \, {}_t p_x$$

$$= \frac{N_x}{D_x}$$

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

Temporary life annuity due:
Value of \$1 for next n years



$$PV = \ddot{a}_{x:\overline{n}|}$$

$$= \sum_{t=0}^{n-1} v^t \, {}_tp_x$$

$$= \frac{N_x - N_{x+n}}{D_x}$$

§430(h) PRESENT VALUE CALCULATION EXAMPLE SEGMENT INTEREST RATES

Participant age x, receives annual benefits

$\ddot{a}_{x:\overline{5}|}$ at 4.0%
$+ (1.045)^{-5}({}_{5}p_{x})\ \ddot{a}_{x+5:\overline{15}|}$ at 4.5%
$+ (1.050)^{-20}({}_{20}p_{x})\ \ddot{a}_{x+20}$ at 5.0%

Using commutation functions

$\ddot{a}_{x:\overline{5}|}$ at 4.0%
$+(D_{x+5}/D_{x \text{ at } 4.5\%})(\ddot{a}_{x+5:\overline{15}|\text{ at } 4.5\%})$
$+(D_{x+20}/D_{x \text{ at } 5.0\%})(\ddot{a}_{x+20 \text{ at } 5.0\%})$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

Participant age x, receives annual benefits

$\ddot{a}_{x:\overline{5|}}$ at 4.0%
$+ (1.045)^{-5}({}_5p_x)\ \ddot{a}_{x+5:\overline{15|}}$ at 4.5%
$+ (1.050)^{-20}({}_{20}p_x)\ \ddot{a}_{x+20}$ at 5.0%

Expression with no annuity symbols:

$(N_x - N_{x+5})/D_x$ at 4.0%
$+ (D_{x+5}/D_x$ at 4.5%$)\{(N_{x+5} - N_{x+20})/D_{x+5}$ at 4.5%$\}$
$+(D_{x+20}/D_x$ at 5.0%$)(N_{x+20}/D_{x+20}$ at 5.0%$)$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

Participant age x, receives annual benefits

$\ddot{a}_{x:\overline{5|}}$ at 4.0%
$+ (1.045)^{-5}({}_5p_x)\ \ddot{a}_{x+5:\overline{15|}}$ at 4.5%
$+ (1.050)^{-20}({}_{20}p_x)\ \ddot{a}_{x+20}$ at 5.0%

Further simplified:

$(N_x - N_{x+5})/D_x$ at 4.0%
$+ (N_{x+5} - N_{x+20})/D_x$ at 4.5%
$+ N_{x+20}/D_x$ at 5.0%

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

Standard approximation for annuities payable more frequently than annually:

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{(m-1)}{2m}$$

Formula for monthly annuity due:

$$\ddot{a}_x^{(12)} \approx \ddot{a}_x - \frac{11}{24}$$

Monthly immediate annuity formulas

$$\begin{aligned} a_x^{(12)} &= \ddot{a}_x^{(12)} - \frac{1}{12} \\ &\approx \ddot{a}_x - \frac{11}{24} - \frac{1}{12} \\ &\approx a_x + 1 - \frac{13}{24} \\ &\approx a_x + \frac{11}{24} \end{aligned}$$

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

Life annuity due of \$1 per year, payable 1/12 at the start of each month

PV = $\ddot{a}_x^{(12)}$

Interpret $N_x^{(12)}$ **as stream of monthly payments for life, starting at age x**

$$N_x^{(12)} = N_x - \frac{11}{24} D_x$$

$$\ddot{a}_x^{(12)} = N_x^{(12)} / D_x$$

PRESENT VALUE CALCULATIONS
COMMUTATION FUNCTIONS

Temporary life annuity due of $1 per year for n years, payable 1/12 at the start of each month

$$PV = \ddot{a}^{(12)}_{x:\overline{n}|}$$

$$= (N^{(12)}_{x} - N^{(12)}_{x+n})/D_{x}$$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

Participant age x, receives monthly benefits

$$\ddot{a}^{(12)}_{x:\overline{5|}} \text{ at } 4.0\%$$
$$+ (1.045)^{-5}({}_{5}p_{x})\ \ddot{a}^{(12)}_{x+5:\overline{15|}} \text{ at } 4.5\%$$
$$+ (1.050)^{-20}({}_{20}p_{x})\ \ddot{a}^{(12)}_{x+20} \text{ at } 5.0\%$$

Using commutation functions

$$\ddot{a}^{(12)}_{x:\overline{5|}} \text{ at } 4.0\%$$
$$+(D_{x+5}/D_{x \text{ at } 4.5\%})(\ddot{a}^{(12)}_{x+5:\overline{15|} \text{ at } 4.5\%})$$
$$+(D_{x+20}/D_{x \text{ at } 5.0\%})(\ddot{a}^{(12)}_{x+20 \text{ at } 5.0\%})$$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

Participant age x, receives monthly benefits

$\ddot{a}^{(12)}_{x:\overline{5|}}$ at 4.0%

$+ (1.045)^{-5}({}_{5}p_{x})\ \ddot{a}^{(12)}_{x+5:\overline{15|}}$ at 4.5%

$+ (1.050)^{-20}({}_{20}p_{x})\ \ddot{a}^{(12)}_{x+20}$ at 5.0%

Expression with no annuity symbols:

$(N^{(12)}_{x} - N^{(12)}_{x+5})/D_{x \text{ at } 4.0\%}$

$+ (D_{x+5}/D_{x \text{ at } 4.5\%})\{(N^{(12)}_{x+5} - N^{(12)}_{x+20})/D_{x+5 \text{ at } 4.5\%}\}$

$+(D_{x+20}/D_{x \text{ at } 5.0\%})(N^{(12)}_{x+20}/D_{x+20 \text{ at } 5.0\%})$

§430(h)
PRESENT VALUE CALCULATION EXAMPLE
SEGMENT INTEREST RATES

Participant age x, receives monthly benefits

$\ddot{a}^{(12)}_{x:\overline{5|}}$ at 4.0%

$+ (1.045)^{-5}({}_5p_x)\ \ddot{a}^{(12)}_{x+5:\overline{15|}}$ at 4.5%

$+ (1.050)^{-20}({}_{20}p_x)\ \ddot{a}^{(12)}_{x+20}$ at 5.0%

Further simplified:

$(N^{(12)}_{x} - N^{(12)}_{x+5})/D_x$ at 4.0%

$+ (N^{(12)}_{x+5} - N^{(12)}_{x+20})/D_x$ at 4.5%

$+ N^{(12)}_{x+20}/D_x$ at 5.0%

§430(h)(2)(D)(ii)
ELECTION TO USE FULL YIELD CURVE

- **Rates vary yearly - no segment rates**
- **No averaging for prior 24 months**
- **Revoke election - need Secretary consent**

§430(h)(2)(D)(ii)
ELECTION TO USE FULL YIELD CURVE

In general,

$$PV = \sum_{t=0}^{\omega} (1+i)^{-t} \, {}_tp_x^{(T)} (\text{Benefit Payment}_{x+t})$$

Yield curve – interest rates vary each year:

$$PV = \sum_{t=0}^{\mu} (1+i_t)^{-t} \, {}_tp_x^{(T)} (\text{Benefit Payment}_{x+t})$$

Note subscript on i in second summation

§430(h)(2)(D)(ii)
ELECTION TO USE FULL YIELD CURVE

Derive forward rates k_t equivalent to the yield curve rates i_t

$(1+i_t)^{-t} = [(1+k_1)(1+k_2)(1+k_3) \ldots (1+k_t)]^{-1}$

$(1+i_1)^{-1} = [(1+k_1)]^{-1}$
$(1+i_2)^{-2} = [(1+k_1)(1+k_2)]^{-1}$
$(1+i_3)^{-3} = [(1+k_1)(1+k_2)(1+k_3)]^{-1}$

Forward rates:

$$\sum_{t=0}^{\mu} [(1+k_1)(1+k_2)\ldots(1+k_t)]^{-1} \, {}_{t}p_{x}^{(T)} (\text{Benefit Payment}_{x+t})$$

Use forward rates in identical manner as select and ultimate rates